UNIVERSITY OF CAMBRIDGE Department of Land Economy

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casa UCL University College London

The Science of Cities

The Size, Shape, & Performance of Cities Using Percolation Theory

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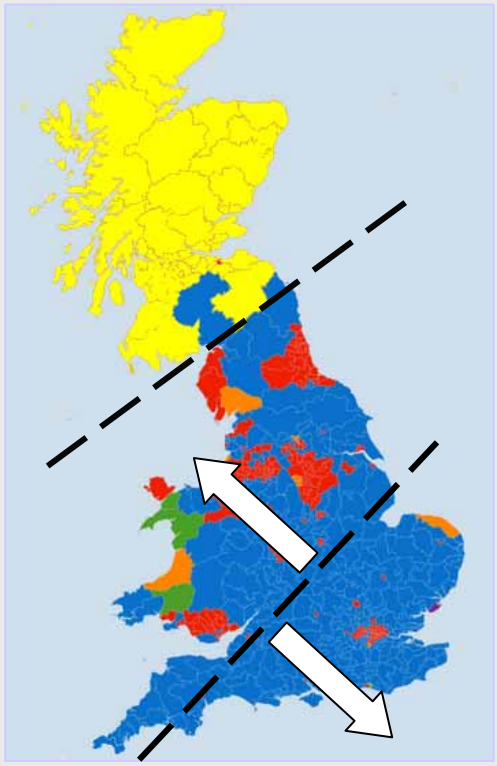
Let me start by telling you a geographical story – based on the recent and not so recent story of British politics

Here is what was happening, 9 months or so ago in May this past year in the UK

2010 ……

2015 ……



where

- Conservatives
- Labour
- Scot Nationalists
- Liberal
- Plaid Cymru
- UKIP

This is an enormous switch – the rise of the Scottish Nationalists

When we saw this happening over the last 12 months since the Referendum on Independence for Scotland, we associated this **polarisation** with one of our projects to define *cities as percolation clusters*. In this, the key element is to use the street network and in doing this, we begin with the entire network for the British island

Let me show you what we found

Essentially we are able to perform a regionalisation of Britain using the street pattern and network that contains all the cultural and social history of the last 2000 years as well as more recent urban growth in the last 200. We knew Britain was fractured – you learn this in high school – the north-south divide – but what we found was more of a surprise, and more of a divide than we expected.

Let me emphasise this point about the street-road network which contains a lot of the economic social and cultural history of Britain, from the Celts, to the Romans, to Anglo-Saxons, to Normans and so on

I will tell you the method we used first and only then tell you the detail of why and how we are doing this.

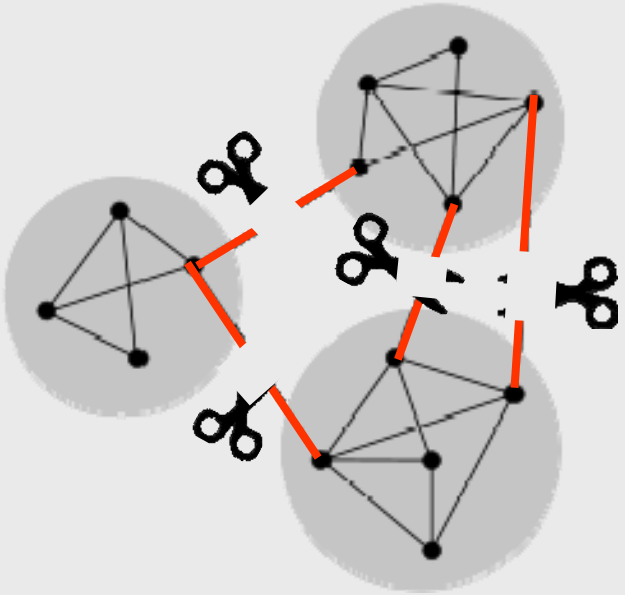
It is part of our project to look at the nature of urban morphology, city size, urban allometry and more particularly the qualitative changes that take place in the economy of cities as they agglomerate

This is a key issue for thinking about a world where every one will be living in cities by the end of this century. But before that, back to Scotland.

Let me digress slightly. You may by be wondering why we are calling this talk a science of cities.

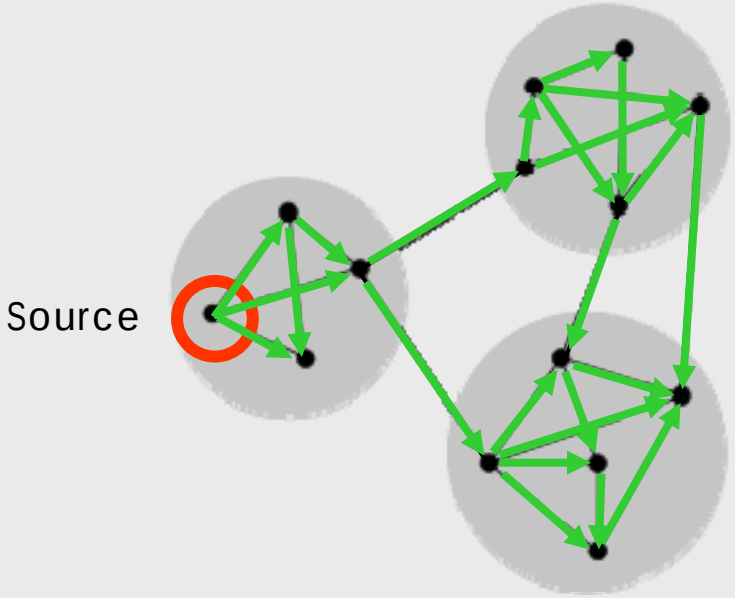
One of the main issues in any science is to define what a city is. That may seem obvious but for millennia since the ancient Greeks, people have speculated on how to define a city. And we need to define it when we talk of them. It may be comparatively easy to define the extent of some cities but not so in many parts of the world. It is hard here for example and it is in the UK

So how do we fracture Britain to predict its nations, regions and cities?
Imagine the network of streets and roads which covers these entire islands.
Let us cut the longest links – those that are greater than a distance threshold – so that we partition the network into clusters, and we keep on doing this .... by tightening the threshold, thus building the hierarchy top down



I had given variants of this talk a few times and then someone asked me after the lecture ***"Just exactly what is percolation?".*** So I had better tell you a little more explicitly

Imagine a process that diffuses from a source one step at a time. If the diffusion takes place on a network, as the diffusion continues, it takes in more and more of the network. At each stage more and more clusters are embraced and the biggest cluster grows in size – this is the giant cluster. Eventually all the network is covered. Think of a forest fire. Here is the little network example again.

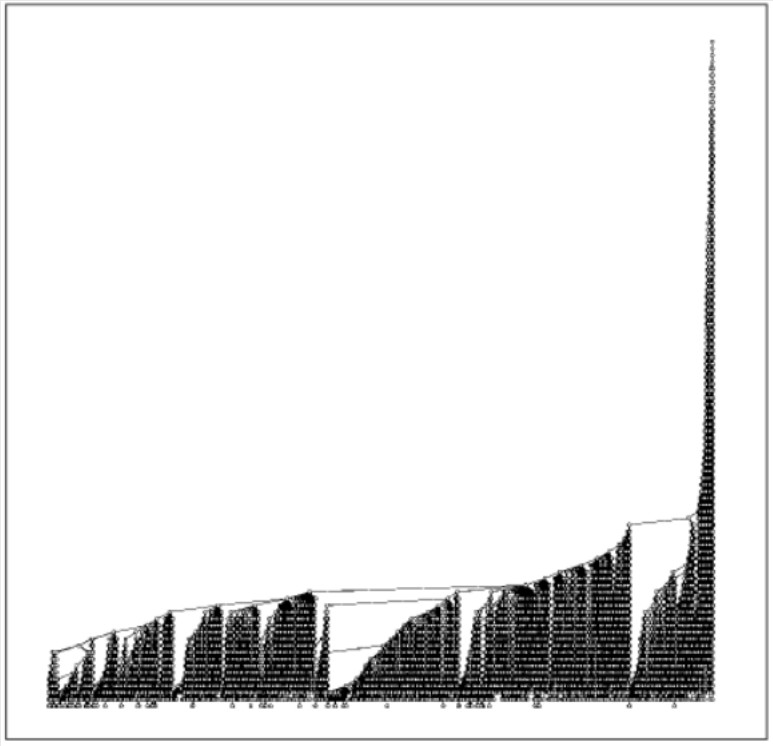


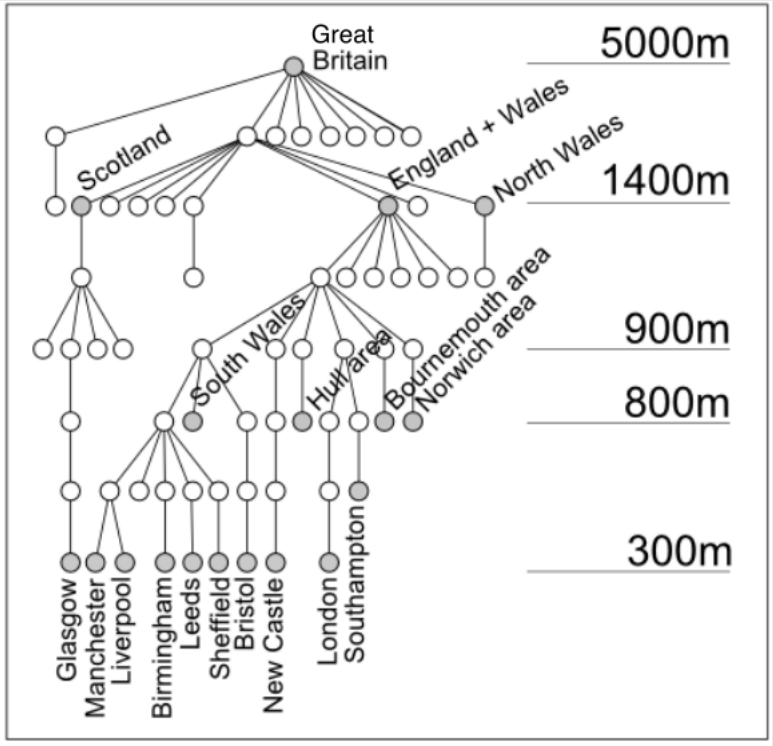
If you play this forwards we build clusters from the bottom up and backwards we build clusters from the top down

I will show the video the other way around from the bottom up because there real power of this is to stop it and look at the regionalisation. And in fact the way we do it is from the bottom up, not the top down; it's easier

Click here

We build or extract a very detailed hierarchy with tens of thousands of clusters

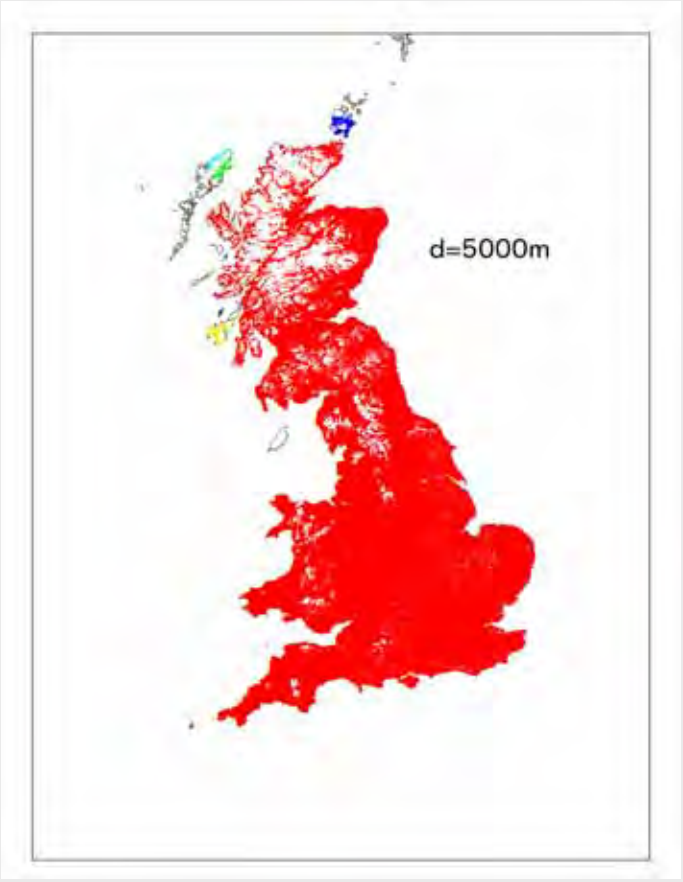




Let us make it a little more relaxed and look at the key stages – the essential issue from the election and what people vote is that *Britain breaks up into its historical cultural pieces*.

Fractured Britain, probably
Fractal Britain

The movie shows it beautifully but so do these frames .....



