GEOGRAPHY RESOURCE & MANAGEMENT
地理與資源管理學系
香港中文大學
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Lectures on Urban Modelling January 2017

About Modelling and Simulation

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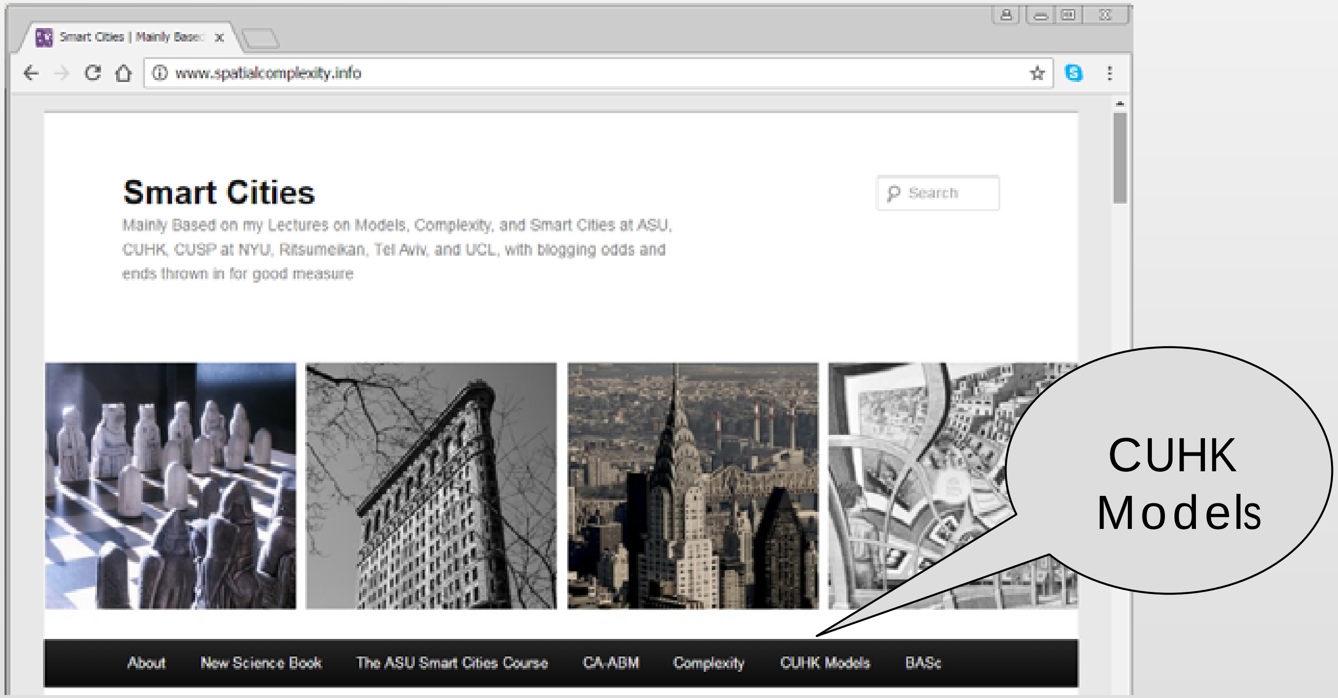
13 January, 2017

http://www.spatialcomplexity.info/

I will post the lectures on my blog

www.spatialcomplexity.info



You will find the link on this blog at

http://www.spatialcomplexity.info/urban-modelling-lectures

Outline

- What are Models? Relationships to Theory
- Definitions of Models
- A Classification: Icons, Analogs, Symbols,
- Aggregate viz Disaggregate Modelling
- Statics viz Dynamics
- The Paradigm Shift: Aggregates to Agents
- The Model-Building Process
- Facts and Theories, Factoids and Stylized Facts
- Verification, Validation, Goodness of Fit
- Calibration and Estimation

What are Models? Relationships to Theory

A <u>theory</u> is an *abstraction* of some phenomena, usually '*real*' but sometimes imagined in a form that makes the *simplification* or abstraction clear. A <u>model</u> is a simplification of *reality* which takes the theoretical abstractions and puts it into a form that we can manipulate. <u>Simulation</u> is often used to characterise this process of implementation.

In everything we do, we theorise, and more and more frequently we build models to demonstrate theory.

This is all fairly obvious – but the focus on theory is important because theory can be implicit as well as explicit. In fact in our growing quest describe the world through models, theory is tending to become part and parcel of models.

The main reason for beginning with theory is that the conventional wisdom of science begins with <u>theory</u> and then *tests* theory against <u>observations</u> – <u>data</u>. It is impossible to approach the world without prior theory and without getting involved in where theory comes from, let us assume that whenever we model a phenomena we have in mind some theory no matter how implicit. Its sometimes hard to even extract the theory we hold but it always there.

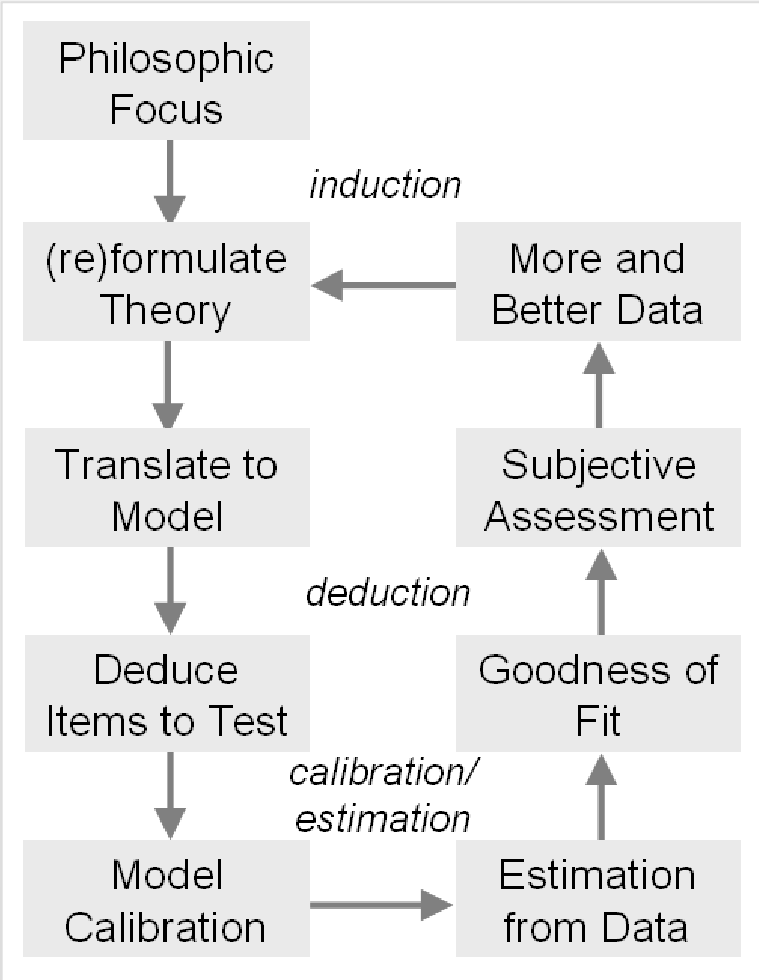
Thus the model- building process is really part and parcel of the scientific process – the <u>scientific method</u> where the current wisdom is that science tests theory by assembling data about reality which is designed to '*falsify*' the theory. This is scientific method *a la Popper* and it suggests that data or observations is the ultimate arbiter of good theory.

The method implies that this process of testing takes place in systems which are controllable in some science, are not volatile, as in experimental lab contexts. In fact as science has progressed, these conditions appear to be increasingly unlikely. Moreover as soon as we take powerful physical theory out of the lab, it becomes subject to volatile influences and can rarely achieve the predictive success it has in the lab

Hence the need for models – for theories in a form other than in the laboratory, where we can perform good testing.

