Extensions and Applications of Land Use Transport Interaction (LUTI) Models

Topic 1: Density, Accessibility, Retail Models

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Outline

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• Singly Constrained Models
• Modular Modelling: Coupled Spatial Interaction
• A Simple Example of Modularity: Lowry’s Model
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• Demand and Supply: Market Clearing
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• Sketch for an Integrated Model
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Spatial Interaction Ideas Again: Unconstrained

Let me begin with spatial interaction models once again and first define key terms. We are going to divide our spatial systems into small zones like Census Tracts which can either be called origins or destinations. Origins are notated using the subscript $i$ and destinations the subscript $j$. Now the original gravity model can be stated as

$$T_{ij} \propto \frac{P_i P_j}{d_{ij}^2} = K \frac{P_i P_j}{d_{ij}^2} = KP_i P_j$$

where we define $T_{ij}$, $P_i$, $P_j$, $d_{ij}^2$, and $K$ as trips, populations, distance squared and a scaling constant.
In fact we can generalise the model first by noting that distance is like in the von Thunen model a measure of generalised travel cost \( c_{ij} \) and the populations are defined as measures of mass or activity as origin and destination activities \( O_i, D_j \). Then

\[
T_{ij} \propto \frac{O_i D_j}{c_{ij}^\beta} = K \frac{O_i D_j}{c_{ij}^\beta} = K O_i D_j c_{ij}^{-\beta}
\]

Where \( \beta \) is the so-called friction of distance parameter controlling the effect of generalised travel cost. When \( \beta \) is large, the effect of distance is great and when it is small it is much less. This gives more trips when it is small than when it is big.
In all our models, we need to estimate these parameters and this is the process of calibration. We need to choose $K$ and $\beta$ so that the predicted trips $T_{ij}$ are as close as possible to the observed trips $T_{ij}^{obs}$.

We can do this in this simplest of models by fitting a linear regression to the logarithmic version of the model and when we take logs we get

$$\log \frac{T_{ij}}{O_iD_j} = \log K - \beta \log c_{ij}$$

We find the parameters by minimising the sum of the squares (squared deviations) between the predicted and observed trips, that is

$$\min \Phi = \min \sum_{ij} (T_{ij} - T_{ij}^{obs})^2$$
The original gravity model has been used for years but in the 1960s and 1970s various researchers cast it in a wider framework – deriving the model by setting up a series of constraints on its form which showed how it might be solved generating consistent models.

The constraints logic led to consistent accounting. The generative logic lead to analogies between utility and entropy maximising and opened a door that has not been much exploited to date between entropy, energy, urban form, physical morphology and economic structure. In particular the economic logic is called choice theory, specifically discrete choice theory.
The key idea is to introduce constraints on the form that the model can take, and these relate to specifying what the model is able to predict. The more constraints we introduce on the model, the more we reduce the model’s predictive power, but the idea of constraints also relates to what we know about the system in comparison with what we want to predict.

The idea of a framework for consistent generation of a model is that we can then handle the constraints systematically as we will now show.
Singly Constrained Models

We must move quite quickly now so let me introduce the basic constraints on spatial interaction and then state various models. The constraints are usually specified as origin constraints and destination constraints as

\[ O_i = \sum_j T_{ij} \]
\[ D_j = \sum_i T_{ij} \]

And we can take our basic gravity model and make it subject to either or both of these constraints or not at all.
So what we get are four possible models

**Unconstrained**  \( T_{ij} = KO_iD_j c_{ij}^{-\beta} \)

**Singly (Origin) Constrained**  \( T_{ij} = AO_iD_j c_{ij}^{-\beta} \)
so that the volume of trips at the origins is conserved

**Singly (Destination) Constrained**  \( T_{ij} = B_j O_i D_j c_{ij}^{-\beta} \)
so that the volume of trips at the destinations is conserved

**Doubly Constrained**  \( T_{ij} = A_i B_j O_i D_j c_{ij}^{-\beta} \)
trip volumes at origins + destinations are conserved