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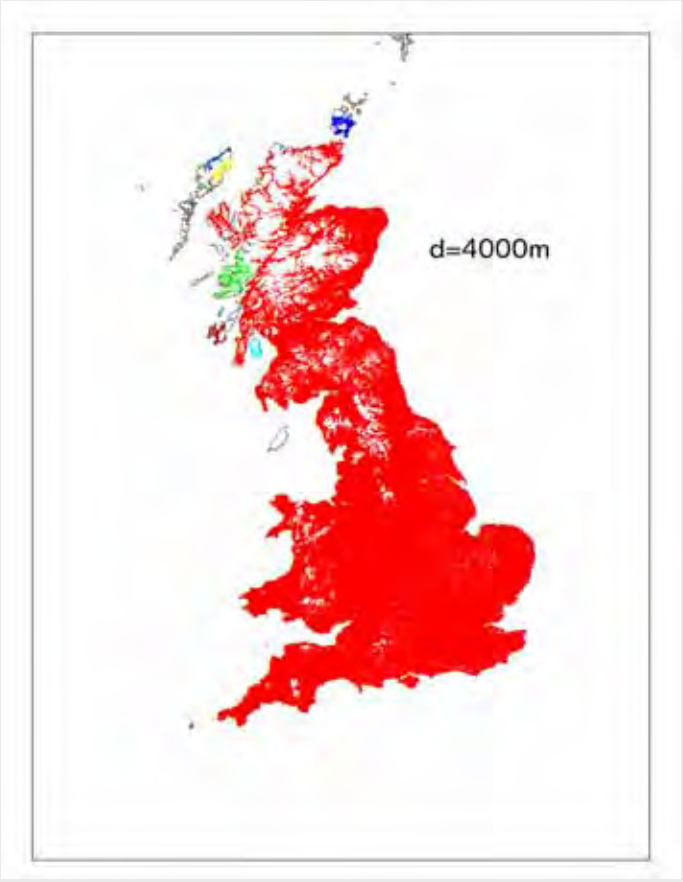

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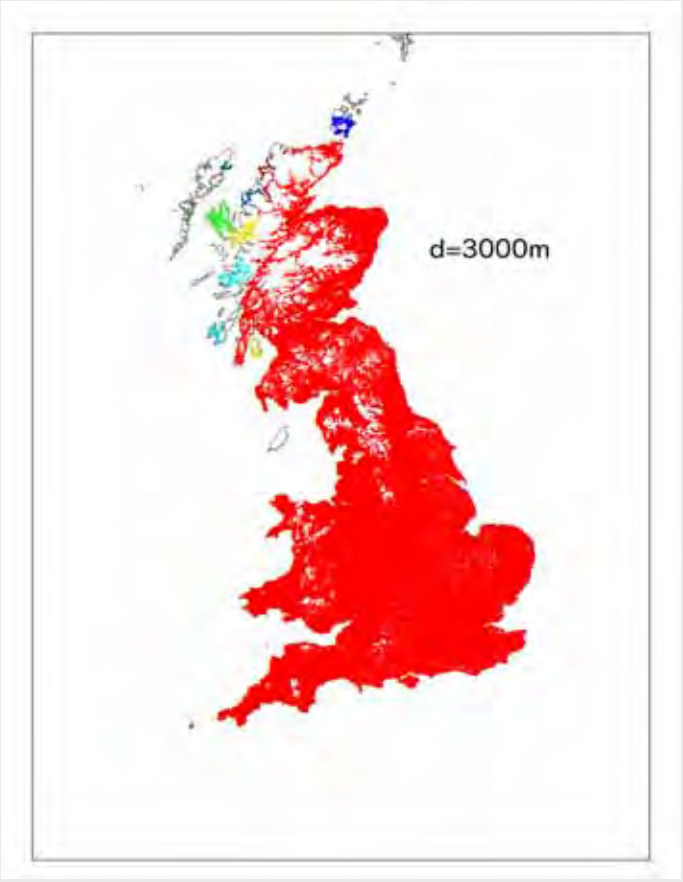

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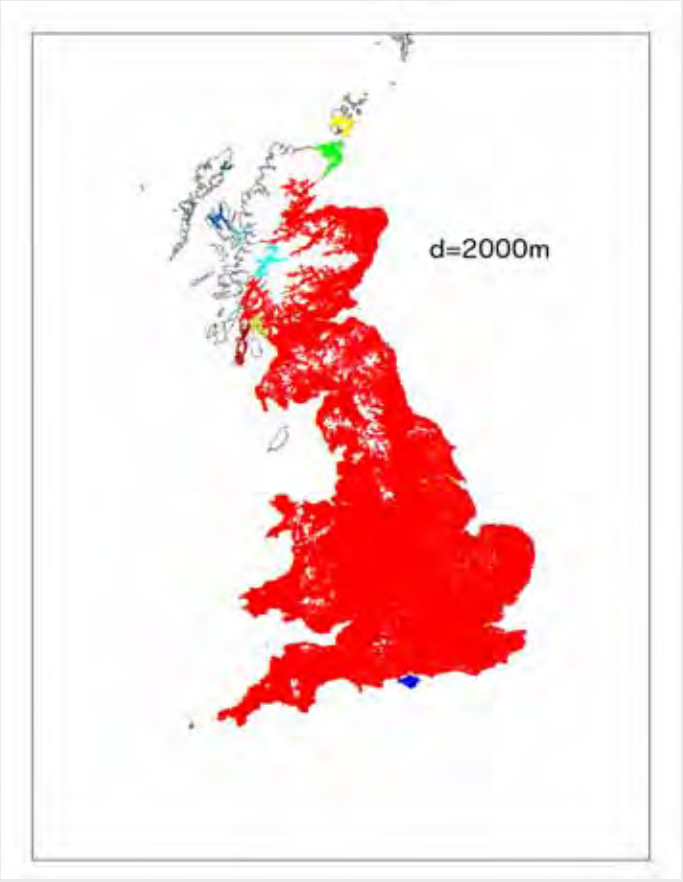

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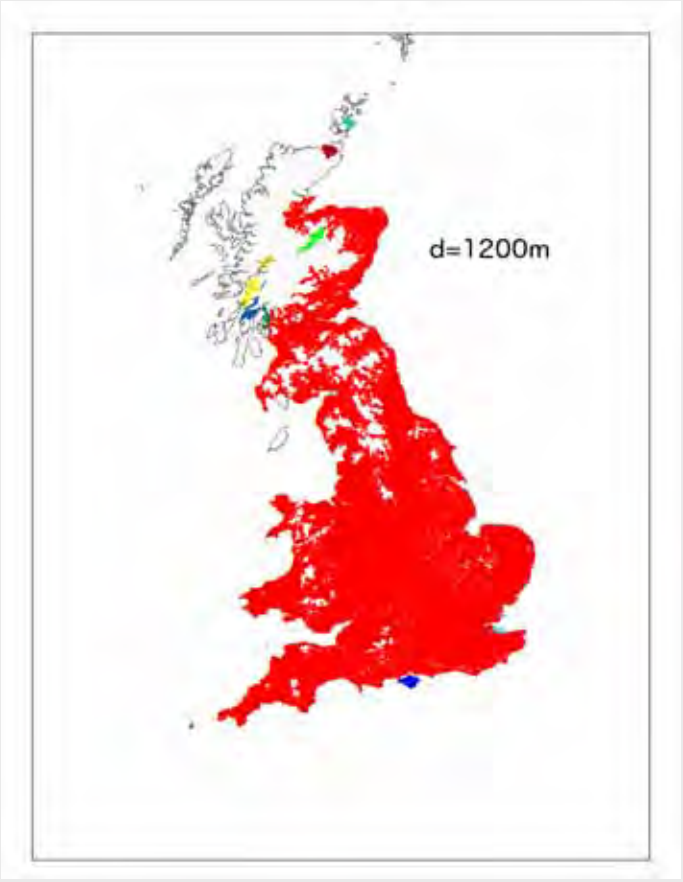

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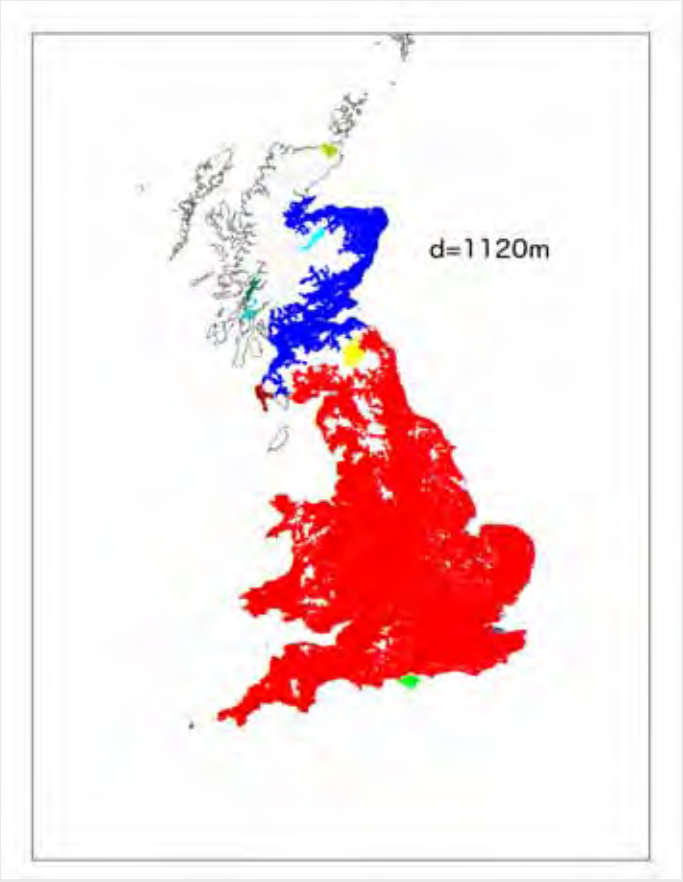

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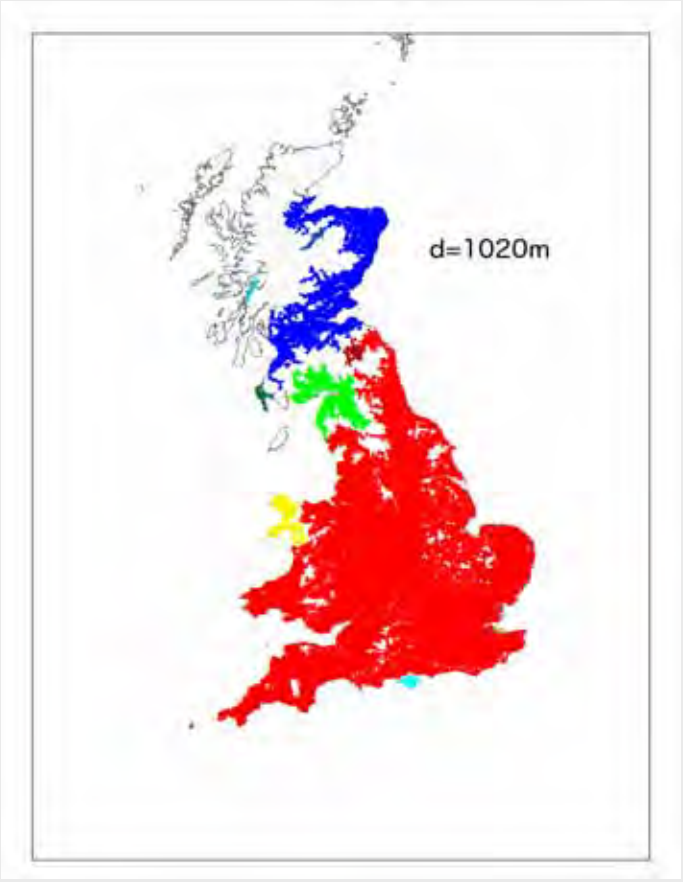

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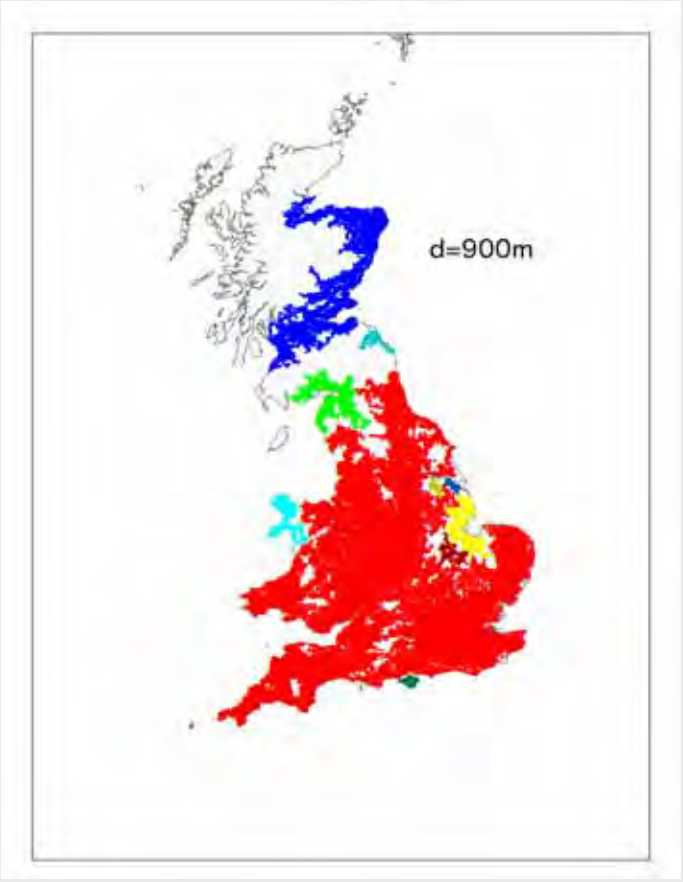

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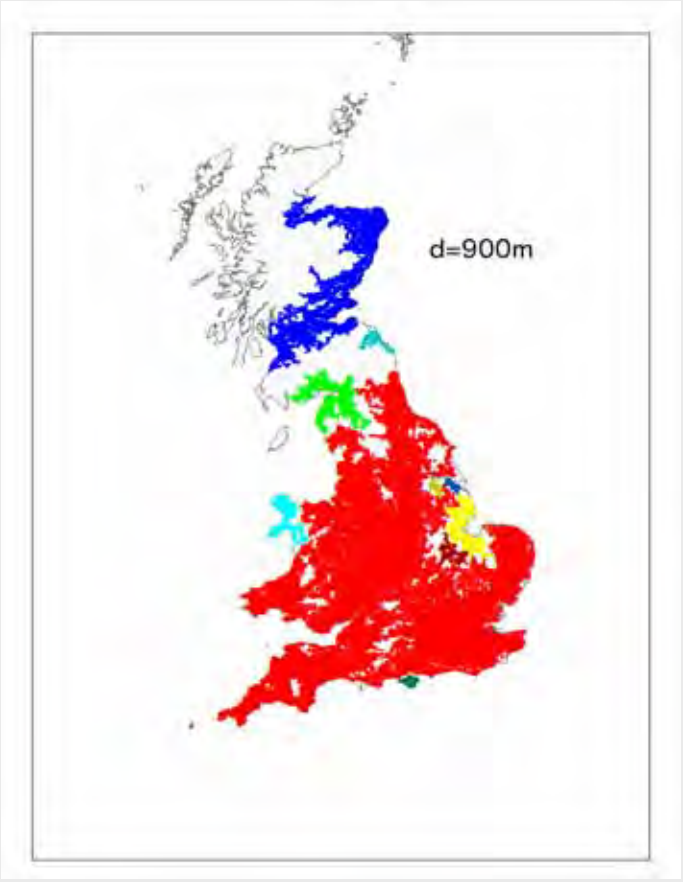