The new form of the laboratory is the computer and instead of experimentation there is simulation. We could and perhaps we should spend time talking about this issue – for by no means all models are simulation models and all science is not based on computers. But increasingly science is intrinsically about computation and this is changing science itself. I also use the term ‘ science’ advisedly, in its most catholic sense ….. another debate perhaps later

Let me get some more terms out of the way – and to do this here is a simple picture of the scientific method.



Definitions of Models

There are of course many types of models and although you may think that here we are only going to deal with mathematical or symbolic models, nothing could be further from the truth. Lowry's 1965 paper – "A Short Course in Model Design" paper that I recommend you read, classifies models, and we will draw loosely on his scheme.

Lowry, I. S. (1965) A Short Course in Model Design, **Journal of the American Institute of Planners**, **31**, 158-165.

There seem to be three or perhaps four different generic ways of abstraction – **iconic**, **analog**, **symbolic** and **logic** but these categories are not mutually exclusive.

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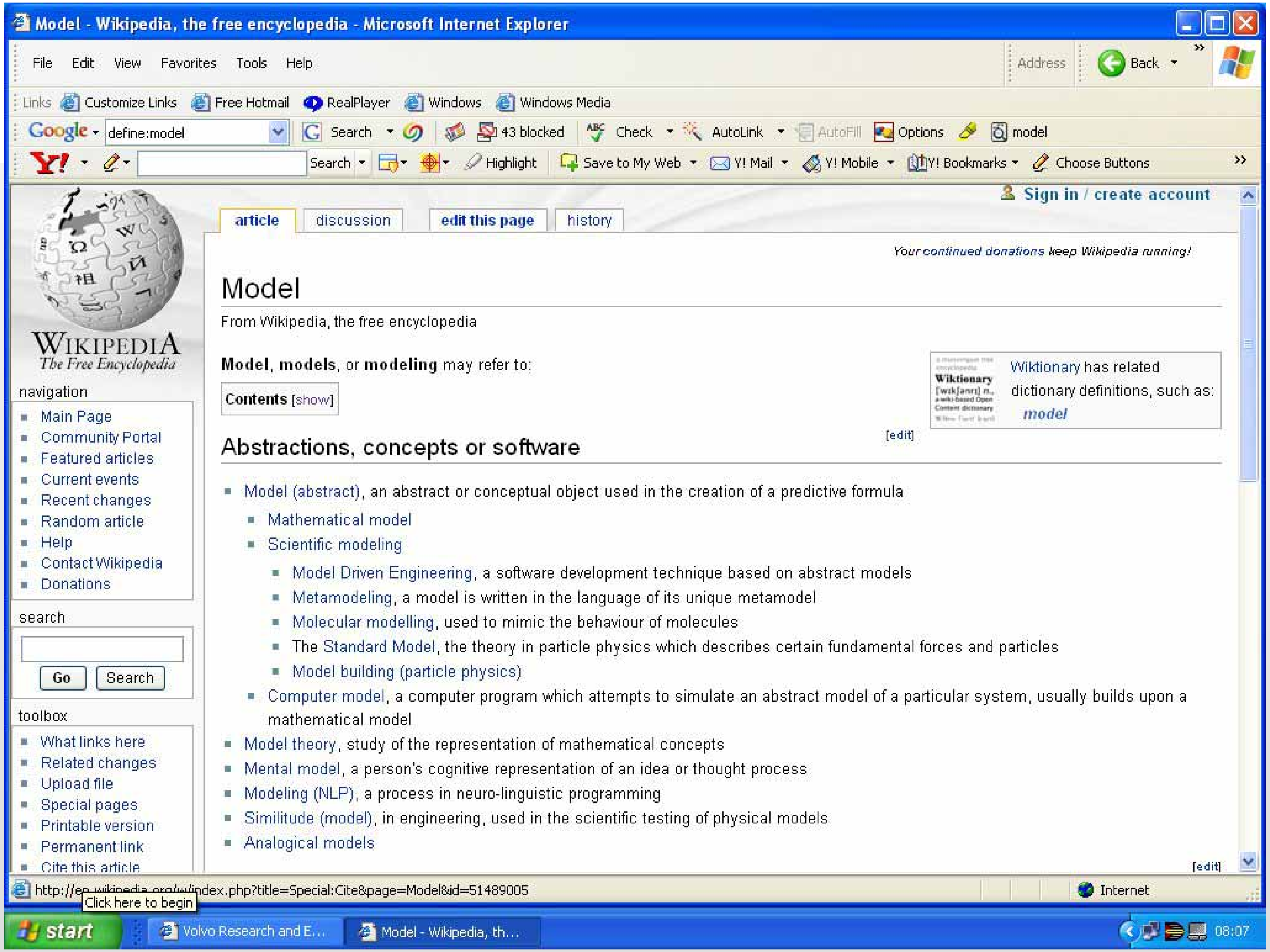
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Model, models, or modeling may refer to:
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Wiktionary has related dictionary definitions, such as: model
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Abstractions, concepts or software
Model (abstract), an abstract or conceptual object used in the creation of a predictive formula
Mathematical model
Scientific modeling
Model Driven Engineering, a software development technique based on abstract models
Metamodeling, a model is written in the language of its unique metamodel
Molecular modelling, used to mimic the behaviour of molecules
The Standard Model, the theory in particle physics which describes certain fundamental forces and particles
Model building (particle physics)
Computer model, a computer program which attempts to simulate an abstract model of a particular system, usually builds upon a mathematical model
Model theory, study of the representation of mathematical concepts
Mental model, a person's cognitive representation of an idea or thought process
Modeling (NLP), a process in neuro-linguistic programming
Similitude (model), in engineering, used in the scientific testing of physical models
Analogical models
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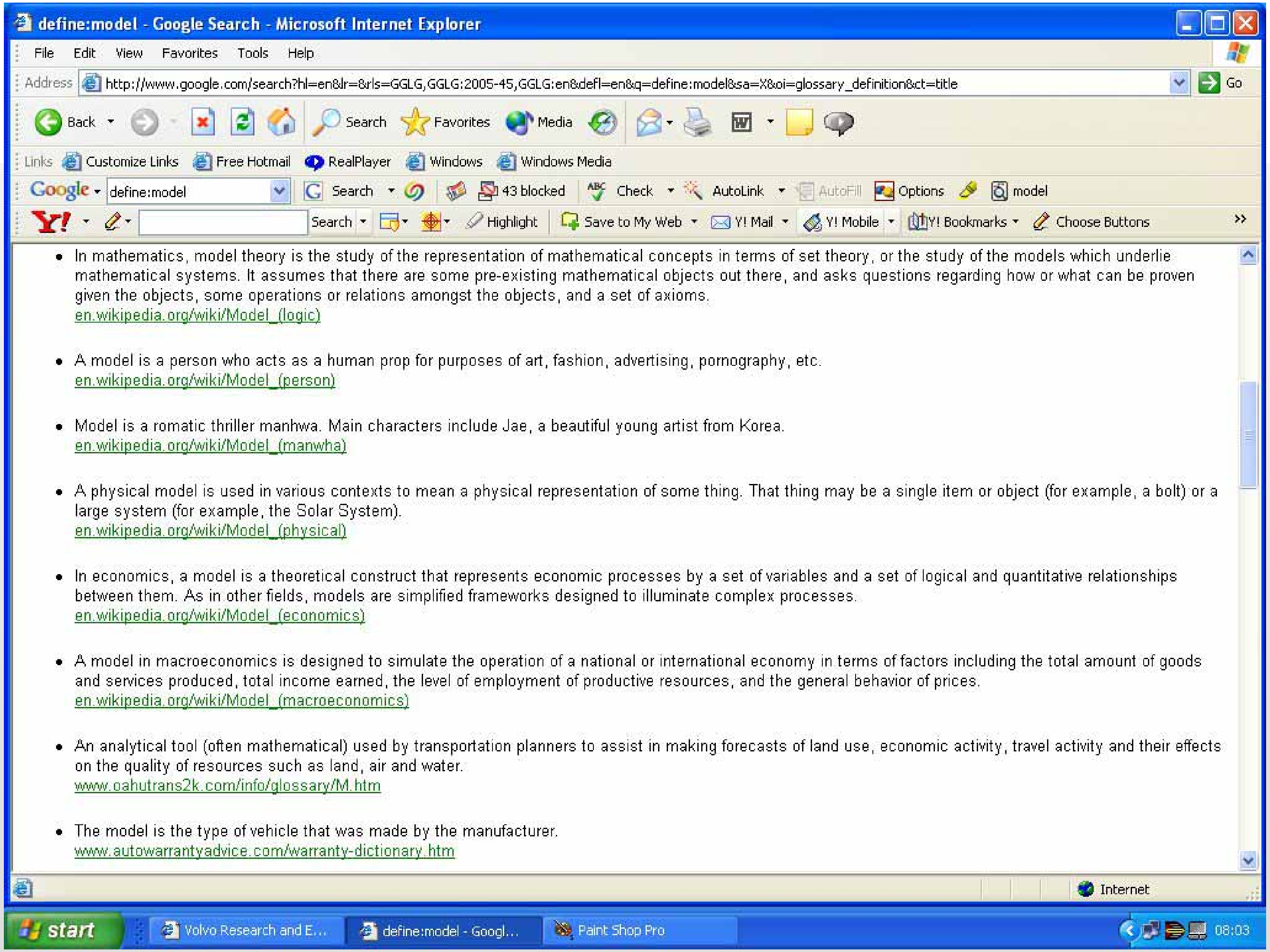
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In mathematics, model theory is the study of the representation of mathematical concepts in terms of set theory, or the study of the models which underlie mathematical systems. It assumes that there are some pre-existing mathematical objects out there, and asks questions regarding how or what can be proven given the objects, some operations or relations amongst the objects, and a set of axioms.
en.wikipedia.org/wiki/Model_(logic)
A model is a person who acts as a human prop for purposes of art, fashion, advertising, pornography, etc.
en.wikipedia.org/wiki/Model_(person)
Model is a romatic thriller manhwa. Main characters include Jae, a beautiful young artist from Korea.
en.wikipedia.org/wiki/Model_(manwha)
A physical model is used in various contexts to mean a physical representation of some thing. That thing may be a single item or object (for example, a bolt) or a large system (for example, the Solar System).
en.wikipedia.org/wiki/Model_(physical)
In economics, a model is a theoretical construct that represents economic processes by a set of variables and a set of logical and quantitative relationships between them. As in other fields, models are simplified frameworks designed to illuminate complex processes.
en.wikipedia.org/wiki/Model_(economics)
A model in macroeconomics is designed to simulate the operation of a national or international economy in terms of factors including the total amount of goods and services produced, total income earned, the level of employment of productive resources, and the general behavior of prices.
en.wikipedia.org/wiki/Model_(macroeconomics)
An analytical tool (often mathematical) used by transportation planners to assist in making forecasts of land use, economic activity, travel activity and their effects on the quality of resources such as land, air and water.
www.oahutrans2k.com/info/glossary/M.htm
The model is the type of vehicle that was made by the manufacturer.
www.autowarrantyadvice.com/warranty-dictionary.htm