The first three are location models, the last is the transportation model
Now the simplest way to work out what the constants mean is to note the constraint equations and then add and factor the model subject to the constraints. Let us show now the singly constrained gravity model which is

$$T_{ij} = A_i O_i D_j c_{ij}^{-\beta} = O_i \frac{D_j c_{ij}^{-\beta}}{\sum_j D_j c_{ij}^{-\beta}} \text{ origin-constrained}$$

You can think of \( D_j c_{ij}^{-\beta} / \sum_j D_j c_{ij}^{-\beta} \) as a probability of working in the origin and going to the destination. If we add up the trips in this equation over \( j \) then we get \( O_i \) --- this of course is the origin constraint.
Now the singly constrained – origin and then destination and the doubly constrained models follow directly and we will simply state their full forms noting that we need to find

\[ T_{ij} = A_i O_i D_j c_{ij}^{-\beta} = O_i \frac{D_j c_{ij}^{-\beta}}{\sum_j D_j c_{ij}^{-\beta}} \]  
origin - constrained

\[ T_{ij} = B_j O_i D_j c_{ij}^{-\beta} = D_j \frac{O_i c_{ij}^{-\beta}}{\sum_j O_i c_{ij}^{-\beta}} \]  
destination constrained

\[ T_{ij} = A_i B_j O_i D_j c_{ij}^{-\beta} \]

\[ A_i = 1/\sum_j B_j D_j c_{ij}^{-\beta} \]  
origin - destination constrained

\[ B_j = 1/\sum_i A_i O_j c_{ij}^{-\beta} \]
Modular Modelling: Coupled Spatial Interaction

So far we have just singled out a module for one kind of interaction - based on a variant of the gravity model - consider stringing these together as more than one kind of spatial interaction: Model 1 ⇄ Model 2 ⇄ Model 3 ⇄ ..... Classically we might model flows from home to work and home to shop but there are many more and in this sense, we can use these as building blocks for wider models. This is for next time too.

What we will now do is illustrate how we might build such a structure taking a journey to work model from Employment to Population and then to Shopping which we structure as --
First we have the journey from work to home model as

\[ T_{ij} = E_i \frac{F_j \exp(-\lambda c_{ij})}{\sum_j F_j \exp(-\lambda c_{ij})} \], \quad \sum_j T_{ij} = E_i \]

\[ P_j = \alpha \sum_i T_{ij} \]

And then the demand from home to shop

\[ S_{jm} = P_j \frac{W_m \exp(-\beta c_{jm})}{\sum_m W_m \exp(-\beta c_{jm})} \], \quad \sum_m S_{jm} = P_j \]

\[ S_m = \sum_j S_{jm} \]

And there is a potential link back to employment from the retail sector

\[ E_m = f(S_m) \]
A Simple Example of Modularity: Lowry’s Model

Lowry’s (1964) model of Pittsburgh was a model of this nature but it also incorporated in it – or rather its derivatives did more formally – a generative sequence of starting with only a portion of employment – basic – and then generating the non-basic that came from this. This non-basic set up demand for more non-basic and so on until all the non-basic employment was generated, and this sequence followed the classic multiplier effect that is central to input-output models.

A block diagram of the model follows
Input data: basic employment, travel time matrix, population-serving and inverse activity rates, parameter values, service and residential attractors, constraints

Allocate increment of employment to residential zones

Calculate increment of residential population

Calculate increment of service employment demanded by residential population

Allocate increment of service employment

Calculate service employment located in service centres

Is increment of service employment or population below acceptable limit?

Alter weight on residential attractor

Is maximum density constraint on population satisfied?

Output data: total population, total employment, work trips, service demand distribution, mean trip lengths and distribution curves

Stop

Fig. 3.3. Generalised flow chart of the Activity Allocation model.
DRAM-EMPAL Style Models

Essentially what we have here is the notion of simultaneous dependence – i.e. one activity generates another but that other activity generates the first one – what came first – the chicken or the egg?

Stephen Putman developed an integrated model to predict residential location DRAM and another to predict employment location EMPAL. In essence different models are used to do each – the employment model tends to be based on very different factors – it is a regression like model of key location factors not a flow model.