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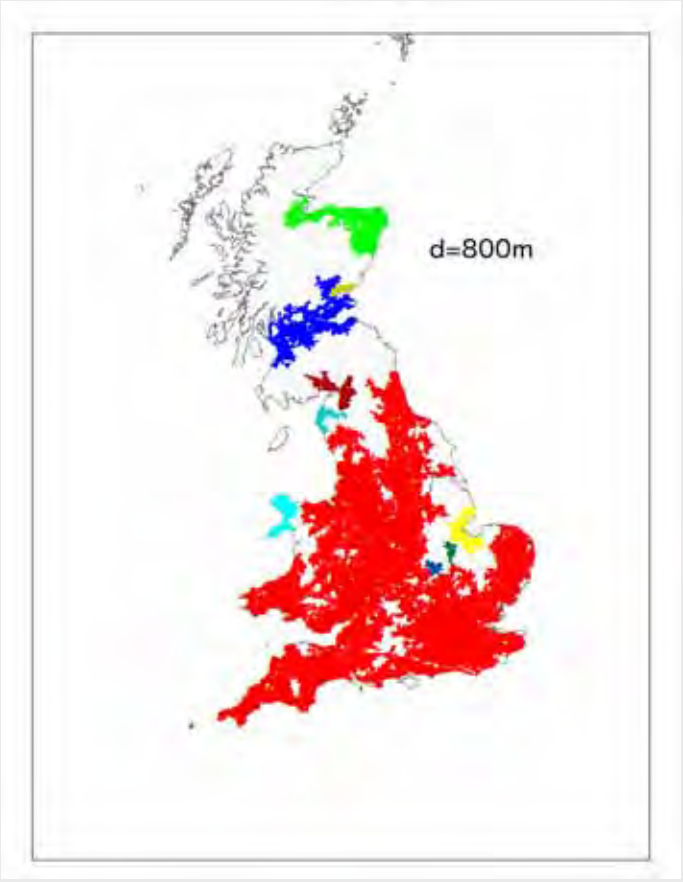

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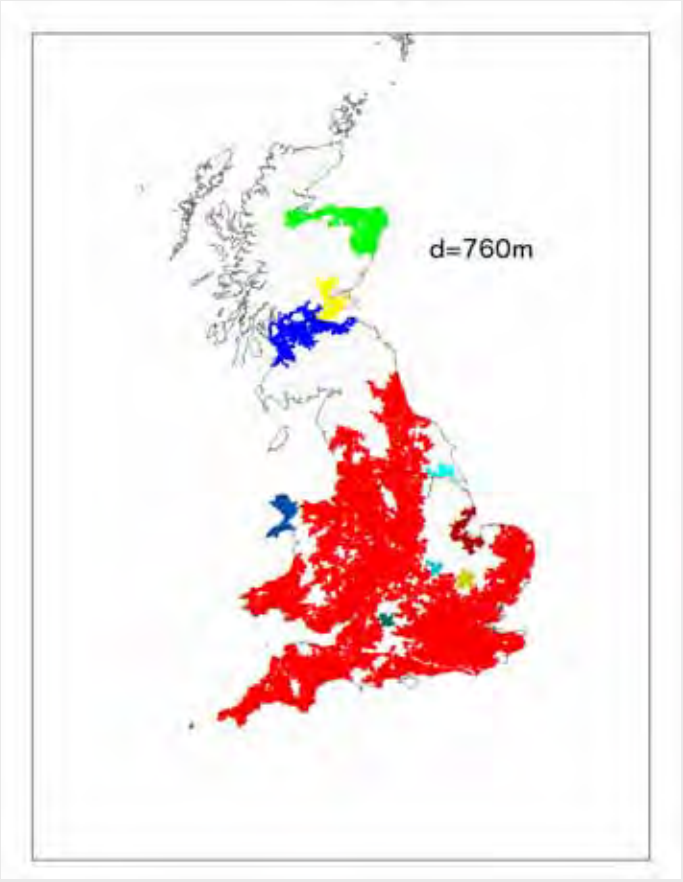

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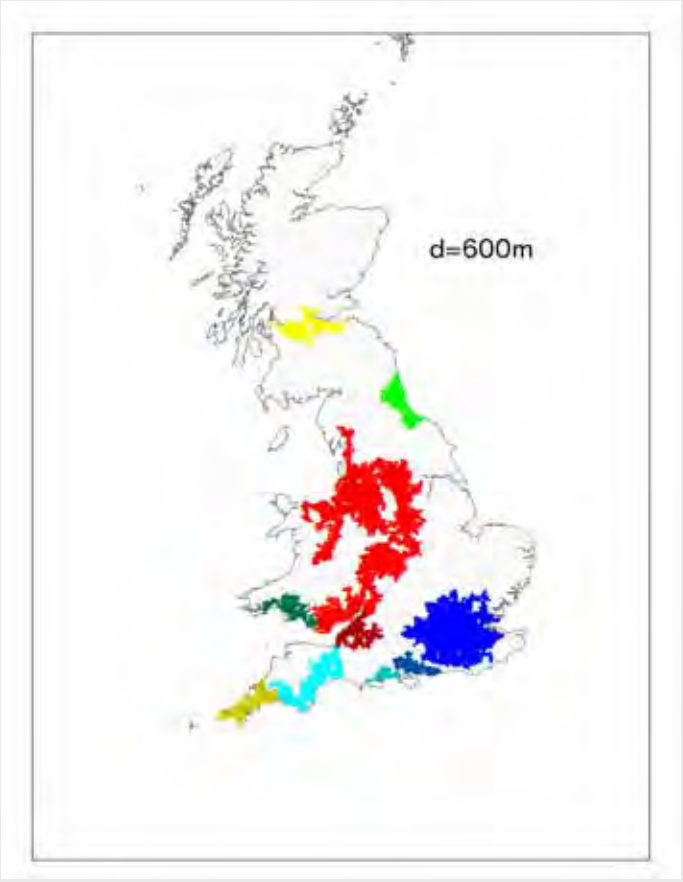

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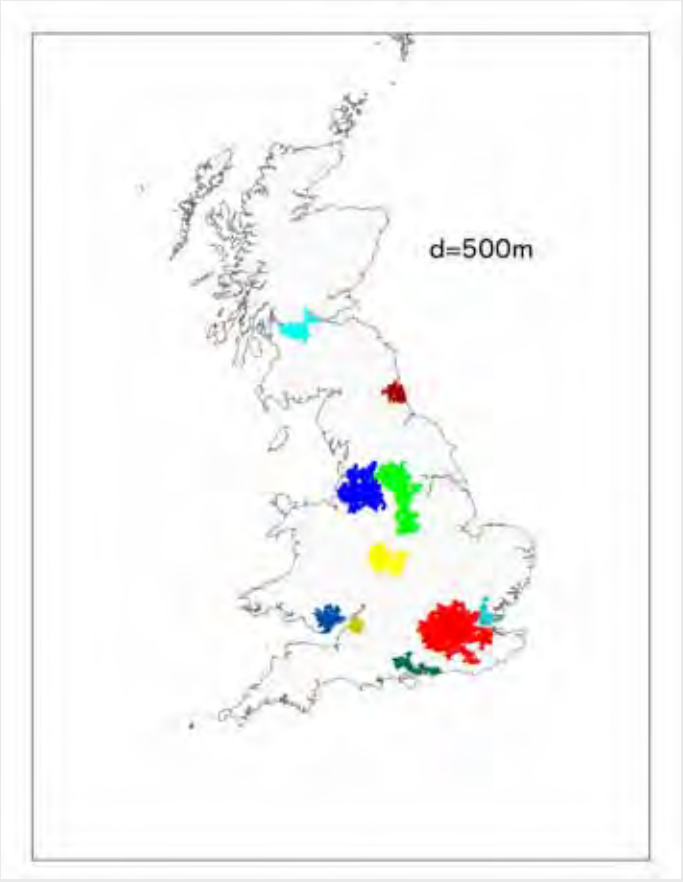

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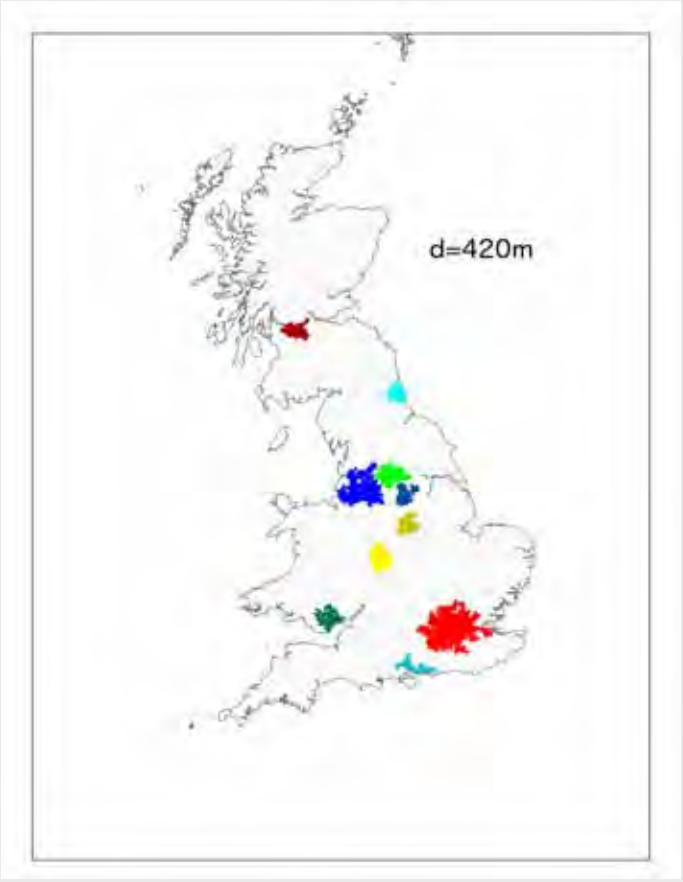

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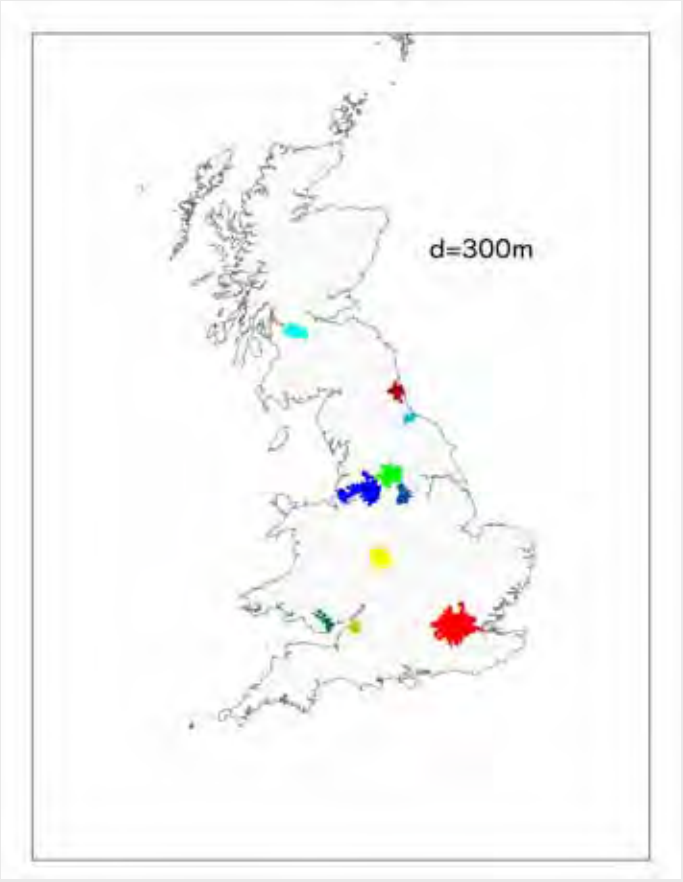

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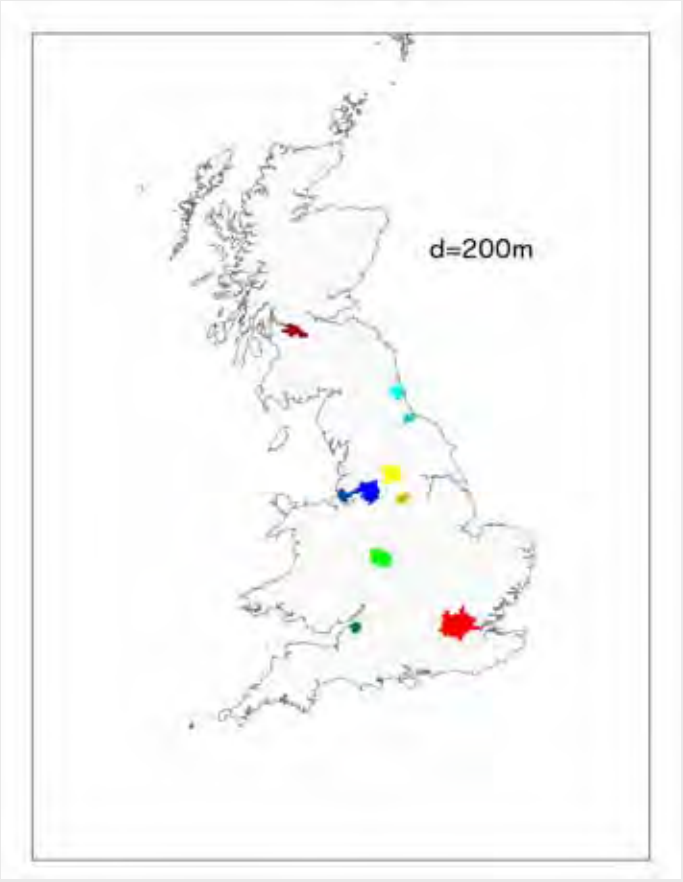