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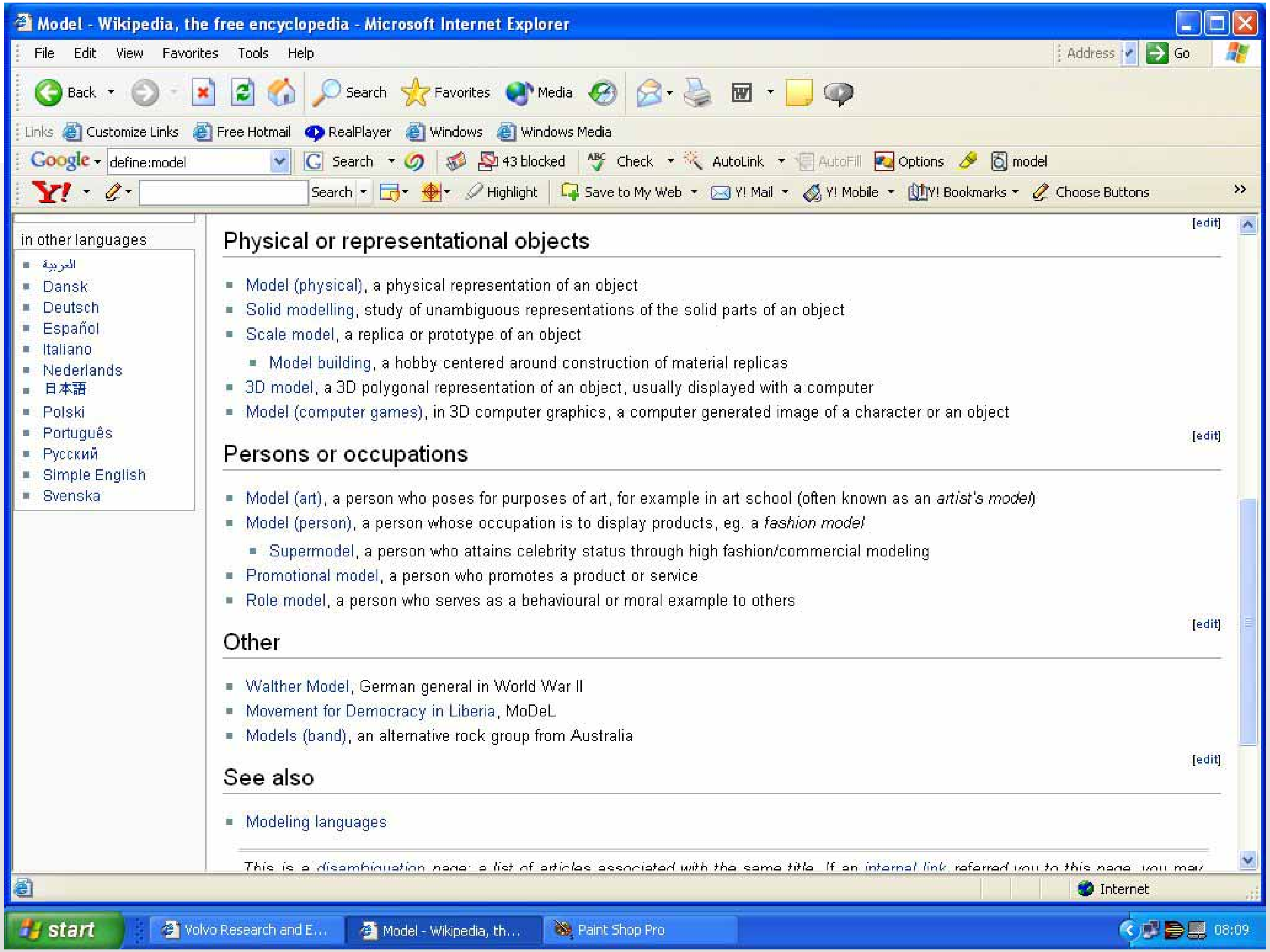
Model - Wikipedia, the free encyclopedia - Microsoft Internet Explorer
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in other languages
العربية
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Русский
Simple English
Svenska
Physical or representational objects
Model (physical), a physical representation of an object
Solid modelling, study of unambiguous representations of the solid parts of an object
Scale model, a replica or prototype of an object
Model building, a hobby centered around construction of material replicas
3D model, a 3D polygonal representation of an object, usually displayed with a computer
Model (computer games), in 3D computer graphics, a computer generated image of a character or an object
Persons or occupations
Model (art), a person who poses for purposes of art, for example in art school (often known as an artist's model)
Model (person), a person whose occupation is to display products, eg. a fashion model
Supermodel, a person who attains celebrity status through high fashion/commercial modeling
Promotional model, a person who promotes a product or service
Role model, a person who serves as a behavioural or moral example to others
Other
Walther Model, German general in World War II
Movement for Democracy in Liberia, MoDeL
Models (band), an alternative rock group from Australia
See also
Modeling languages
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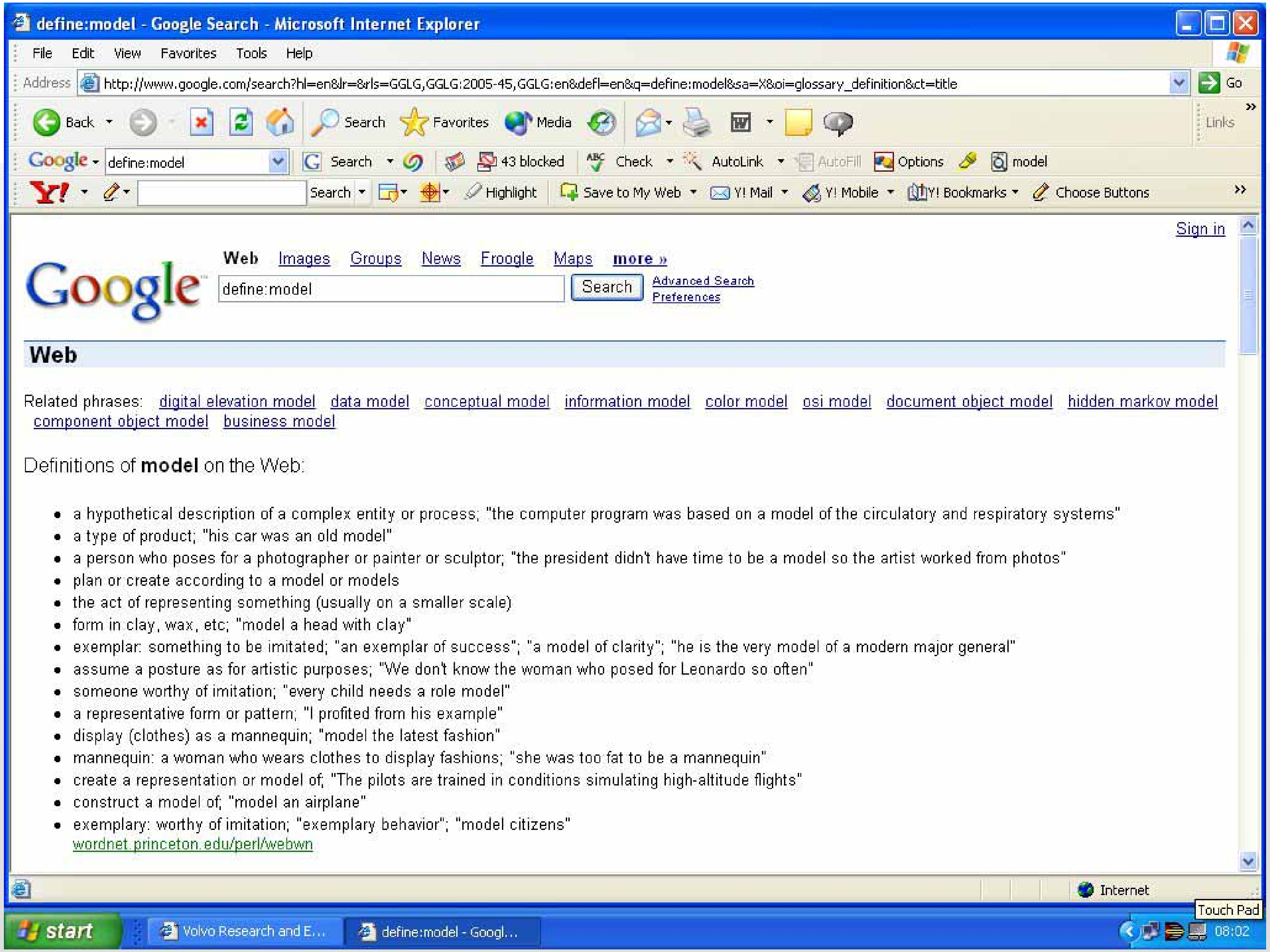
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Web Images Groups News Froogle Maps more »
define:model
Search
Advanced Search
Preferences
Sign in
Web
Related phrases: digital elevation model data model conceptual model information model color model osi model document object model hidden markov model component object model business model
Definitions of model on the Web:
a hypothetical description of a complex entity or process; "the computer program was based on a model of the circulatory and respiratory systems"
a type of product; "his car was an old model"
a person who poses for a photographer or painter or sculptor; "the president didn't have time to be a model so the artist worked from photos"
plan or create according to a model or models
the act of representing something (usually on a smaller scale)
form in clay, wax, etc; "model a head with clay"
exemplar: something to be imitated; "an exemplar of success"; "a model of clarity"; "he is the very model of a modern major general"
assume a posture as for artistic purposes; "We don't know the woman who posed for Leonardo so often"
someone worthy of imitation; "every child needs a role model"
a representative form or pattern; "I profited from his example"
display (clothes) as a mannequin; "model the latest fashion"
mannequin: a woman who wears clothes to display fashions; "she was too fat to be a mannequin"
create a representation or model of; "The pilots are trained in conditions simulating high-altitude flights"
construct a model of; "model an airplane"
exemplary: worthy of imitation; "exemplary behavior"; "model citizens"
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A Classification: Icons, Analogs, Symbols