Now some models take the transport component out and use accessibility, then interfacing with a transport model that is built externally.
Demand and Supply: Market Clearing

So far most of these models have been articulated from the demand side – they are models of travel demand and locational demand – they say nothing about supply although we did introduce the notion that in simulating trips and assigning these to the network, we need to invoke supply.

When demand and supply are in balance, then the usual signal of this is the price that is charged. In one sense the DRAM EMPAL model configures residential location as demand and employment location as supply but most models tend to treat supply as being relatively fixed, given, non-modellable.
However several models that couple more than one activity together treat supply as being balanced with demand, often starting with demand, seeing if demand is met, if not changing the basis of demand and so on until equilibrium is ascertained. Sometimes prices determines the signal of this balance. If demand is too high, price rises and demand falls until supply is met and vice versa. Often this is done simply to ensure demand is not greater than supply.

Most urban models do not attempt to model supply for supply side modelling is much harder and less subject to generalisable behaviour.

A strategy for ensuring balance is as follows for a model with two sectors – like the one we illustrated earlier.
In the following slide, we have two submodels—first residential location and second retail location.

In each submodel, we first have interaction (trip distribution) and then location.

The first loops in terms of interaction are for capacity constraints on supply, the second are for capacity constraints on location.

The second set of red loops involve reiterating the interaction and location so that we can get balance within the entire submodel.

The thick black loop in the middle couples the residential to the retail model, the thick black loop around the two models is used if retail predictions are to influence employment.
The decision to nest what loop inside what other loop is a big issue that makes these models non-unique.

If the supply side is modelled separately then the way this is incorporated further complicates the sequence of model operations.

In the large scale integrated models that we will deal with next, these are crucial issues.

There is one further structural issue we will deal and this involves extending the models sectorally and the Echenique input-output formulation is a good example of this extension.
Input-Output: The Echenique Models

So far we have only developed couplings between models that are added together in ordered sequences that string sectors together apart from reference last time to the Lowry model which organised this sequence around the basic-non-basic employment multiplier.

We can extend this to a series of linked causal multipliers between different sectors by extending this chain to an input-output model framework. In essence we define many different sectors involving households, labour, industries, services and so on and build the model so that there are consistent economic relations between each
Echenique’s MEPLAN models are structured in this fashion. So too is the TRANUS model. We can introduce these as follows.

Essentially the system is divided into production and consumption based on activities $m$ that are produced in zone $I$, $X_i^m$, and consumed as activities $n$ in zone $j$, $Y_j^n$. These are organised as in an input output table but noting that they are spatially specific.

\[
X_i^m = \sum_j \sum_n T_{ij}^{mn}
\]
\[
Y_j^n = \sum_i \sum_m T_{ij}^{mn}
\]

Here is the typical I-O table
- Section A of the matrix $T^{mn}$ represents the transactions between factors. This area is normally included in standard input–output models (Leontief, 1951). It represents the sales from sector $m$ to sector $n$.
- Section B of the matrix $T^{mn}$ represents the transactions between factor $m$ and the household group $n$, in other words the consumption by the households of products or services $m$.
- Section C of the matrix $T^{mn}$ represents the transactions between factors $m$ to be exported to outside the area in consideration. Normally, both sections B and C are considered the final demand in standard input–output models that also includes investments and government consumption. It is described as the exogenous sector, that is to say, it is determined outside the model.
- Section D represents the sale of labor or other income received by socio-economic groups $m$ from the factor $n$ (e.g. dividends).
- Section E represents the sale of labor from socio-economic groups $m$ to households in socio-economic groups $n$ (e.g. domestic labor).
- Section F represents the sale of labor or other income received from the exogenous factor, such as pensions and other payments from government, etc.
- Section G represents the imports from outside the area and payments to the exogenous factor such as taxes to the government. In this sector, rental of property or land is sometimes included.
- Section H represents the payments by the households factor such as taxes, rental, etc.
- Section I represents payments by the exogenous factor to itself, such as imports for the government or for investments.
The flows are based on spatial interaction models of the form

\[ T_{ij}^{mn} = Y_j^n \frac{\exp(-\beta^m c_{ij}^m)}{\sum_i \exp(-\beta^m c_{ij}^m)} \]