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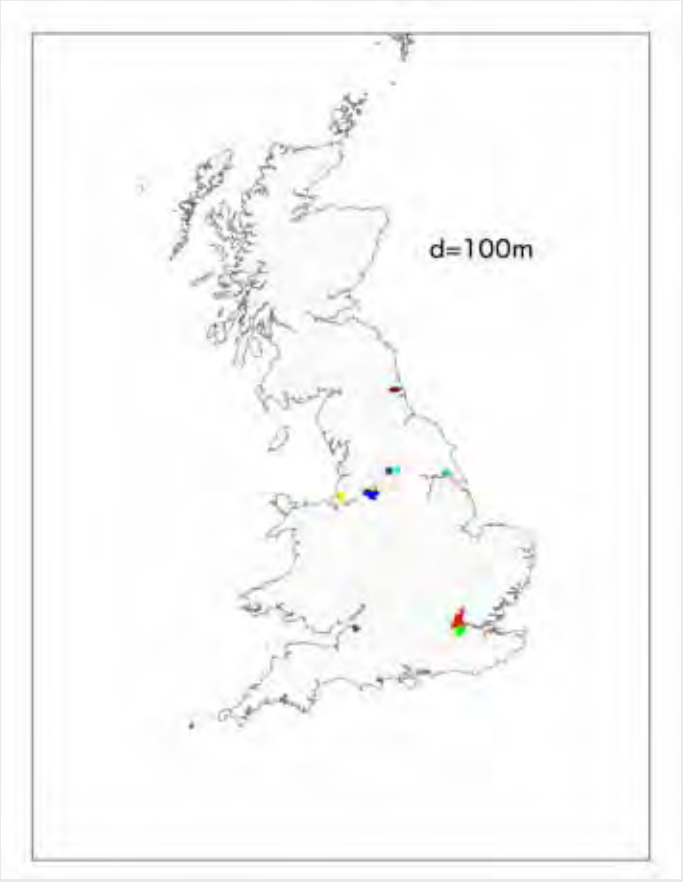
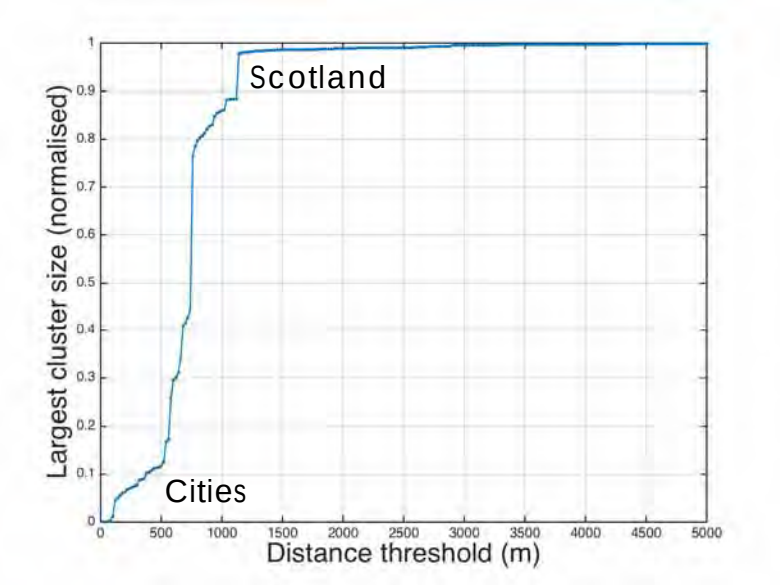
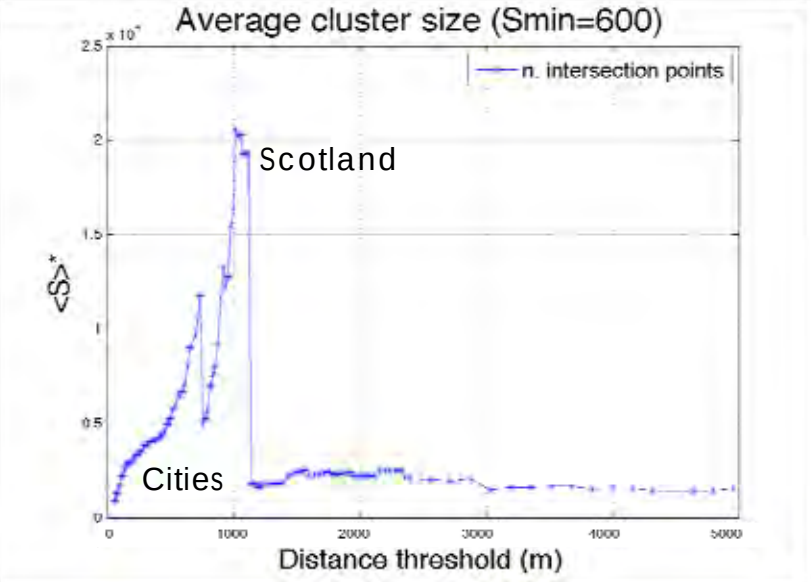



Scotland
Cities
Largest cluster size (normalised)
Distance threshold (m)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000


Average cluster size (Smin=600)
n. intersection points
Scotland
Cities
<S>*
Distance threshold (m)
0
0.5
1
1.5
2
2.5
0
1000
2000
3000
4000
5000

So what is this all about

We are testing a theory that is as old as the hills: and that is that as cities get bigger, they get more prosperous, *ceteris paribus*

This is due to Alfred Marshall of course who coined the term urban agglomeration, but it is also key to allometry which is the study of qualitative change in living things as they get bigger.

The people who are most into this are at Santa Fe. Geoff West and Luis Bettencourt in particular.

They have shown that using MSA data and definitions for 357 cities in the US, the income grows more than proportionately with city size; in short if Y_{city} is income and P_{city} is population, then

$$Y_{city} = kP_{city}^{\beta} \sim kP_{city}^{1.10}$$

This they call superlinear scaling $\beta > 1$

PNAS

Growth, innovation, scaling, and the pace of life in cities

Luis M. A. Bettencourt*†, José Lobo‡, Dirk Helbing§, Christian Kühnert§, and Geoffrey B. West*¶

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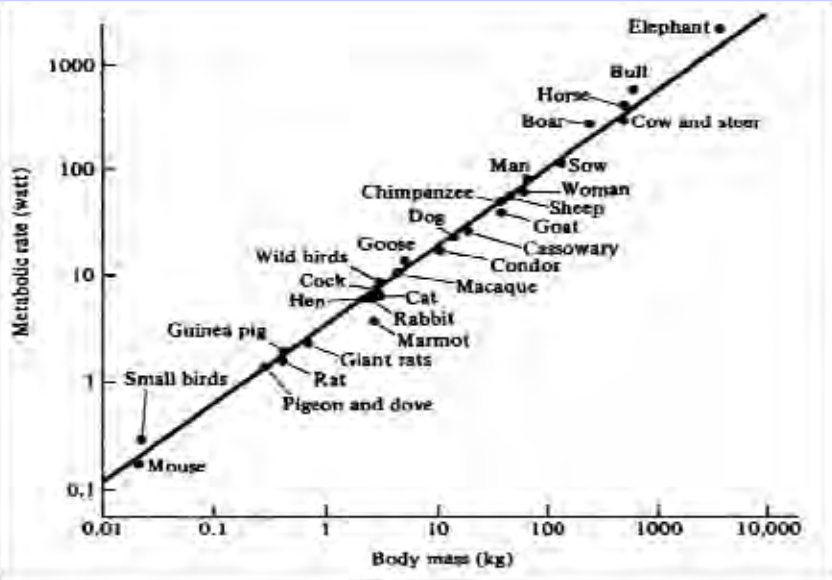
Edited by Simon Gittins, Indiana University, Bloomington, IN, and approved March 6, 2007 (received for review November 16, 2006)



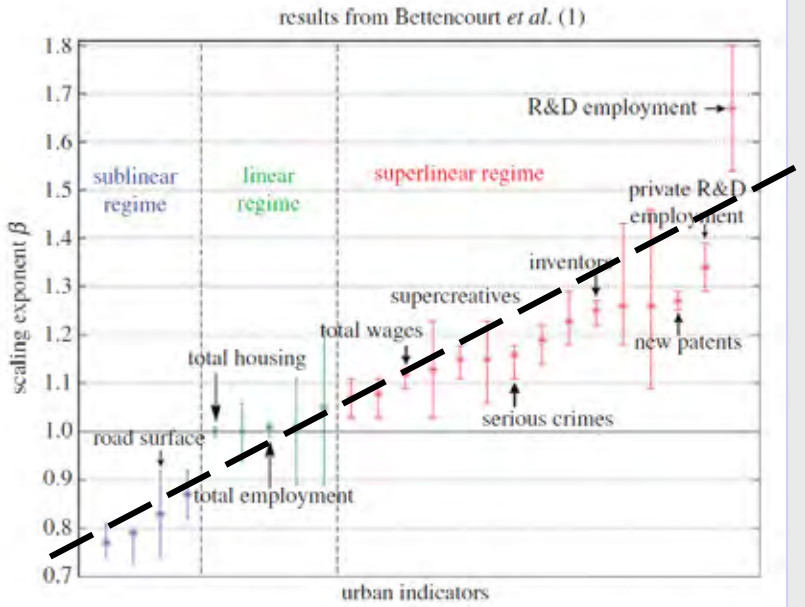


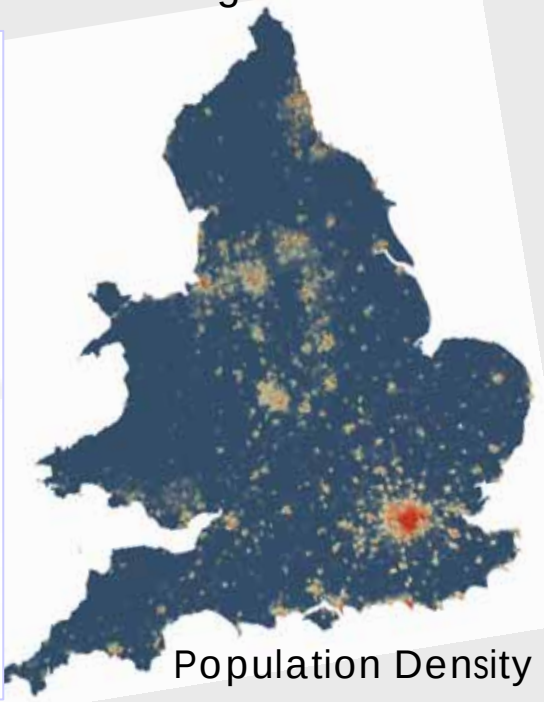
Figure 1. Exponents with 95% CI for different urban indicators found for the USA, Germany and China in reference [14]. These are colour-coded according

Our group considers the West-Bettencourt results to be problematic for the UK. We do not think that income scales superlinearly with population. We cannot reproduce this result from standard definitions of UK cities and we believe that the *globalisation of cities* destroys this kind of simplicity.

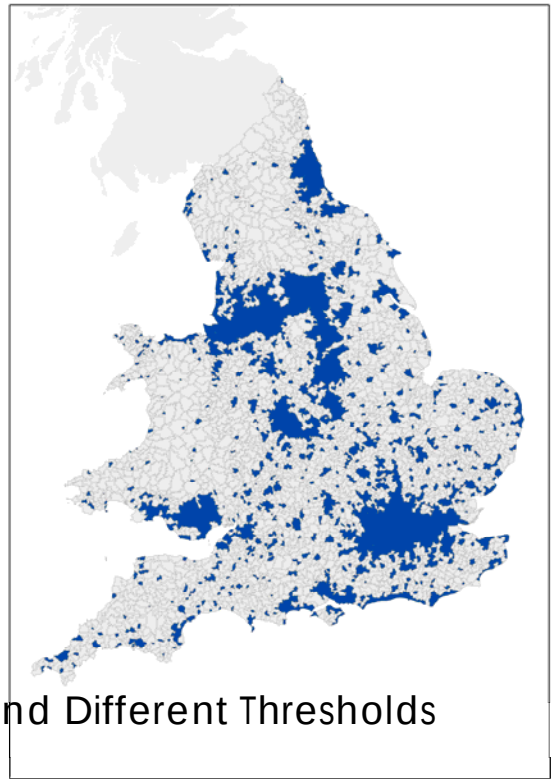
In short, everything turns on what we define as a city. In terms of MSAs, many are simply not good definitions of where people ***locate*** & particularly ***interact*** to do city things. So what we are doing is ***defining cities*** and to do this, we consider percolation theory a good potential approach. Let us begin however with some standard tests of this hypothesis for the UK or rather E & W

There are some poor definitions of cities in the EU such as LUZ – large urban zones and we should not use these but we will use density then commuting thresholds