Iconic models are representations that visually convey what the real things looks like – maps are the classic example – these are largely representations – they may have some symbology but they are scaled down versions of the real thing.

Symbolic models represents system in terms of the way they functions, often through time and over space – these models are invariably mathematical.

Analog models are a half way house between iconic and symbolic. The key issue is that they take a representational and/or functional form of one system and apply it to another.

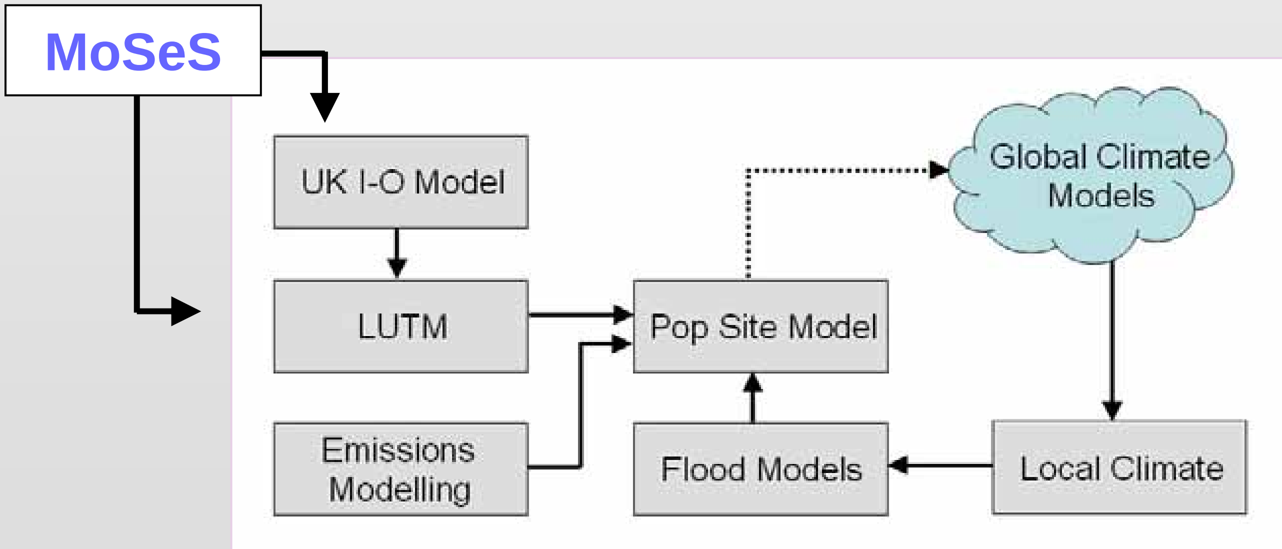
e.g. analogies between physical and human systems – the flow of blood in analogy to hydrodynamics developed for models of the atmosphere, traffic flow as an analog of an electrical network, and so on.