Where the generalised interaction costs also include other costs such as prices of good m at l

\[ c_{ij}^m = p_i^m + t_{ij}^m + w_{ij}^m \]

The order in which these equations are solved and linked together is given in the following flow chart. Note that prices are determined from spatial interactions as

\[ p_i^m = \frac{1}{\beta} \log \sum_i \exp(-\beta^m c_{ij}^m) \]
And then linked back to the prices of goods produced as

\[ p_j^n = \sum_m a^{mn} p_j^m \]

\[ a^{mn} = \frac{\sum_i \sum_j T_{ij}^{mn}}{\sum_j Y_j^n} \]

The precise details of how the model works are extremely hard to figure out from the papers but the following flow chart goes some way to showing how the various elements are configured.

This is a general point. In models that are coupled in this fashion – integrated, then it is often hard to figure out the precise ordering or the structure. I am just reading a PhD on the TRANUS model and this is a very complicated feature – what is solved first – the order.
Figure 3. An integrated spatial system.
I will simply point you in the right directions here – the Handbook I referred you to in the last lecture contains several very good papers on these issues and I will briefly present some notes from Miller’s article.
Here is a summary from his article of the key structure of such models and also their requirements.

### Box 1
**Integrated urban model design issues**

- **Physical system representation**
  - Time
  - Space (land)
  - Building stock
  - Transportation networks
  - Services

- **Representation of processes**
  - Land development
  - Location choices
  - Job market
  - Demographics
  - Regional economics
  - Automobile holdings
  - Activity/travel demand
  - Network performance

- **Representation of decision-makers**
  - Persons
  - Households
  - Private firms
  - Public authorities

- “Generic issues”
  - Level of aggregation/disaggregation
  - Endogenous versus exogenous treatment
  - Level of “process type”
  - Model specification

- **Implementation issues**
  - Data requirements
  - Computational requirements
  - Technical support requirements

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**Figure 1.** Integrated urban modeling system framework.
<table>
<thead>
<tr>
<th>Software</th>
<th>Developer</th>
<th>Operational history</th>
<th>Platform</th>
<th>Commercial availability</th>
<th>Support</th>
</tr>
</thead>
<tbody>
<tr>
<td>ITLUP</td>
<td>S. H. Putman</td>
<td>Developed over the last 25 years; operationally applied in many US cities plus selected overseas (40 plus calibrations)</td>
<td>Originated in FORTRAN for mainframe/workstation. PC version (METROPILUS) in ArcView shell, which provides linkage to ArcView GIS (Windows compatible)</td>
<td>Yes</td>
<td>Consulting firm, with commercial documentation and technical support (user's manual, newsletter, user group)</td>
</tr>
<tr>
<td>MEPLAN</td>
<td>M. Echenique</td>
<td>Much shared history over 25-year development. Operational applications throughout the world, including the USA (Sacramento for both; Washington State for MEPLAN; Oregon State and Baltimore for TRANUS)</td>
<td>MEPLAN originated in FORTRAN for mainframe; now PC based</td>
<td>Yes</td>
<td>Consulting firm, with commercial documentation and technical support (user's manual, newsletter)</td>
</tr>
<tr>
<td>TRANUS</td>
<td>T. de la Barra</td>
<td>TRANUS developed directly for PC (Windows orientation)</td>
<td>Yes</td>
<td>Consulting firm, with commercial documentation and technical support (user's manual)</td>
<td></td>
</tr>
<tr>
<td>MUSSA</td>
<td>F. Martinez</td>
<td>Operational in Santiago, Chile. Developed over last 8-10 years</td>
<td>PC based; runs under Windows. Interfaces with a relational database management system (Access). GUI and GIS</td>
<td>Yes</td>
<td>University-based research team in collaboration with the Government of Chile</td>
</tr>
<tr>
<td>NYMTC-LUM</td>
<td>A. Anas</td>
<td>Currently being implemented in New York City. Based upon previous models (CATLASS, CPHMM, NYSIM) developed in Chicago and New York over the last 20 years</td>
<td>PC or workstation. FORTRAN program</td>
<td>Yes</td>
<td>Alex Anas &amp; Associates (a small firm). Limited documentation</td>
</tr>
<tr>
<td>UrbanSim</td>
<td>P. Waddell</td>
<td>Currently being implemented in Honolulu, Eugene/Springfield and Salt Lake City. Historical validation performed in Oregon</td>
<td>Platform independent, written in Java. Viewer currently implemented in MapObjects GIS on Windows 95/NT</td>
<td>Yes; public domain via website (<a href="http://www.urbanism.org">www.urbanism.org</a>)</td>
<td>University of Washington. Limited documentation currently. Reference manual, user guide, software available at website (<a href="http://www.urbanism.org">www.urbanism.org</a>)</td>
</tr>
</tbody>
</table>