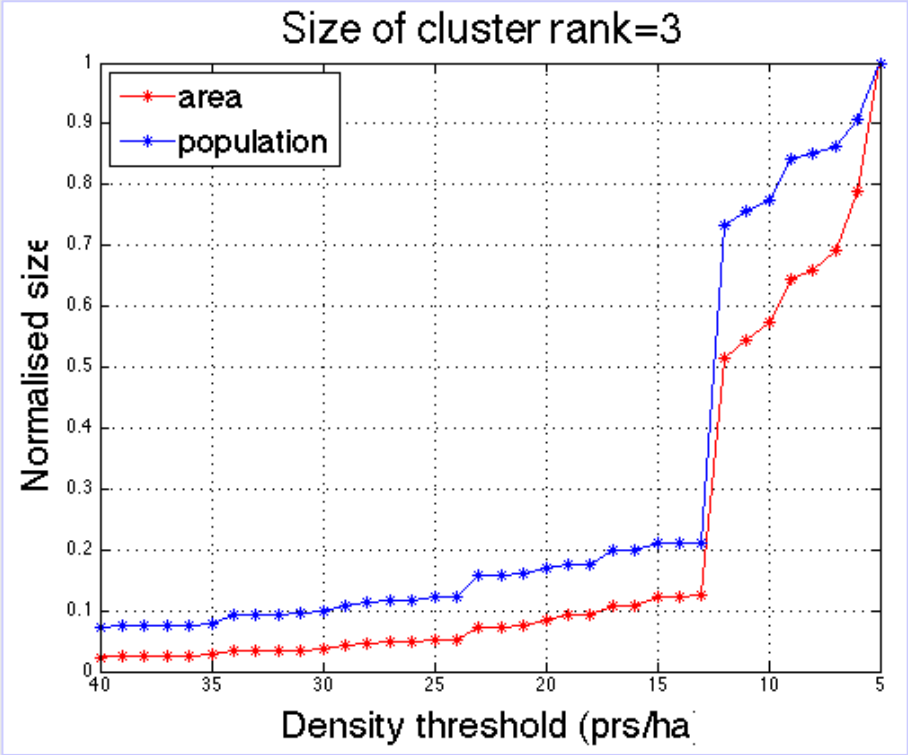
LUZs

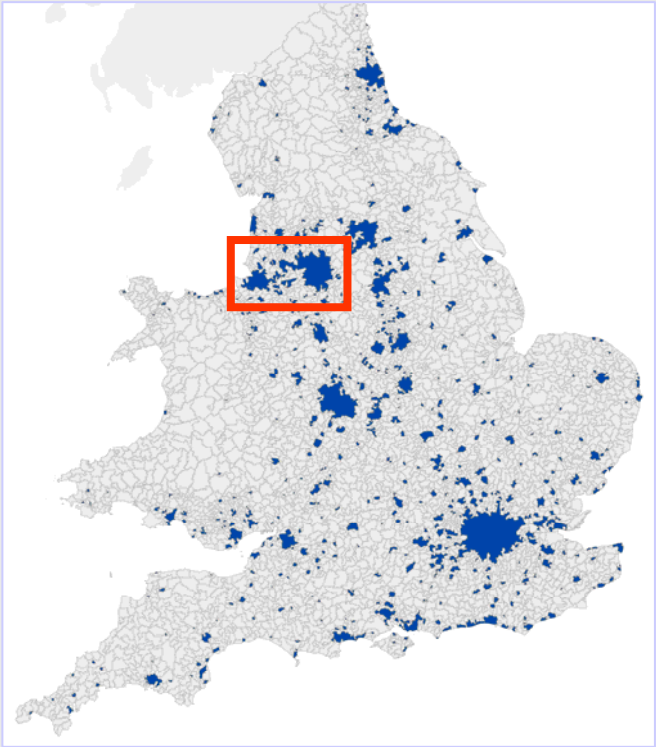


Population Density and Different Thresholds

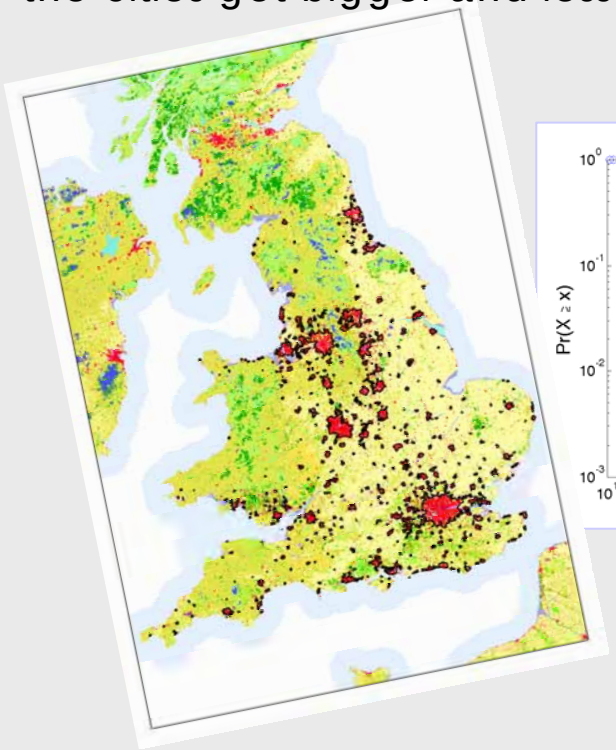


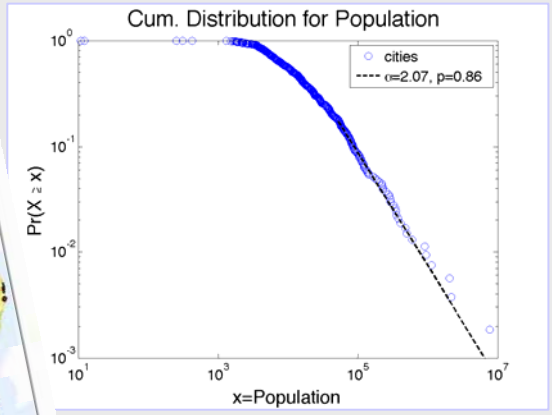
We consider 14persons per acre to be the right density cut-off ; here Liverpool and Manchester separate

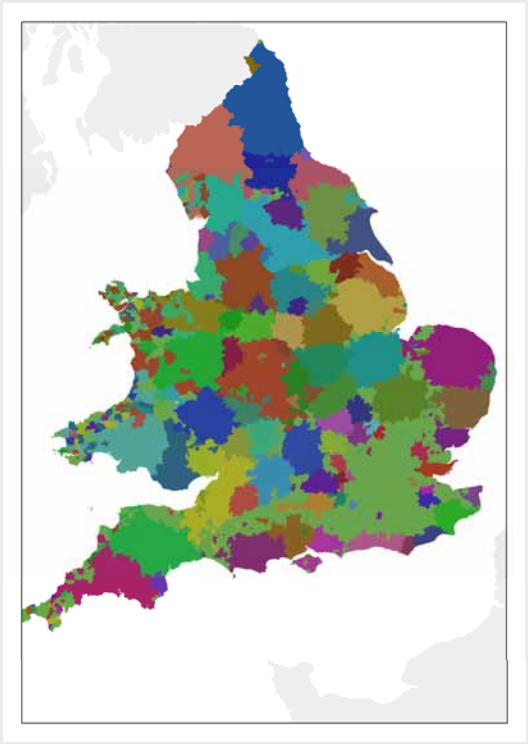




The correlation with the EU Corine RS data is good and Zipf's Law looks stable but we need to look at commuting fields – as they get larger around any place then the cities get bigger and less well-defined







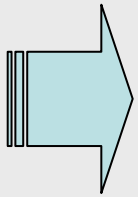
When we put all these definitions together defining cities at ***14pph and with the 30% commuting*** thresholds, and we examine a range of indicators wrt to city size, ***we do not get superlinear scaling*** for income against the population cities.

The bigger the city does not mean the richer in E&W and London is an enormous outlier.

We show these scaling relations and they are quite different from the Bettencourt-West results

We now need to do sensitivity analysis on these definitions varying the density ρ and commuting cut-offs τ and seeing what happens to β

$$\begin{array}{c} \\ \tau = 0 \\ \\ \\ \tau = 100 \end{array} \begin{array}{c} \begin{array}{cccc} \rho = 1 & & \ldots & \rho = 40 \end{array} \\ \begin{bmatrix} \beta_{11} & \beta_{12} & \ldots & \beta_{1n} \\ \beta_{21} & & & \\ & & & \\ \beta_{n1} & & \ldots & \beta_{nn} \end{bmatrix} \end{array}$$



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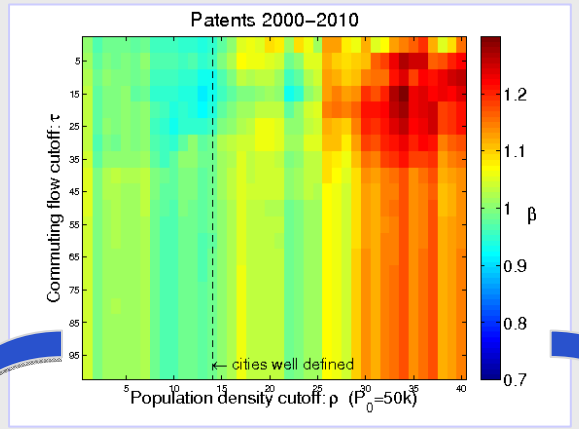
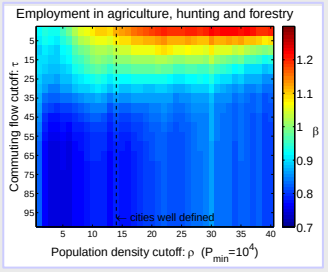
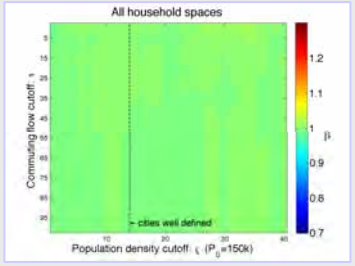
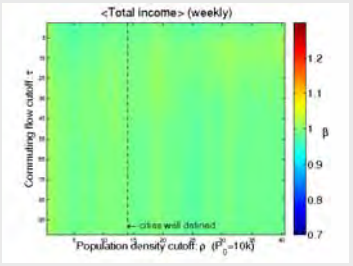
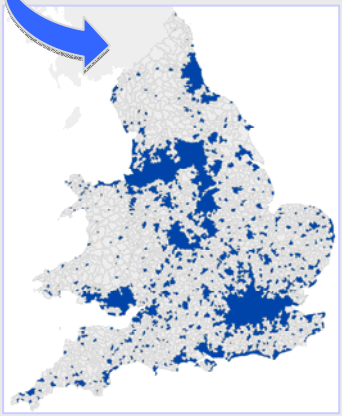
Warm colors – high and superlinear are reds to colder colors – low and sublinear blue

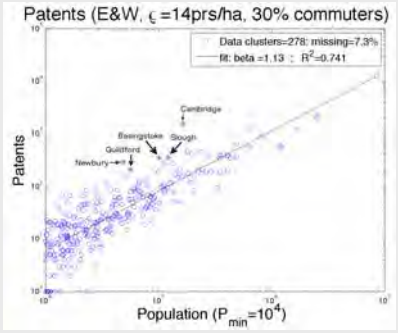

Patents 2000–2010
Commuting flow cutoff: τ
← cities well defined
Population density cutoff: ρ (P0=50k)
β

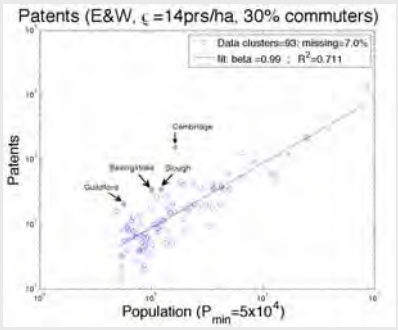

Employment in agriculture, hunting and forestry
Commuting flow cutoff: τ
← cities well defined
Population density cutoff: ρ (Pmin=10^4)
β


All household spaces
Commuting flow cutoff: τ
← cities well defined
Population density cutoff: ρ (P0=150k)
β

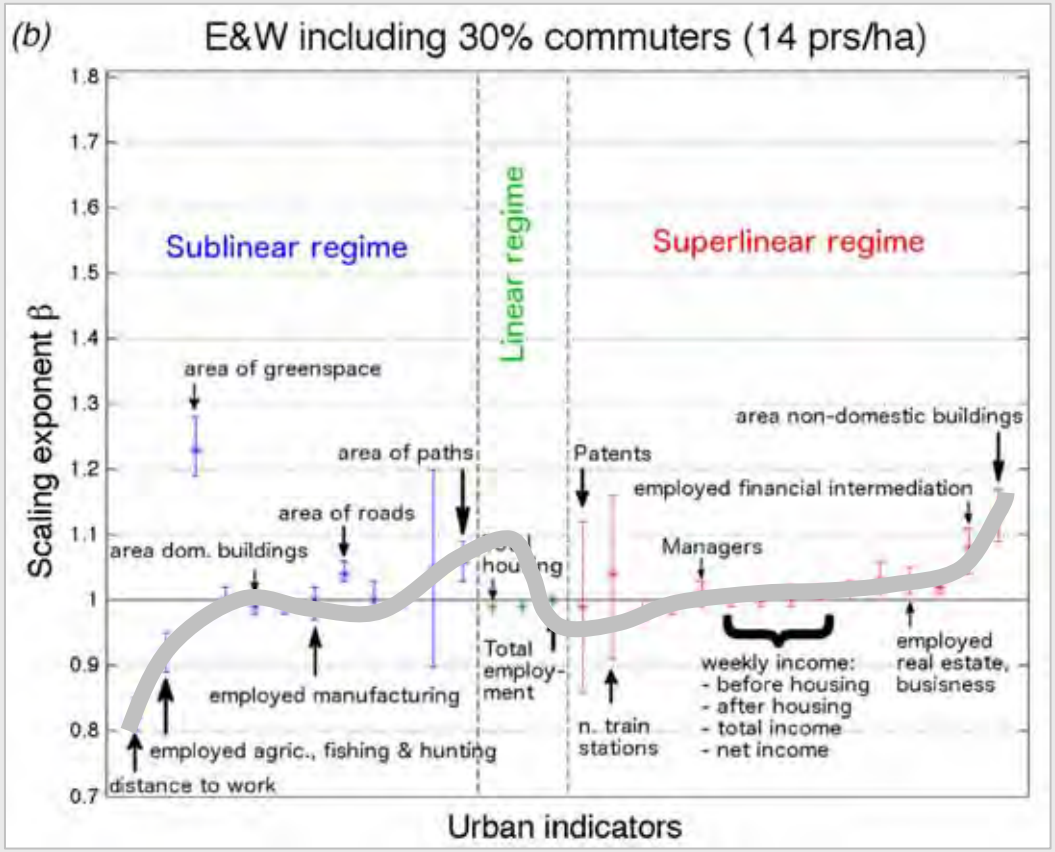

<Total income> (weekly)
Commuting flow cutoff: τ
← cities well defined
Population density cutoff: ρ (P0=10k)
β



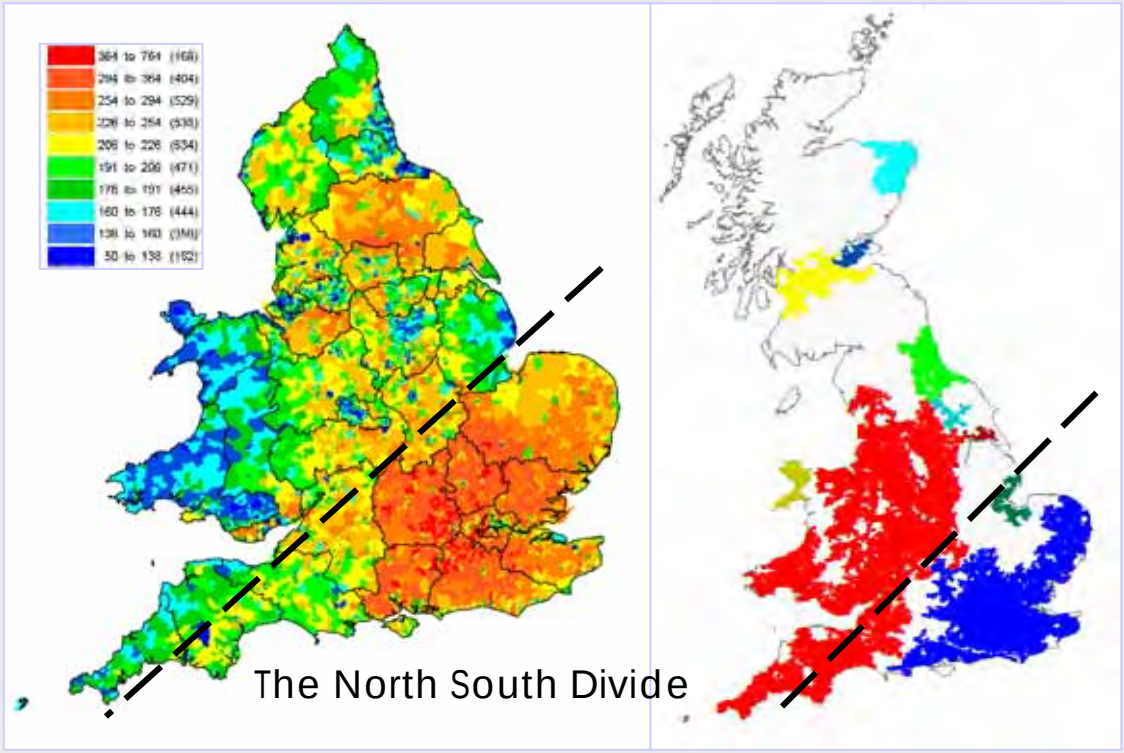

Patents (E&W, ρ =14prs/ha, 30% commuters)
Data clusters=278; missing=7.3%
fit: beta =1.13 ; R2=0.741
Patents
Cambridge
Basingstoke
Slough
Guildford
Newbury
Population (Pmin=10^4)