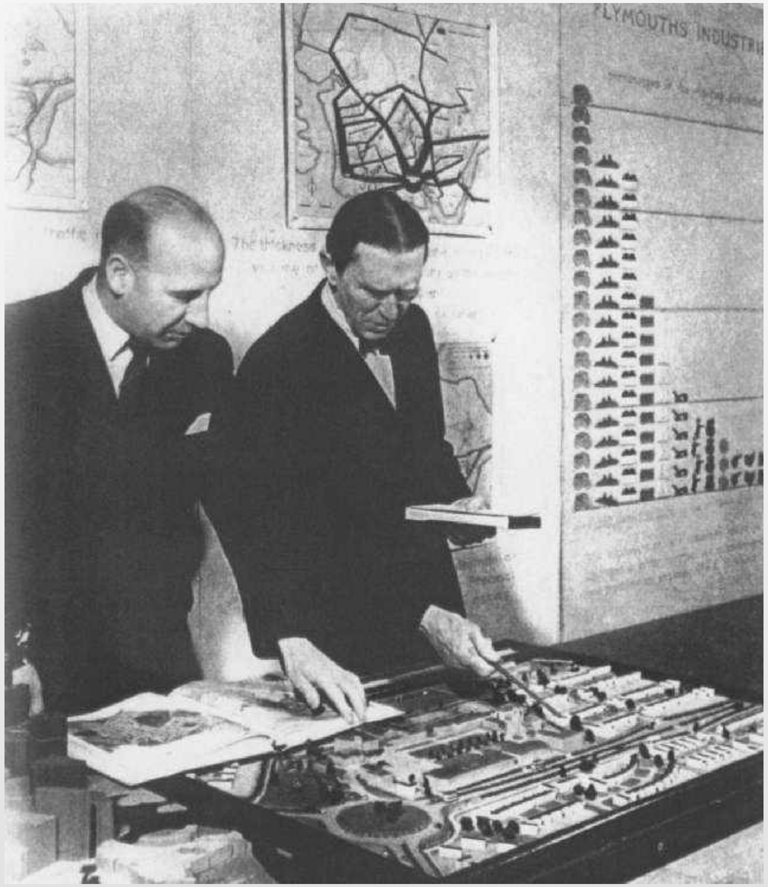
Logical models are symbolic in a sense but are based on causal connections composed of rules. We can mix, of course, any of these four types.

To this I am going to add Data-driven models. We can look at models existing on a spectrum from the data we collected about a situation that can contain elements of prediction within to fully predictive models that attempt to forecast the future or rather forecast events that have not yet happened or we have not yet observed

And I now want to introduce the idea of <u>integrated models</u> – different sectors and scales with different types of model – from iconic to symbolic.

Here is an example of how symbolic models are chained together – I don't want to spell out the maths and at this level the chains are like loops of spaghetti but flow diagrams are better



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The reason why the term model has become so significant is that computers are increasingly being used as the ‘<u>container</u>’ or ‘<u>media</u>’ for many models as our world becomes digital.

Computers mean that iconic, analog, symbolic and logical models merge into one another, so for example we can have iconic models but built of mathematical structures as in GIS

And computer models are being generalised to all sorts of other things that we never used to call models – to plans, to processes of participation etc. Its is one of the most overworked words of he last half of the 20th century and in this one it is widespread.

Aggregate viz Disaggregate Modelling

50 years ago when models first became identifiable as a distinct activity in science, and as the social sciences embraced them, they were usually statistical summaries or aggregations of elemental units.

Good examples were economic models based on macro economics, e.g. Keynesian models, econometric models

Population models, models based on social physics

There has always been a quest however to disaggregate – meaning that the model needs to be specified in more detail. Let me take an example – models of retail systems, called shopping models

Shopping trips = f (Population, Floorspace, Distance)

from zone i where people live to zone j where they shop	*zone i where people live*	*zone j where people shop*	*from zone i to zone j*

We might want to disaggregate the data into detailed types of population and detailed types of shopping, different transport networks and so on.

As computers have become ever faster and larger in terms of processing power, such models have become more and more disaggregate – in principle although data remains a constraints. We will show this a lot in the next two lectures

In fact as disaggregation has proceeded, models have changed in focus and a new stream of model where the fundamental elements themselves can be represented have become popular.

These are based on objects – or agents – where every element can be simulated – and we will say a lot more about these in later lectures.

Statics viz Dynamics

In passing, it would be remiss not to make the distinction between statics and dynamics. Models in social systems have tended to be static – comparative static or cross sectional as they are called in economics – with assumptions about that systems tend towards equilibrium.

In the last 20 years, all this has been thrown up in the air and dynamics has come onto the agenda in a big way. This has important implications for spatial systems where time has not been a popular feature of representations and models. The smart cities movement has introduced time into the agenda rather than just space.

The Paradigm Shift: Aggregates to Agents

I am not going to talk this morning about this paradigm shift to other kinds of models but just to flag these ideas, we will need to note models that we wont be introducing, namely

- temporally dynamic models on fine scale spaces called <u>cellular automata</u> or <u>CA</u> models
- temporally dynamic models where individuals or objects move in space – agent-based models <u>ABM</u> or multi-agent models <u>MAS</u>

The Model-Building Process

In later discussions, we will return to the model building process and examine processes for defining a problem, theorising about the problem, formulating a model, operationalising the model, confronting the model with data, calibrating the model to the data, testing the model's fit, taking the model elsewhere to truly test it, improving the model by extending the theory, and reiterating the process in this way.

But here we need to say something about facts and how we fit models to facts which come from data and we can distinguish the following.

Facts and Theories, Factoids & Stylized Facts

- Generally observations/data of the system being modelled or simulated are assembled and the model's predictions are compared against these 'facts'
- Facts are publicly agreed sets of observations over which there is 'no' disagreement
- Facts can range in quality from well defined observations to highly speculative pieces of data.
- Factoids and stylized facts are two types of observation that are sometimes also used in testing a model's predictive abilities

Factoids

1. A piece of unverified or inaccurate information that is presented in the press as factual, often as part of a publicity effort, and that is then accepted as true because of frequent repetition.
2. combining the word "fact" and the ending "-oid" to mean "like a fact".
3. Factoid has since developed a second meaning, that of a brief, somewhat interesting fact, that might better have been called a ‘factette’.
4. A 'factlet' is a fact that is tiny and trivial, and also correct.

Stylized Fact

In social sciences, especially economics, a stylized fact is a simplified presentation of an empirical finding. While results in statistics can only be shown to be highly probable, in a stylized fact, they are presented as true.

A stylized fact is often a broad generalisation, which although essentially true may have inaccuracies in the detail.

Highly applicable to the assumptions of agent-based models which may not be verifiable but plausible

Verification (a model matches its design)

To check, confirm or prove the truth of something. To establish, prove, substantiate, attest, corroborate, support, confirm. In modelling it is usually used to see if the model is working properly

Validation (a model matches the data)

To meet some criterion/criteria associated with the model and or the data/observations. In general, validation is the process of checking if something satisfies a certain criterion. Examples would be: checking if a statement is true, if an appliance works as intended, if a computer system is secure, or if computer data is compliant with an open standard. This should not be confused with verification.