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URL: http://dx.doi.org/10.1080/0144164052000336470
Sketch for an Integrated Model

I am very quickly going to sketch an integrated model which builds on the ideas so far - I will not disaggregate the model into m employment types and n housing types but we can assume that this is a complicating feature that simply makes the presentation trickier - so we will simply deal with the aggregate version.

The model has three sectors - employment, retailing and residential location with a link from retailing into part employment. Three different models are built for each sector - spatial interaction for residential and retailing and a linear model of land development for employment.
We begin with the residential, then retail sector, then trips' capacities, and finally employment.

\[ T_{ij}^k = E_i \left( \frac{R_j \exp\left(-\lambda^k c_{ij}^k\right)}{\sum_{jk} \exp\left(-\lambda^k c_{ij}^k\right)} \right) \]

\[ P_j = \sum_{ik} T_{ij}^k \]

Residential location

If \( P_j > P_j^{\text{max}} \), then \( R_j^* = R_j \cdot \frac{P_j}{P_j^{\text{max}}} \)

\[ S_{jz}^k = P_j \left( \frac{W_z \exp\left(-\lambda^k c_{jz}^k\right)}{\sum_{zk} \exp\left(-\lambda^k c_{jz}^k\right)} \right) \]

\[ S_z = \sum_{jz} S_{jz}^k \]

Retail location

If \( S_z > S_z^{\text{max}} \), then \( W_z^* = W_z \cdot \frac{S_z}{S_z^{\text{max}}} \)

If \( F_{ij}^k = T_{ij}^k + S_{ij}^k > \text{CAP}_{ij}^k \), then \( c_{ij}^{k^*} = \frac{T_{ij}^k + S_{ij}^k}{\text{CAP}_{ij}^k} \)

Capacitated Transport Constraints

In the next slides, we show the loops which need to be invoked to balance demand and supply and to couple the submodels.

\[ E_i = X_i + \phi S_i \]

Employment location

\[ X_i = X \left( \sum_q \sum_i \alpha_q x_{qi} \right) \]

\[ E_i^* = X_i + \phi S_i \]
We begin with the residential, then retail sector, then trips capacities, and finally employment.

\[ T_{ij}^k = E_i \frac{R_j \exp(-\lambda_c^k c_{ij}^k)}{\sum_{jk} \exp(-\lambda_c^k c_{ij}^k)} \]

\[ P_j = \sum_{ik} T_{ij}^k \]

If \( P_j > P_j^{max} \) \( \rightarrow R_j^* = R_j \frac{P_j}{P_j^{max}} \)

\[ S_{jz}^k = P_j \frac{W_z \exp(-\lambda_c^k c_{jz}^k)}{\sum_{zk} \exp(-\lambda_c^k c_{jz}^k)} \]

\[ S_z = \sum_{jz} S_{jz}^k \]

If \( S_z > S_z^{max} \) \( \rightarrow W_z^* = W_z \frac{S_z}{S_z^{max}} \)

\[ F_{ij}^k = T_{ij}^k + S_{ij}^k > \text{CAP}_{ij} \]

\[ c_{ij}^k = \frac{T_{ij}^k + S_{ij}^k}{\text{CAP}_{ij}} \]

\[ E_i = X_i + \phi S_i \]

\[ X_i = X \frac{\sum_q \alpha_q x_{qi}}{\sum_q \sum_i \alpha_q x_{qi}} \]

\[ E_i^* = X_i + \phi S_i \]
We begin with the residential, then retail sector, then trips capacities, and finally employment.