Patents (E&W, ρ =14prs/ha, 30% commuters)
Data clusters=93; missing=7.0%
fit: beta =0.99 ; R2=0.711
Patents
Cambridge
Basingstoke
Slough
Guildford
Population (Pmin=5x10^4)

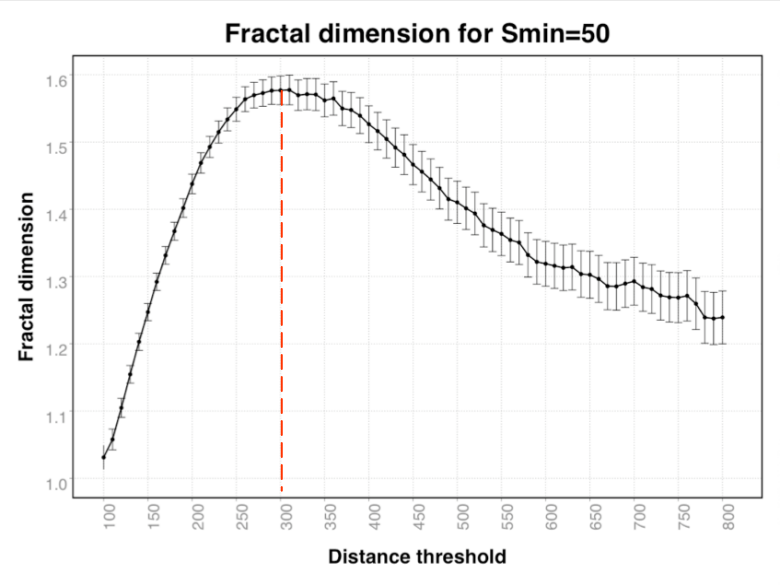
Here is the regime equivalent to the West-Bettencourt analysis and as you the linearity of the indicators is not what the W-B results for the USA imply

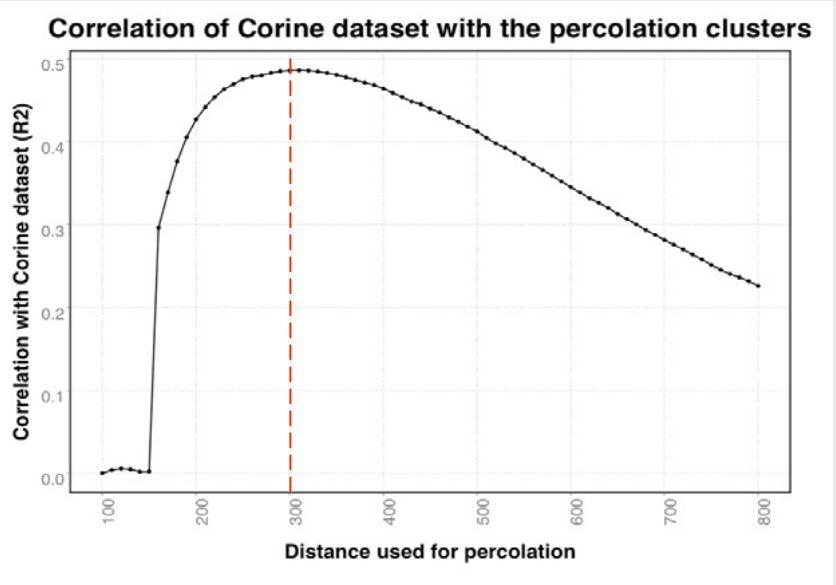


A little more on percolation ..... From the earlier work



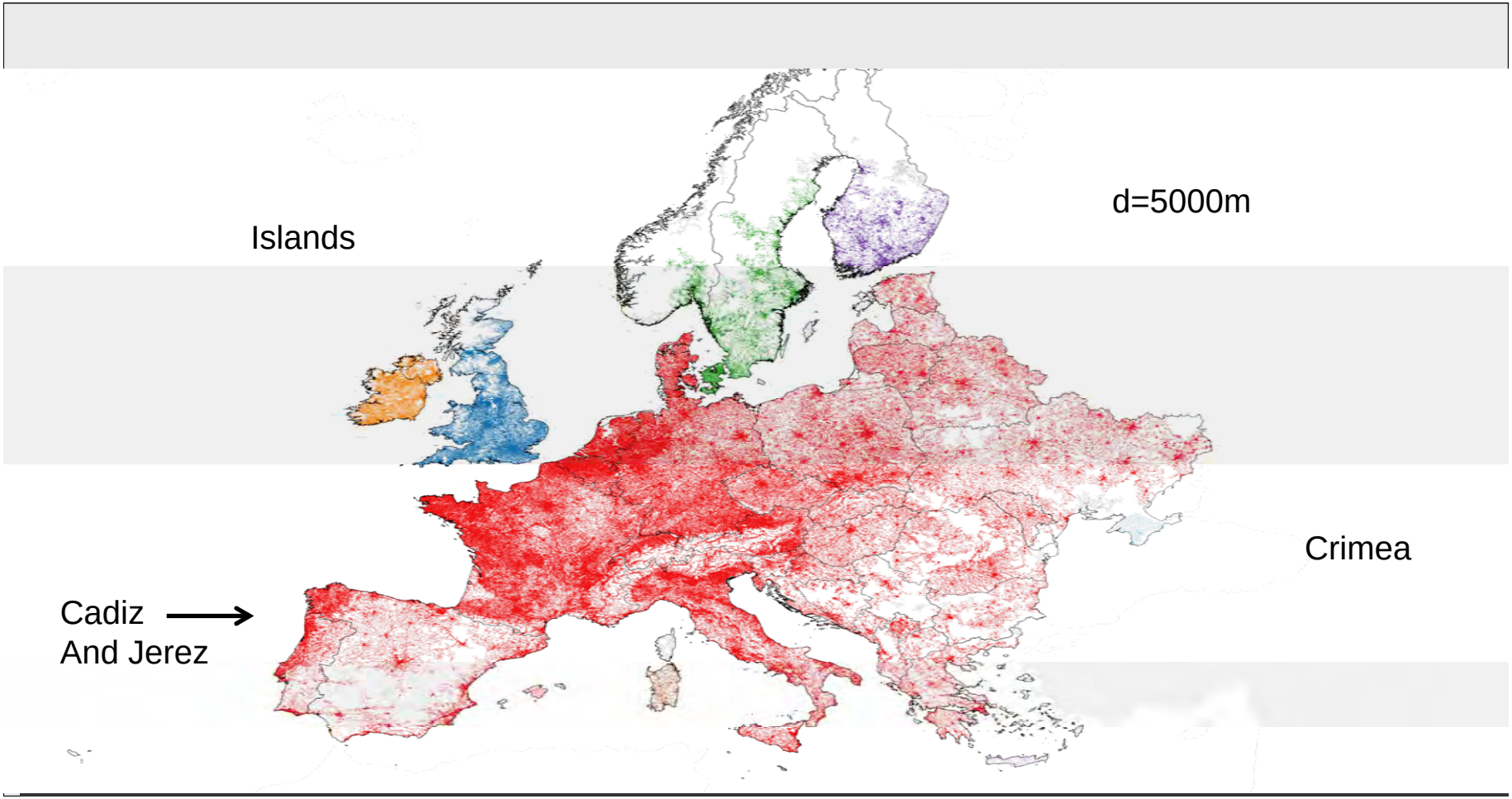
Defining cities: fractal dimension – maximum space filling; and correlation with RS imagery interpretations