Goodness of Fit

A well defined measure of how the model's predictions match the known observations of facts, typically some measure of difference between predictions and observations.

Predictions are any outcome of the model, past, present or future

Calibration and Estimation

Calibration is the generic process of validation and verification. Estimation is the process or method of generating a precise estimate of some parameter characterising the model.

Other Issues

consistency and reliability –with reliability is the consistency of your measurement,

I don’t think there is a coherent discussion of all these issues *per se* as they are pieced together from multiple sources.

Sensitivity Testing
Process Modelling
Parsimony v richness
Scale, aggregation …. Space and time

Background Reading

I will put this material up on the web tomorrow and there are five papers worth looking at

Batty, M. (2009) Urban Modeling, in R. Kitchin and N. Thrift (Eds) **International Encyclopedia of Human Geography**, Volume 12, Elsevier, Oxford, 51–58.

Batty, M. (2008) Spatial Interaction, in K. K. Kemp (Editor) **International Encyclopedia of Geographic Information Science**, Sage. Los Angeles, CA, 416-418.

Batty, M. and Torrens, P. (2005) Modelling and Prediction in a Complex World, **Futures**, **37** (7), 745-766.

Lowry, I. S. (1965) A Short Course in Model Design, **Journal of the American Institute of Planners**, **31**, 158-165.

Vanderleeuw, S. E. (2004) Why Model? **Cybernetics and Systems: An International Journal**, **35**, 117-128

And if you want some old background you can download my old book **Urban Modelling** (1976) from our web site at www.casa.ucl.ac.uk/urbanmodelling/

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Lectures on Urban Modelling January 2017

Gravitation and Spatial Interaction
Preliminary Ideas

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13 January, 2017

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Outline

This second part of the lecture introduces specific models of spatial interaction

- Spatial Structure: Profit and Cost, Distance, Agglomeration, Accessibility
- Von Thunen and DLA
- Gravitation: The Basic Models
- Potential and Accessibility
- Spatial Interaction and Trip Distribution: Constraints on Volume & Location

Spatial Structure: Profit and Cost, Distance, Agglomeration, Accessibility

We don't have time for a detailed development of theories of spatial structure but benefits and costs – profits and losses - are key to location. These define the pull and push factors of selecting any location defining agglomeration and dis-agglomeration economies, & accessibility and inaccessibility

We will define two very different approaches that lead to the same sort of structures

First from urban economics, and second the same phenomena from physical movement

Von Thunen and then DLA

Essentially benefits (or profits) depend on nearness to market which under normal profits we measure as rent payable. This is balanced against the cost of transport to the market. This might relate to space that one might get – as one moves further away from a point, more space is accessible

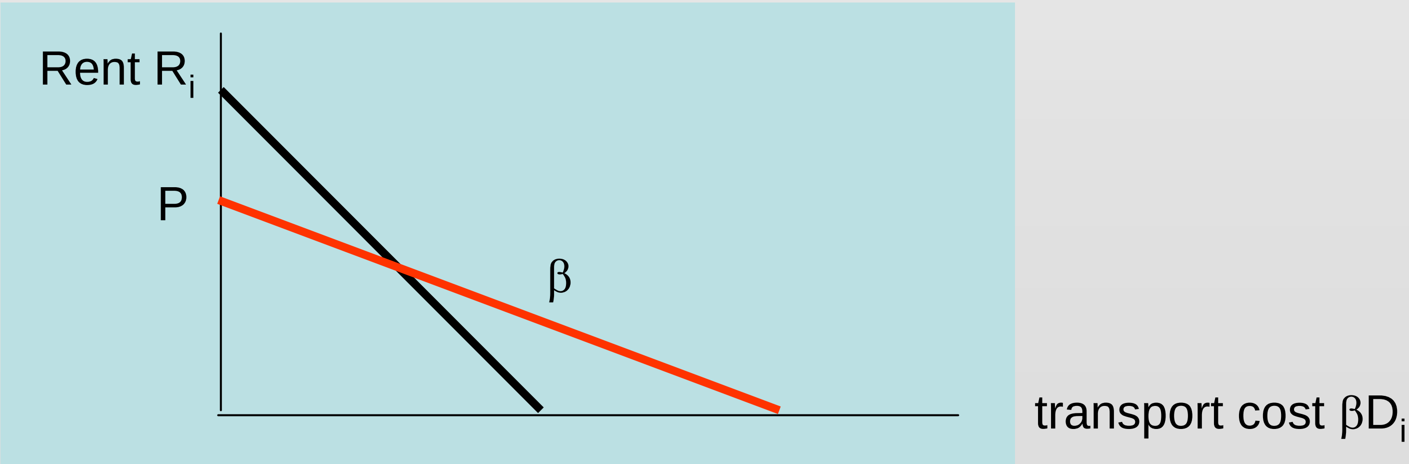
Who locates where depends on trade off of benefits versus transport cost

We define profit or yield at the market P, distance D_i at location i from the market or centre, transport cost per unit of distance β and then rent payable at i which is R_i

We thus define the key equation for the costs and benefits of location as

$$R_i = P - \beta D_i$$

This is a linear equation where we can think of profit or yield as the intercept and slope as the transport cost per unit of distance. As we vary these the slope of the line will vary as