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T_{ij}^k = E_i \frac{R_j \exp(-\lambda^k c_{ij}^k)}{\sum_j \exp(-\lambda^k c_{ij}^k)}
\]

\[
P_j = \sum_{ik} T_{ij}^k
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if \( P_j > P_j^{\text{max}} \) \( \rightarrow R_j^* = R_j \frac{P_j}{P_j^{\text{max}}} \)

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F_{ij}^k (= T_{ij}^k + S_{ij}^k) > \text{CAP}_{ij}^k \rightarrow c_{ij}^{k^*} = c_{ij}^k \frac{T_{ij}^k + S_{ij}^k}{\text{CAP}_{ij}^k}
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T_{ij}^k = E_i \frac{R_j \exp(-\lambda^k c_{ij}^k)}{\sum_{jk} \exp(-\lambda^k c_{ij}^k)} \\
S_{jz}^k = P_j \frac{W_z \exp(-\lambda^k c_{jz}^k)}{\sum_{zk} \exp(-\lambda^k c_{jz}^k)} \\
P_j = \sum_{ik} T_{ij}^k \\
S_z = \sum_{jk} S_{jz}^k \\
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\[
P_j = \sum_{ik} T_{ij}^k
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if \( P_j > P_j^{\text{max}} \) \( \rightarrow R_j^* = R_j \frac{P_j}{P_j^{\text{max}}} \)

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\]

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S_z = \sum_{jz} S_{jz}^k
\]

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F_{ij}^k = (T_{ij}^k + S_{ij}^k) > \text{CAP}_{ij}^k \rightarrow c_{ij}^{k^*} = c_{ij}^k \frac{T_{ij}^k + S_{ij}^k}{\text{CAP}_{ij}^k}
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P_j = \sum_{ik} T_{ij}^k
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\[
\text{if } P_j > P_j^{\text{max}} \rightarrow R_j^* = R_j \frac{P_j}{P_j^{\text{max}}}
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S_{jz}^k = P_j \frac{W_z \exp(-\lambda^k c_{jz}^k)}{\sum_{zk} \exp(-\lambda^k c_{jz}^k)}
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\]

\[
\text{if } F_{ij}^k (= T_{ij}^k + S_{ij}^k) > CAP_{ij}^k \rightarrow c_{ij}^{k^*} = c_{ij}^k \frac{T_{ij}^k + S_{ij}^k}{CAP_{ij}^k}
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S_{jz}^k = P_j \frac{W_z \exp(-\lambda^k c_{jz}^k)}{\sum_{zk} \exp(-\lambda^k c_{jz}^k)}
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\]

if \( S_z > S_z^{\text{max}} \) \( \rightarrow W_z^\ast = W_z \frac{S_z}{S_z^{\text{max}}} \)

\text{if } F_{ij}^k (= T_{ij}^k + S_{ij}^k) > \text{CAP}_{ij}^k \rightarrow c_{ij}^k = \frac{T_{ij}^k + S_{ij}^k}{\text{CAP}_{ij}^k}

\[
E_i = X_i + \phi S_i
\]

\[
X_i^\ast = X \frac{\sum_q \alpha_q x_{qi}}{\sum_q \sum_i \alpha_q x_{qi}}
\]

\[
E_i^\ast = X_i + \phi S_i
\]
We begin with the residential, then retail sector, then trips capacities, and finally employment.