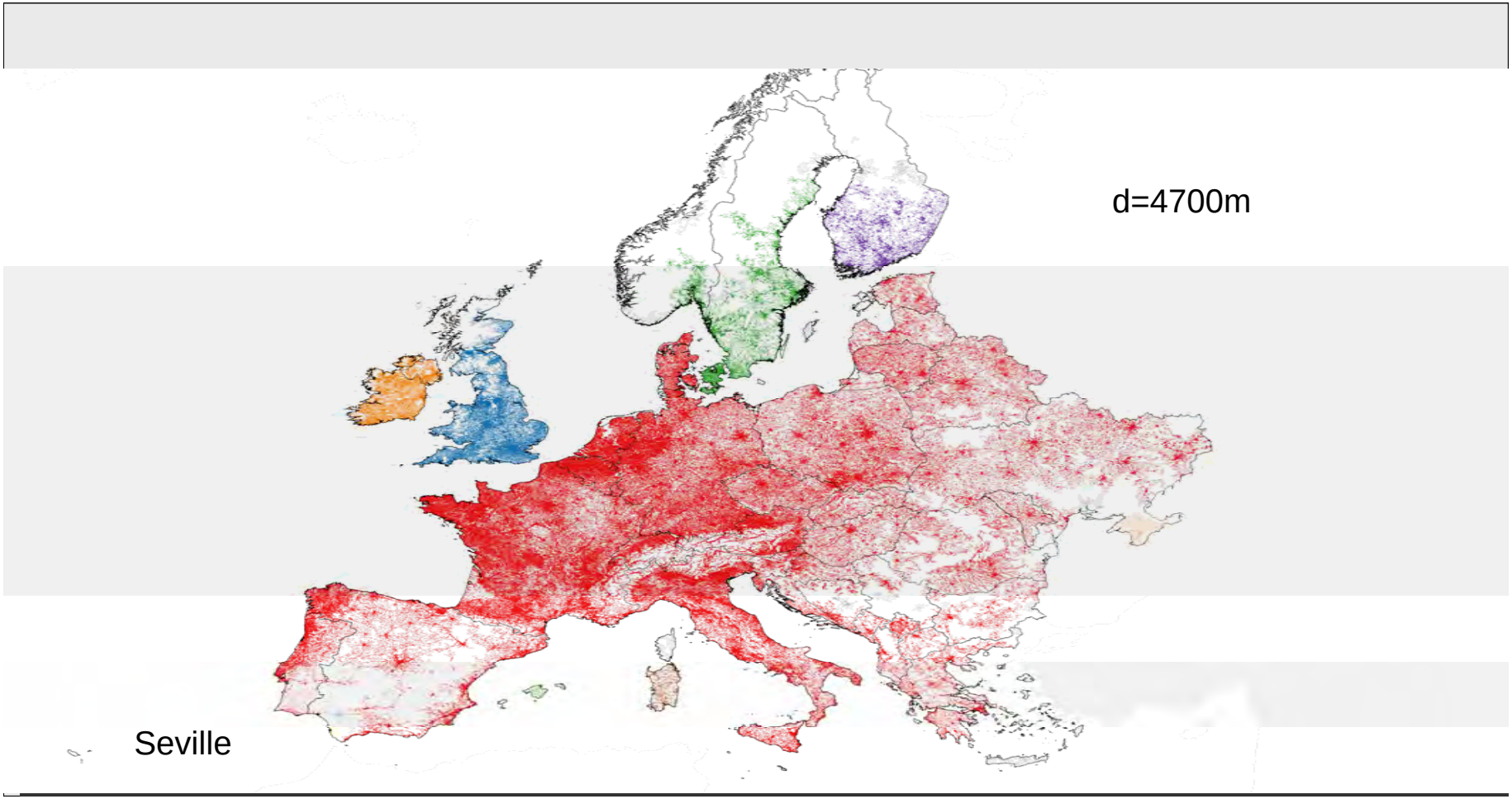


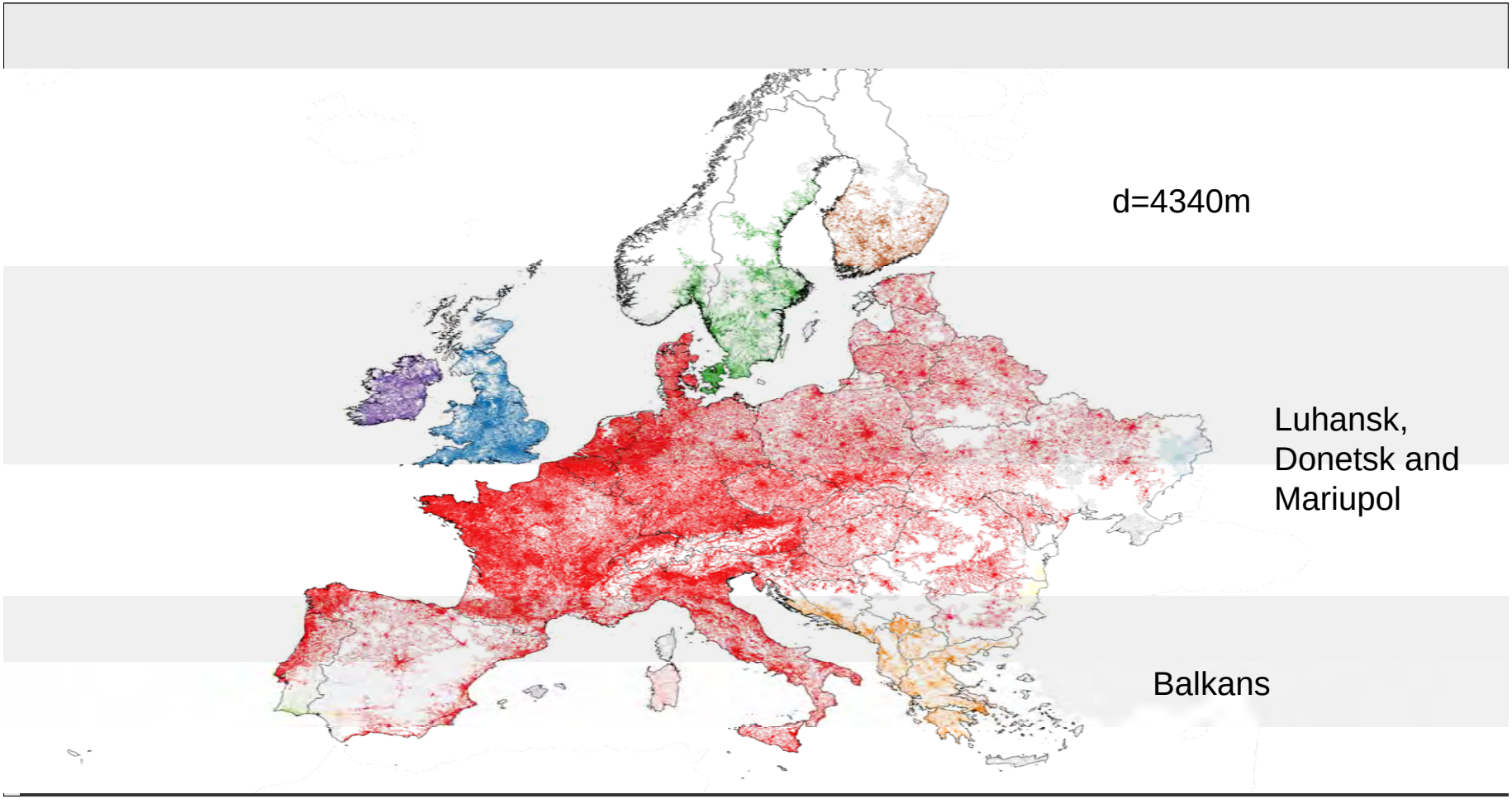


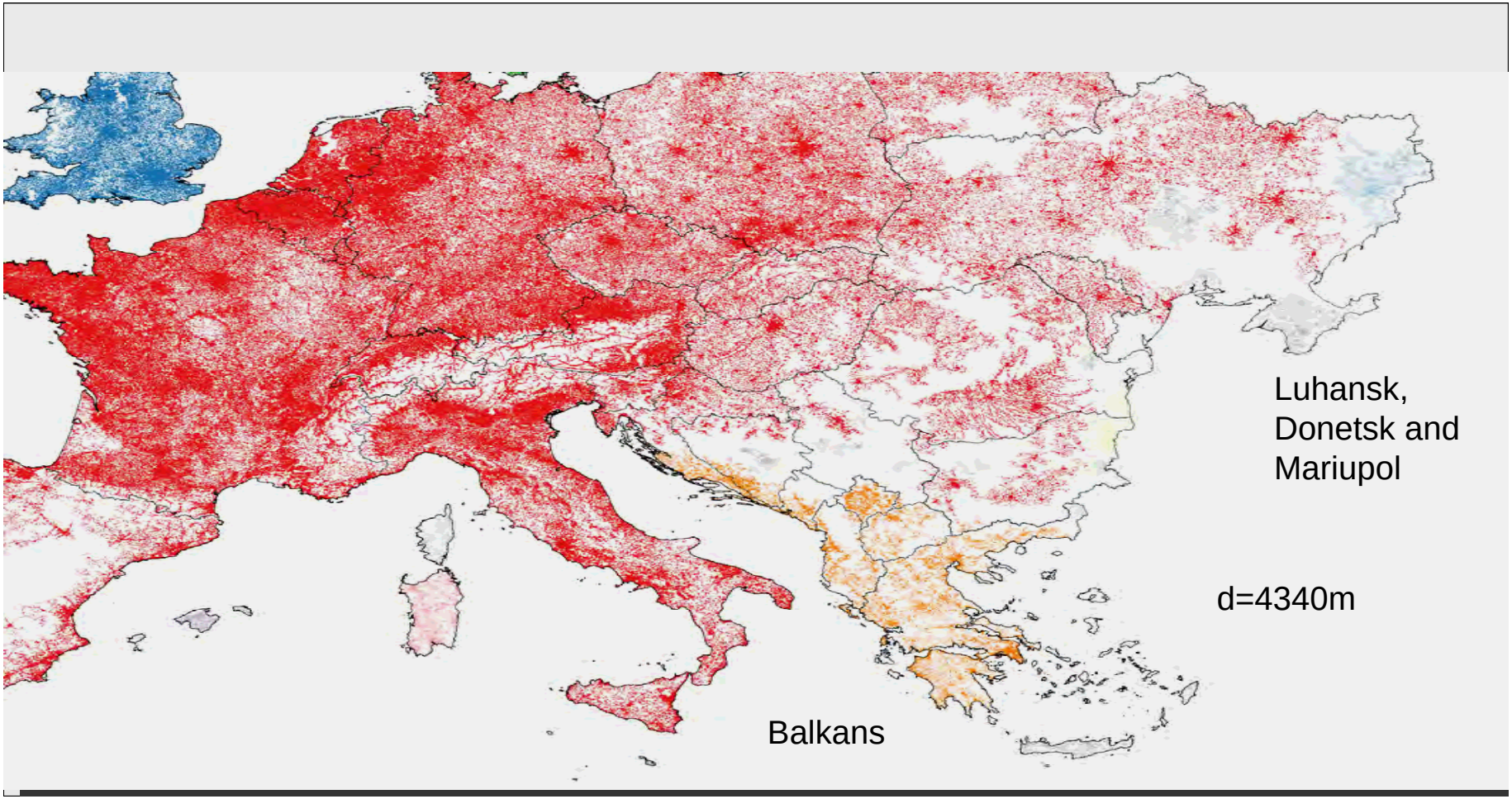
What we now want to do is:

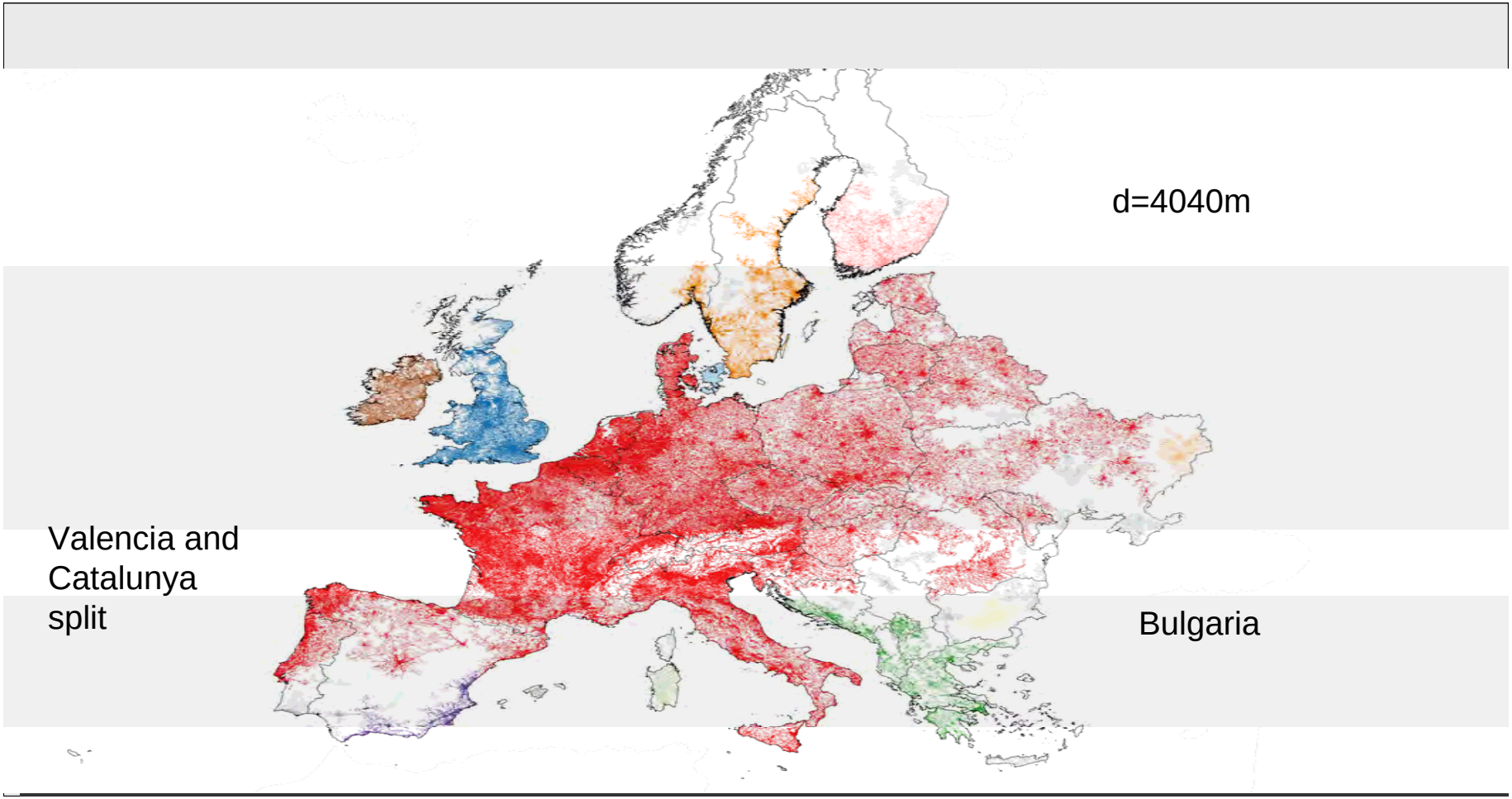
- Develop percolation theory for defining cities and although we have not yet reproduced the E&W results for a definition of cities based on percolation – we know that the clusters at a certain level are similar to density/commuting thresholds
- Do the same for Europe and America – I will show Europe
- Work out new measures of performance and extend the percolation to weighted measures
- Begin to do this at the intra-urban level – to define how cities are formed and evolve from the bottom up – I will show preliminary results for London