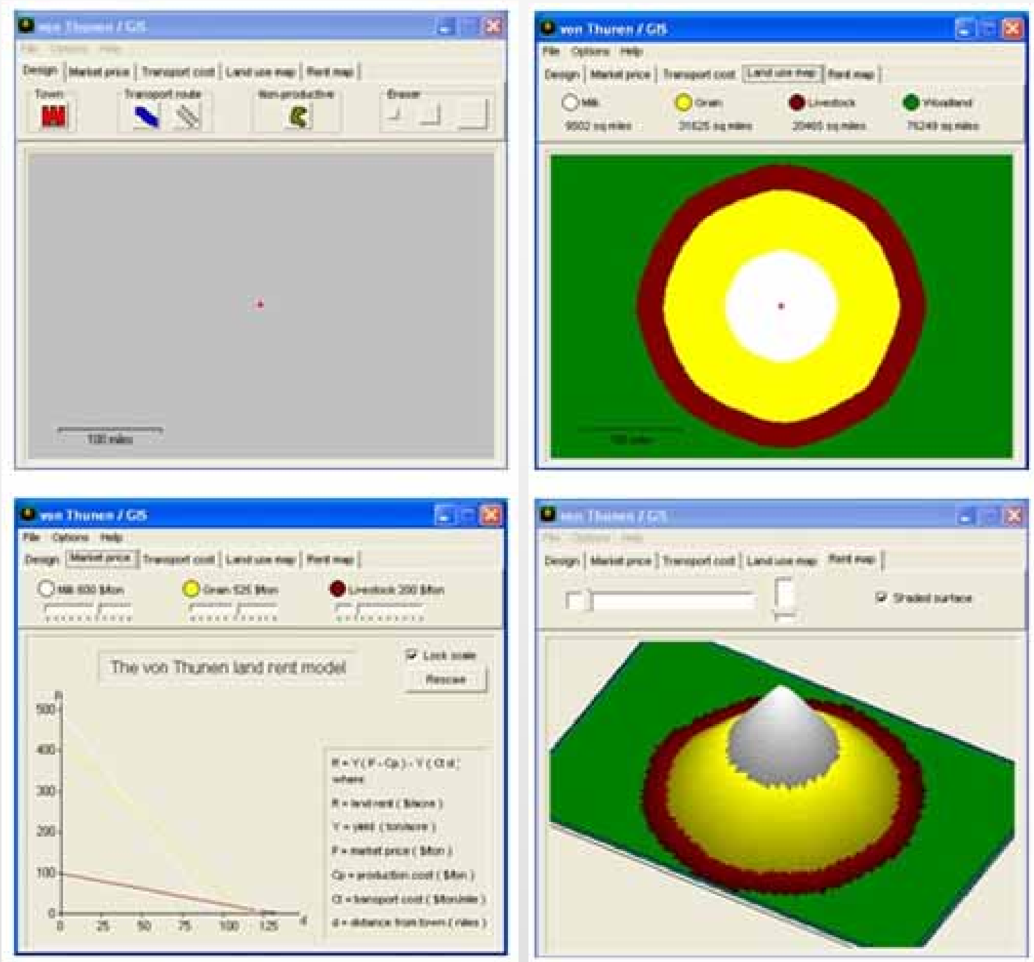


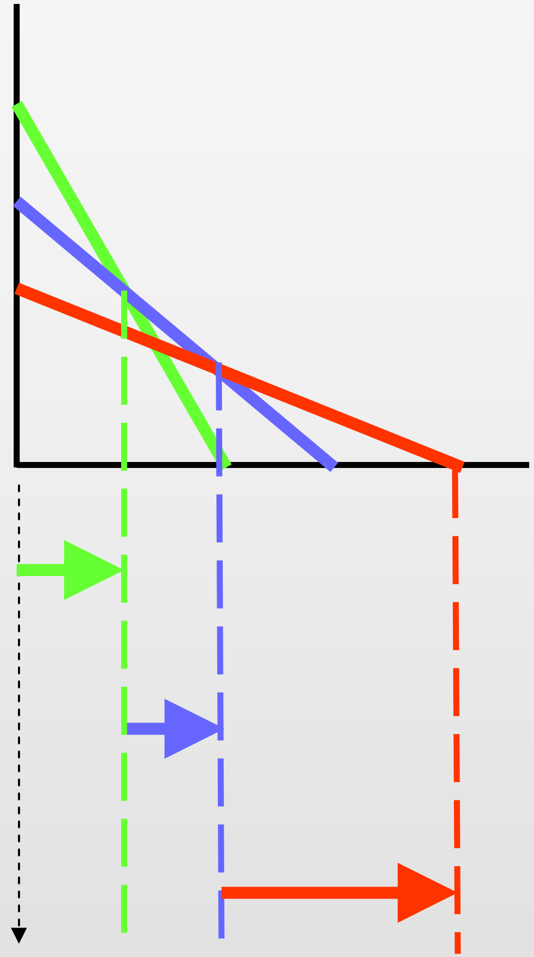
Let me load the von Thunen model and show you how this works. Go to our web site and download to save or run the model from

http://www.casa.ucl.ac.uk/software/vonthunen.asp

What you see is the following which I will show before we run it: a simply canvas on which you plant a market, you can adjust the benefits and cost equation for a series of different land uses – all agricultural which imply different types of production and transport of goods to market

And market clearing takes place where one land use outbids in terms of rent any other: this assumes normal profits





Land use are determined by the bid rent curves in terms of their dominance and the circular pattern of land use falls out from this – I will run the model

The Diffusion-Limited Aggregation Model

My second model is quite different. This is a physical model in which an agent wants to locate at a town to get economies of scale but also wants to be as far from the town as possible.

Thus the agent wants to realise agglomeration, clustering but also economies of getting as much space as possible. Thus the issues are to balance centripetal forces with centrifugal.

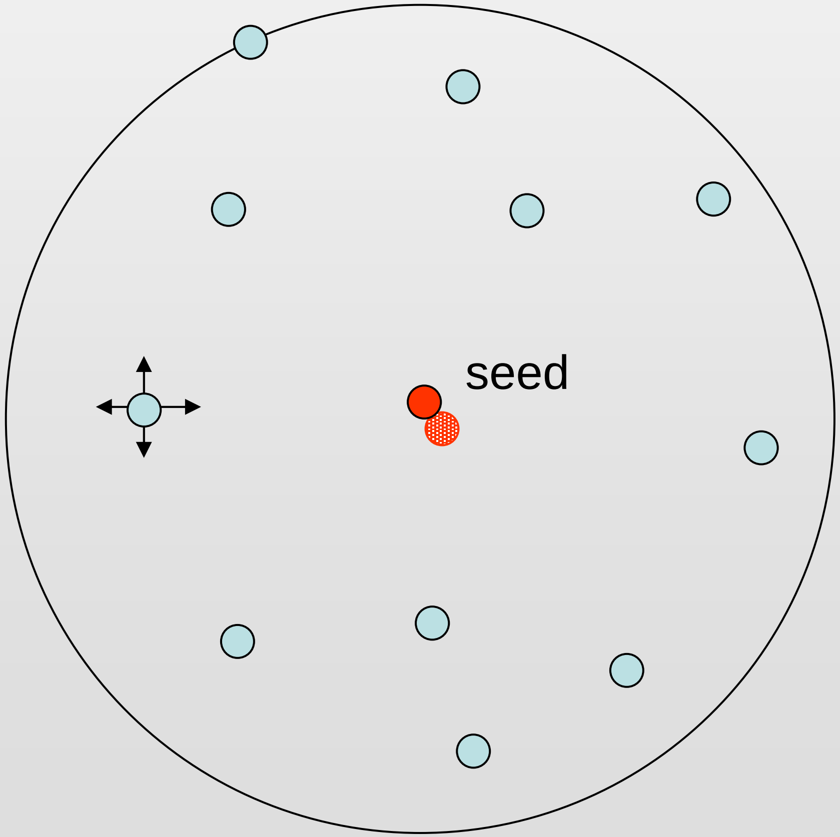
We can set up a model to show how this occurs. Plant a seed and let many agents wander randomly in a circular region around the seed. The rule for fixing location are dead simple

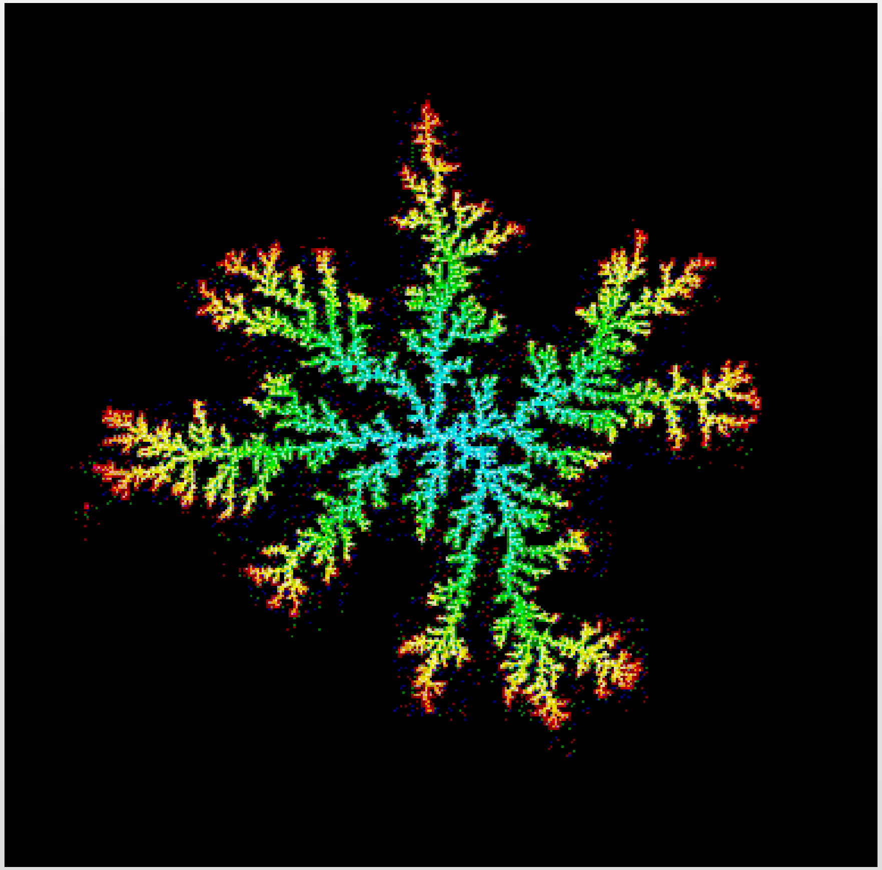
Ok, let me show you the simplest possible model of an organically growing city – based on two simple principles