Here are all the loops:

\[ T_{ij}^k = E_i \frac{R_j \exp(-\lambda^k c_{ij}^k)}{\sum_j \exp(-\lambda^k c_{ij}^k)} \]

\[ P_j = \sum_{ij} T_{ij}^k \]

If \( P_j > P_j^{\text{max}} \) → \( R_j = R_j \frac{P_j}{P_j^{\text{max}}} \)

\[ S_{jz} = \sum_{zk} W_z \exp(-\lambda^k c_{jz}^k) \]

If \( S_z > S_z^{\text{max}} \) → \( W_z^* = W_z \frac{S_z}{S_z^{\text{max}}} \)

If \( F_{ij}^k = (T_{ij}^k + S_{ij}^k) > \text{CAP}_{ij}^k \) → \( c_{ij}^{k*} = \frac{T_{ij}^k + S_{ij}^k}{\text{CAP}_{ij}^k} \)

\[ E_i = X_i + \phi S_i \]

\[ X_i = X \frac{\sum_q \alpha_q x_{qi}}{\sum_q \sum_i \alpha_q x_{qi}} \]

\[ E_i^* = X_i + \phi S_i \]

Lectures on Urban Modelling
Integrated Urban Models
Topic 2: Applications

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Demonstrating an Aggregate Large Scale Model

We have broadened our residential location model for London to Greater London and the outer metropolitan area and we will demonstrate this in a moment.

Our current model is more disaggregate, more extensive and is really a suite of model types, it has an explicit money sector as house prices and wages are quite important. We do not attempt to model markets - this is quite impossible in London as the market hardly follows any known theory. We simply use transport costs, wages and prices to determine residential location.
Let me give you a quick summary of its structure:

\[ t = 1 \]

- Employment: \( E_i(t) \)
- Population: \( P_j(t) \)
- Economy: \( S_{ji}(t) \)
- Demography: \( T_{ij}(t) \)
Let me give you a quick summary of its structure:

**Origins**
- Wages and Revenues: $w_i \& v_i$

**Destinations**
- House prices: $p_j$
- Population: $P_j(t)$
- Employment: $E_{ij}(t)$

Other flows, than people or money, materials and information?
To illustrate very briefly the sort of data that we have in the money sector that is driving this variant of the model and also the residential location equations.

And then we put wages, prices and transport costs together in the interaction model as follows.
with travel as a difference or variance $\sigma^2$ between these two sets of costs. Then, the system must satisfy the constraint

$$\sum_i \sum_j T_{ij} \left[(h_i + t_r) - (c_{ij} + \rho_j)\right]^2 = \sigma^2$$  \hspace{1cm} \text{(11)}$$

The model that is generated from this constraint and which is the alternative residential location model in the current model variant is

$$T_{ij} = \frac{A_i \exp(-\lambda \left[(h_i + t_r) - (c_{ij} + \rho_j)\right]^2)}{\sum_j A_j \exp(-\lambda \left[(h_i + t_r) - (c_{ij} + \rho_j)\right]^2)}$$,  \hspace{1cm} \text{(12)}$$

which is subject to the usual origin constraint, generating population from equation (2) with (12) replacing equation (1).
This is the order in which the operations take place:

- Sequence of Model Functions
- Activity Totals
- Map Graphics
- Parameter Values
- Goodness of Fit Statistics: Deviations & $r^2$
- Graphical Functions
- Graph Data
- Logo
I will run the model as it works very quickly on the desktop.
Applications

Very Rapid Prototyping of Aggregate Models

A New Retail Centre in Dubai
Where did we get the data – in a data poor environment?

- Dubai Business Density derived from Google Places API
- Dubai Built-up area derived from Landsat 8 imagery
- Dubai Business Diversity Density Index
Predicting Urban Futures for Dubai
Simulating Land Use, Population, Employment, Retailing, and Transportation

Here we simulate the impact of large changes in urban structure on the population and employment distributions in 220 communities which define the Emirate of Dubai. The population and employment which are linked together through the transportation system and flows of trips. The model we use is heavily data driven as the data mirrors how people locate and interact in the city.

This is a simple demonstration to indicate the features of such a simulation model. If we were building this model for operational use in planning Dubai, we would have many different sectors describing different types of population distinguishing particularly between guest workers and the local population, and between retailing, construction, financial services and related industrial activities. We would also define transport by different modes.
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Applications

A New Retail Centre in Dubai
Reading about integrated models is more tricky as these models are convoluted – involved – that clear statements are hard to find. Two papers are relevant.