- *A city is connected in that its units of development are physically adjacent*
- *Each unit of development wants as much space around it as it needs for its function.*

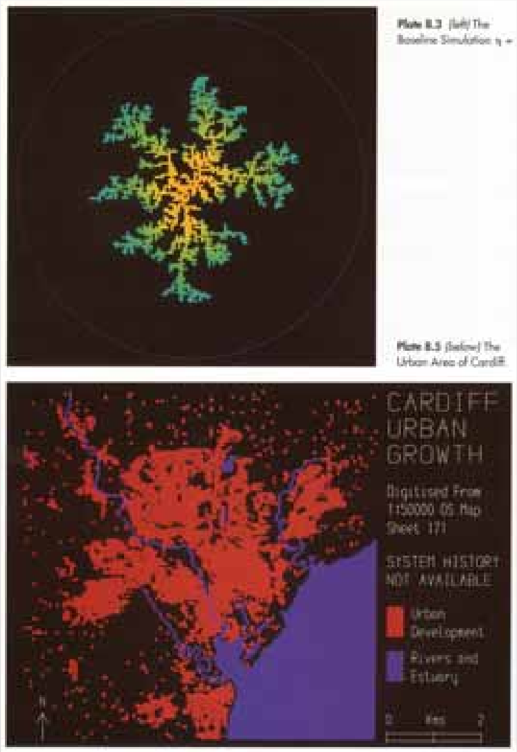
We start with a seed at the centre of a space and simply let actors or agents randomly walk in search of others who have settled. When they find someone, they stick. That is all.

In essence, this is random walk in space which is can be likened to the diffusion of particles around a source but limited to remain within the influence of the source – the city





What we get is the following dendritic pattern: this is a model called diffusion limited aggregation, DLA where the diffusion is limited or constrained



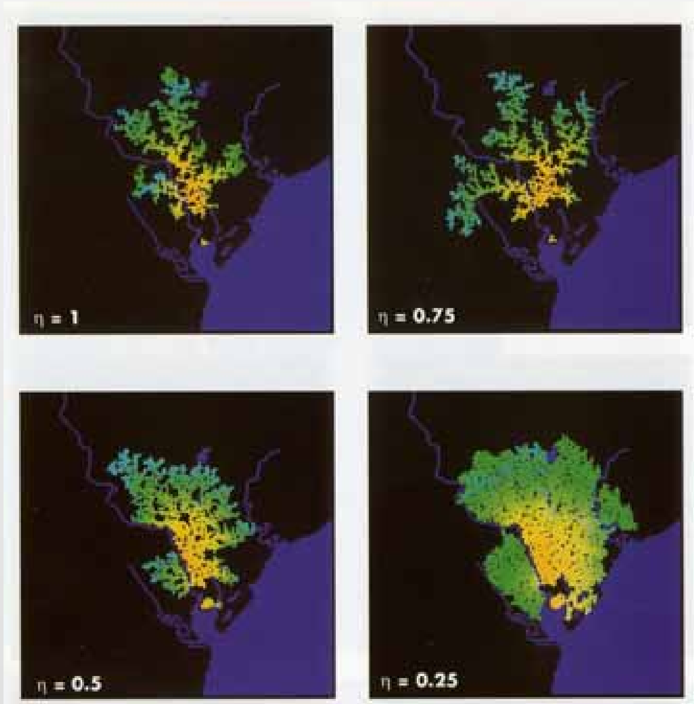


Plate 8.6 Simulating the Urban Growth of Cardiff

There are many examples on the internet that you can search for yourselves

Gravitation: The Basic Models

Ok, we can incorporate these ideas in the basic model of forces which was first articulated by Newton as his second law of motion – force is proportional to mass times acceleration

In more conventional terms we might write the force between two bodies as

$$F_{12} \sim M_1 M_2 / (d^2)_{12}$$

There is a very long history of analogies between force and social interaction going back to Newton himself. There are many references and I will add some to the web page

But let me immediately generalise this and say that we need to define many interactions – we break our system in to areas or points which we define as origins and destinations i and j

And then we measure the distance as in von Thunen not as distance per se but as travel cost or rather generalised cost c_{ij}

We also define the mass at the origins and destinations as O_i and D_j and we then write the conventional spatial interaction of gravity model as

$$T_{ij} \sim \frac{P_i P_j}{c_{ij}^2} = K \frac{P_i P_j}{c_{ij}^2}$$

Where K is the gravitational constant

Potential and Accessibility

In the 1940, the astronomer John Stewart suggested that a measure of potential could be produced from the gravity model that was an overall measure of the force of an object on all others. He defined this from the GM equation as

$$V_i = \sum_j T_{ij} \sim P_i \sum_j \frac{P_j}{c_{ij}^2}$$

$$v_i \sim \frac{V_i}{P_i} = \sum_j \frac{P_j}{c_{ij}^2}$$

This is essentially accessibility or nearness and it was first used as the basis for a simple urban model by

By Walter Hanson in the late 1950s in a paper called "How Accessibility Shapes Land Use". There he said that the residential development in a place was a simple function of accessibility i.e.

$$R_i \sim \frac{V_i}{P_i} = \sum_j \frac{P_j}{c_{ij}^2}$$

In fact is total residential development is R, then the equation can be written as

$$R_i = R \frac{(V_i / P_i)}{\sum_k (V_k / P_k)}$$

The original gravity model has been used for years but in the 1960s and 1970s various researchers cast it in a wider framework – deriving the model by setting up a series of constraints on its form which showed how it might be solved generating consistent models. The *constraints logic* led to **consistent accounting** The *generative logic* lead to analogies between **utility and entropy maximising** and opened a door that has not been much exploited to date between entropy, energy, urban form physical morphology and economic structure. In particular the economic logic is called choice theory, specifically discrete choice

Now to introduce all this, we need to define some more terms. We will refer to the size of volume of origins and destinations not as population P but as O_i and D_j assuming they are different from one another. We will also assume that the inverse square law on distance or travel cost does not apply and that whenever c_{ij} appears it will be parameterised with a value that varies which we call λ. We will assume trips are as we have defined them as T_{ij} but we will also normalise trips by their total volume T to produce probabilities

$$p_{ij} = \frac{T_{ij}}{T} = \frac{T_{ij}}{\sum_{ij} T_{ij}}$$

Note that we use summation extensively in what follows

Trip Distribution: Constraints on Volume & Location

We must move quite quickly now so let me introduce the basic constraints on spatial interaction and then state various models

The constraints are usually specified as origin constraints and destination constraints as

$$O_i = \sum_j T_{ij}$$

$$D_j = \sum_i T_{ij}$$

And we can take our basic gravity model and make it subject to either or both of these constraints or not at all

So what we get are four possible models

Unconstrained

$$T_{ij} = KO_i D_j c_{ij}^{-\lambda}$$

Singly (Origin) Constrained so that the volume of trips at the origins is conserved

$$T_{ij} = A_i O_i D_j c_{ij}^{-\lambda}$$

Singly (Destination) Constrained so that the volume of trips at the destinations is conserved

$$T_{ij} = B_j O_i D_j c_{ij}^{-\lambda}$$

Doubly Constrained trip volumes at origins + destinations are conserved

$$T_{ij} = A_i B_j O_i D_j c_{ij}^{-\lambda}$$

The first three are location models, the last is a traffic model

I am going to stop at this point and once again point you to my old book if you want to get some more ideas on this – but we will start it again with this and develop it in the next lecture

Lectures on Urban Modelling January 2017

Thanks – More Next Monday, same time

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