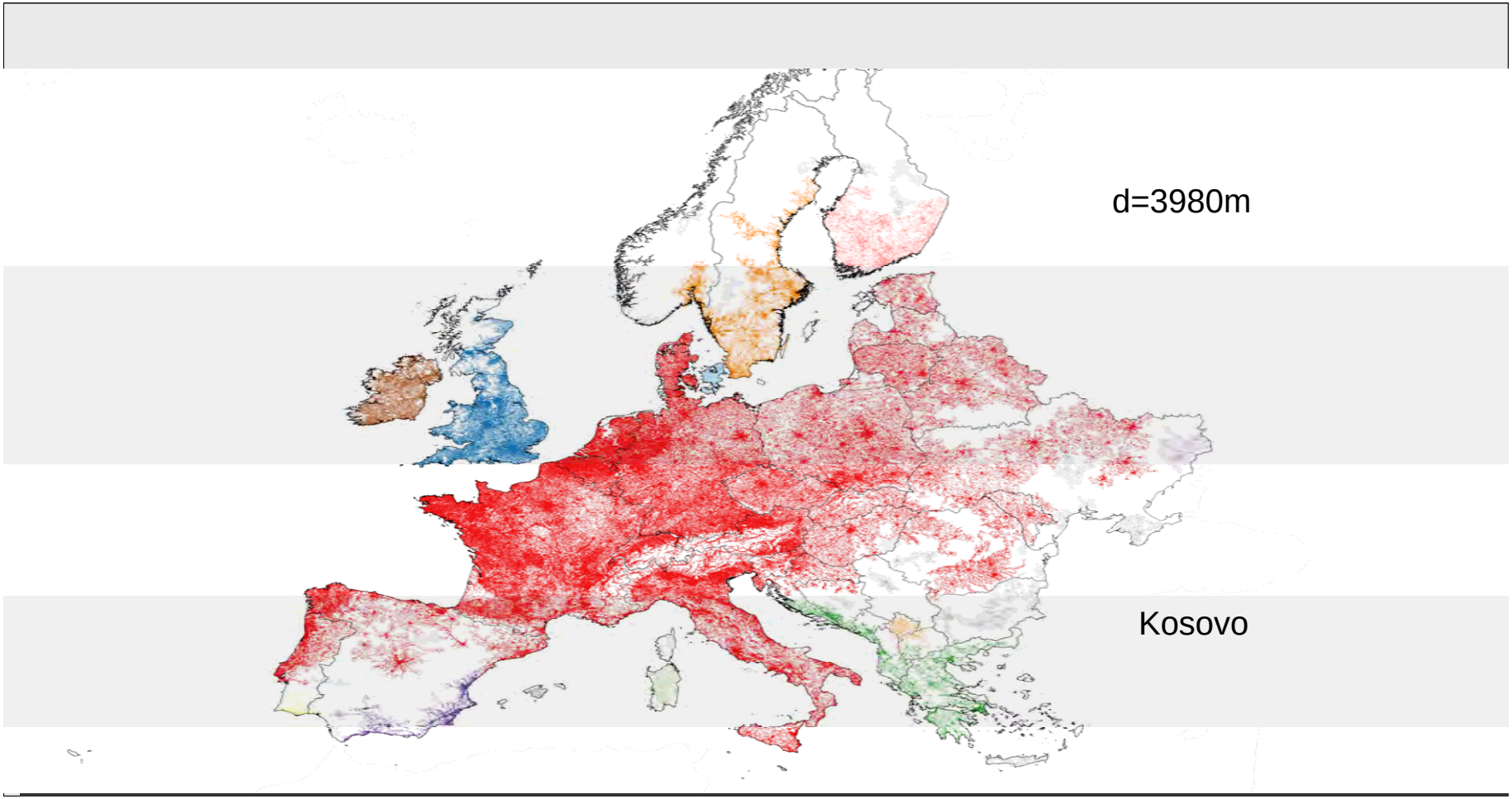

d=5000m
Islands
Crimea
Cadiz
And Jerez

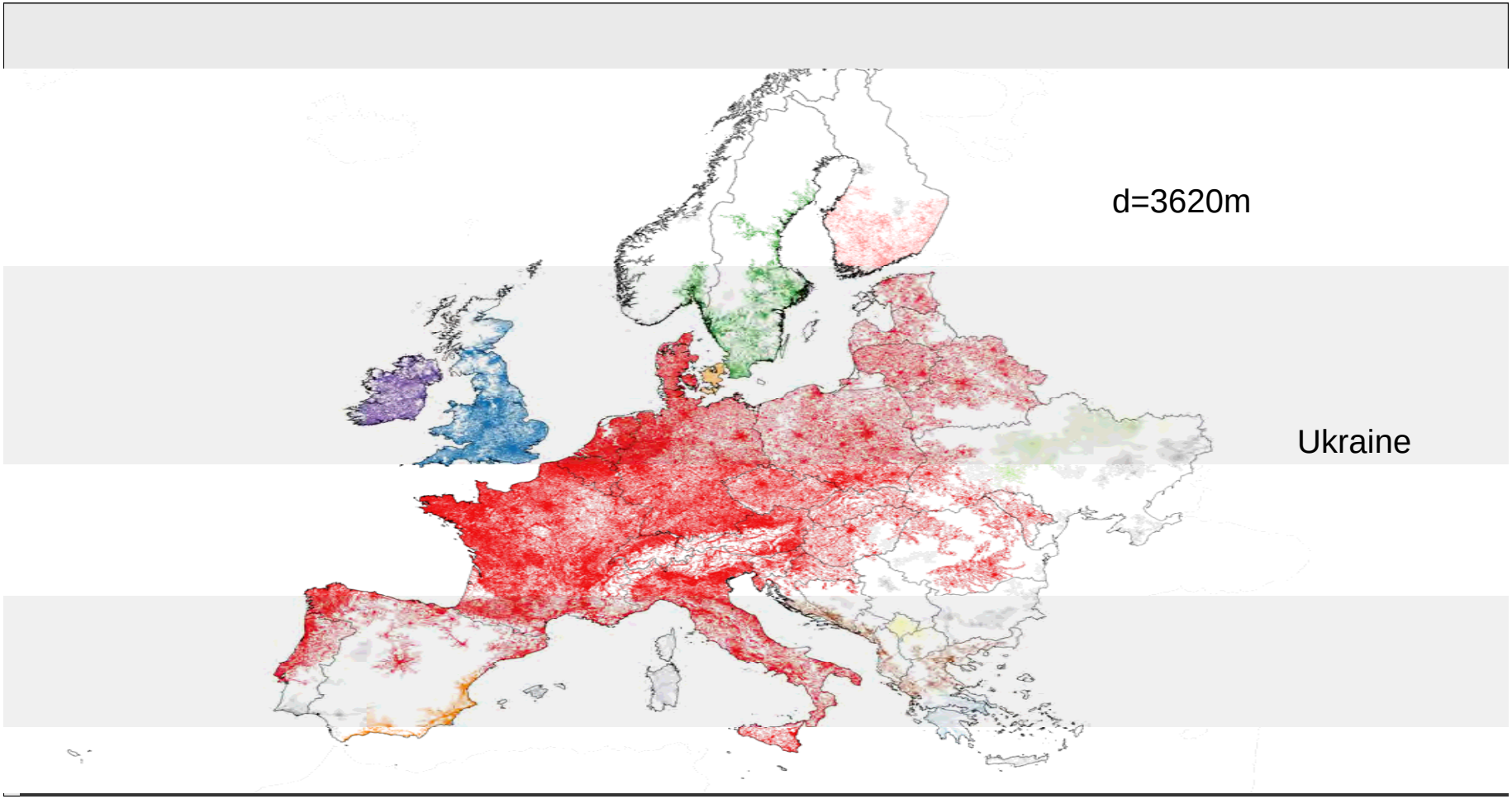

d=4700m
Seville

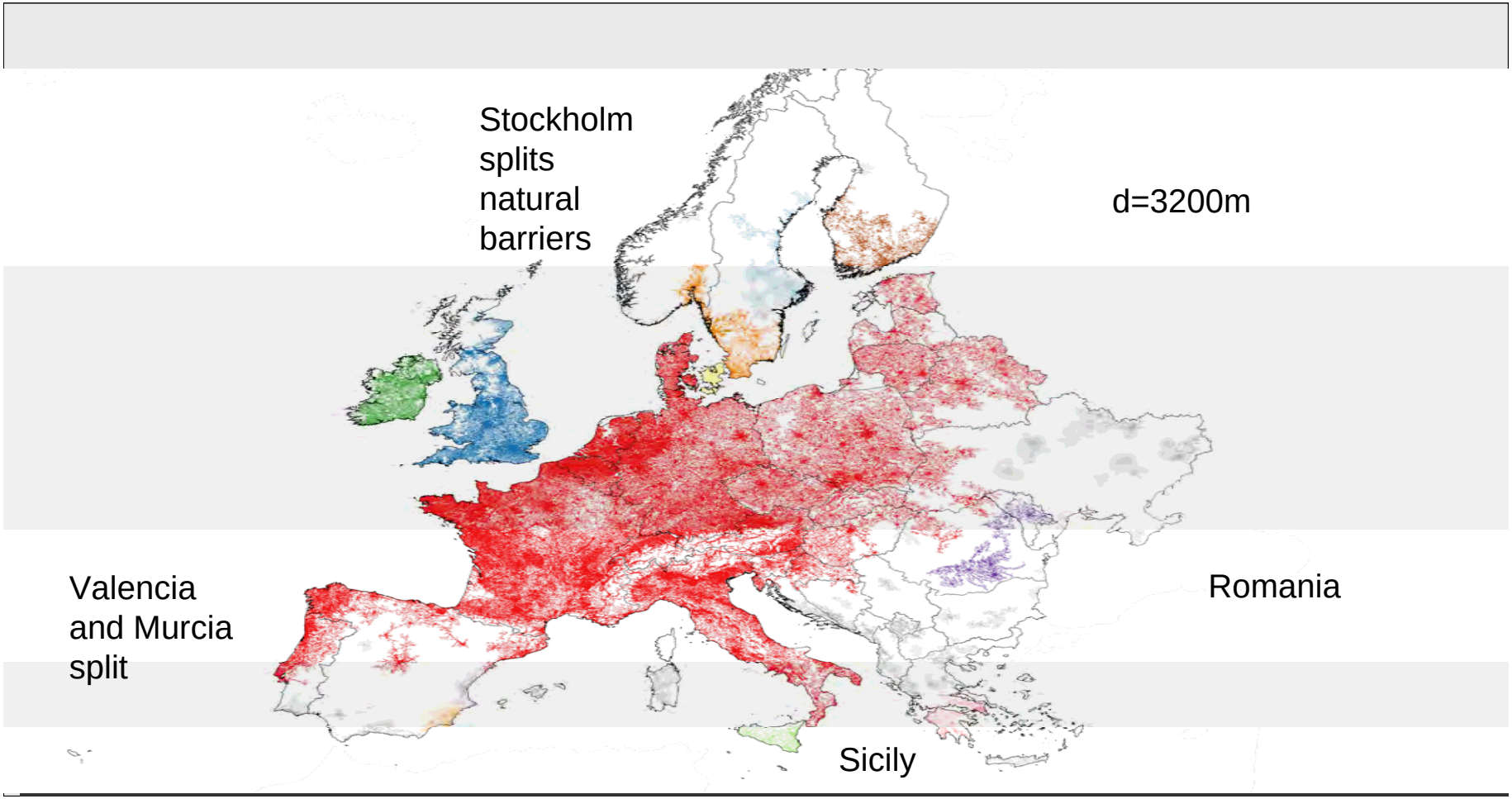

d=4340m
Luhansk,
Donetsk and
Mariupol
Balkans

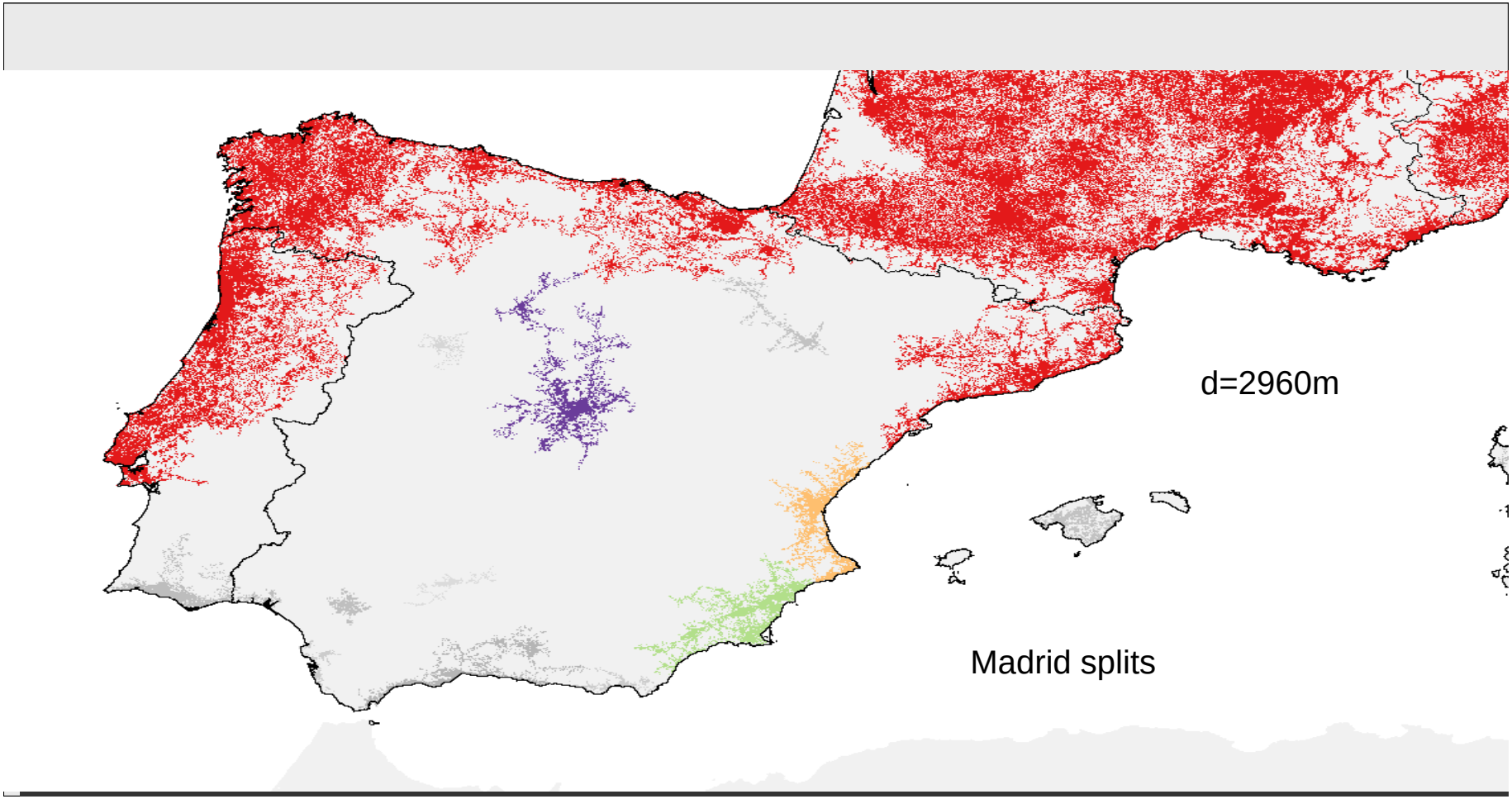

Luhansk,
Donetsk and
Mariupol
d=4340m
Balkans

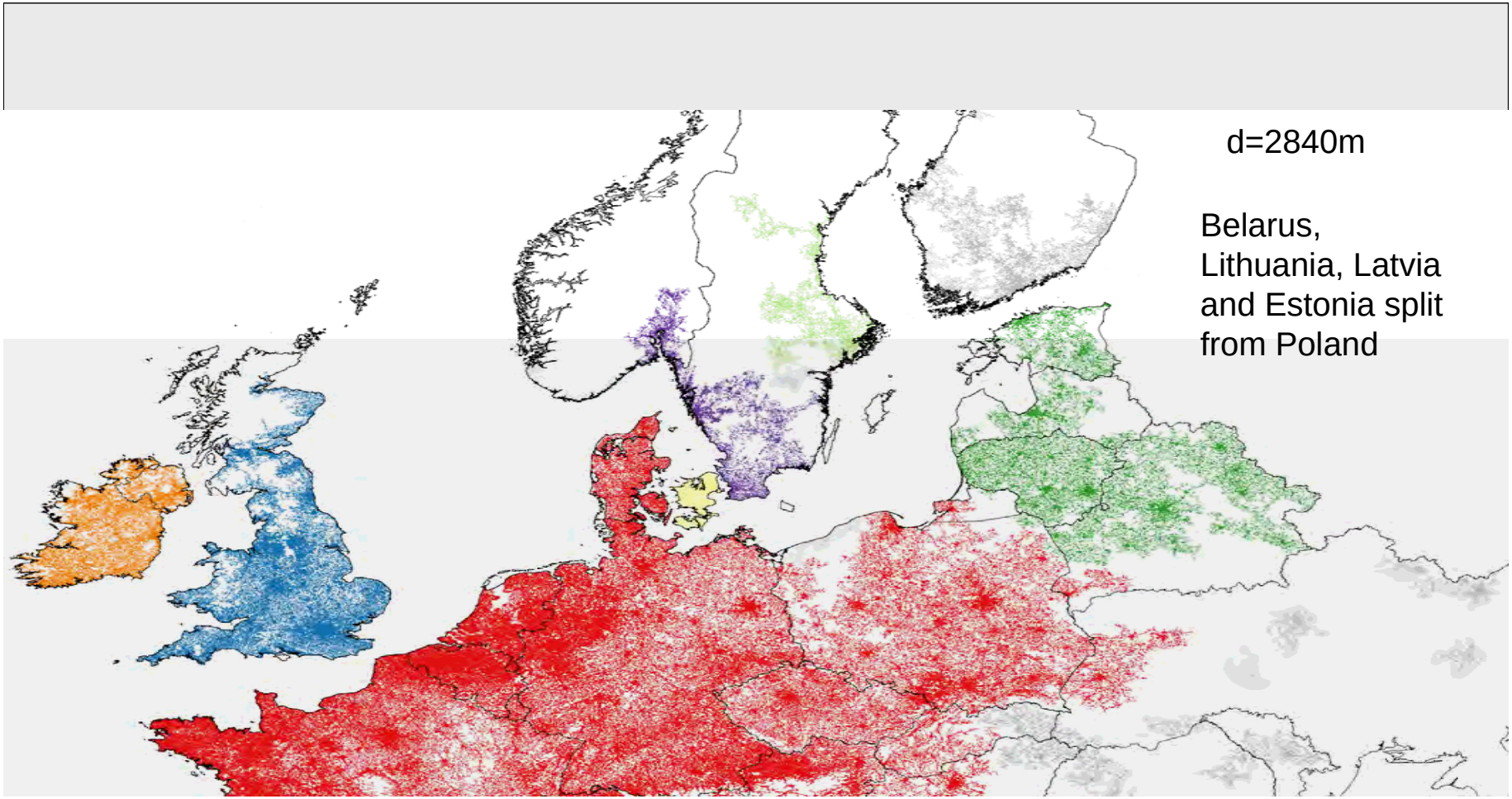

d=4040m
Valencia and
Catalunya
split
Bulgaria

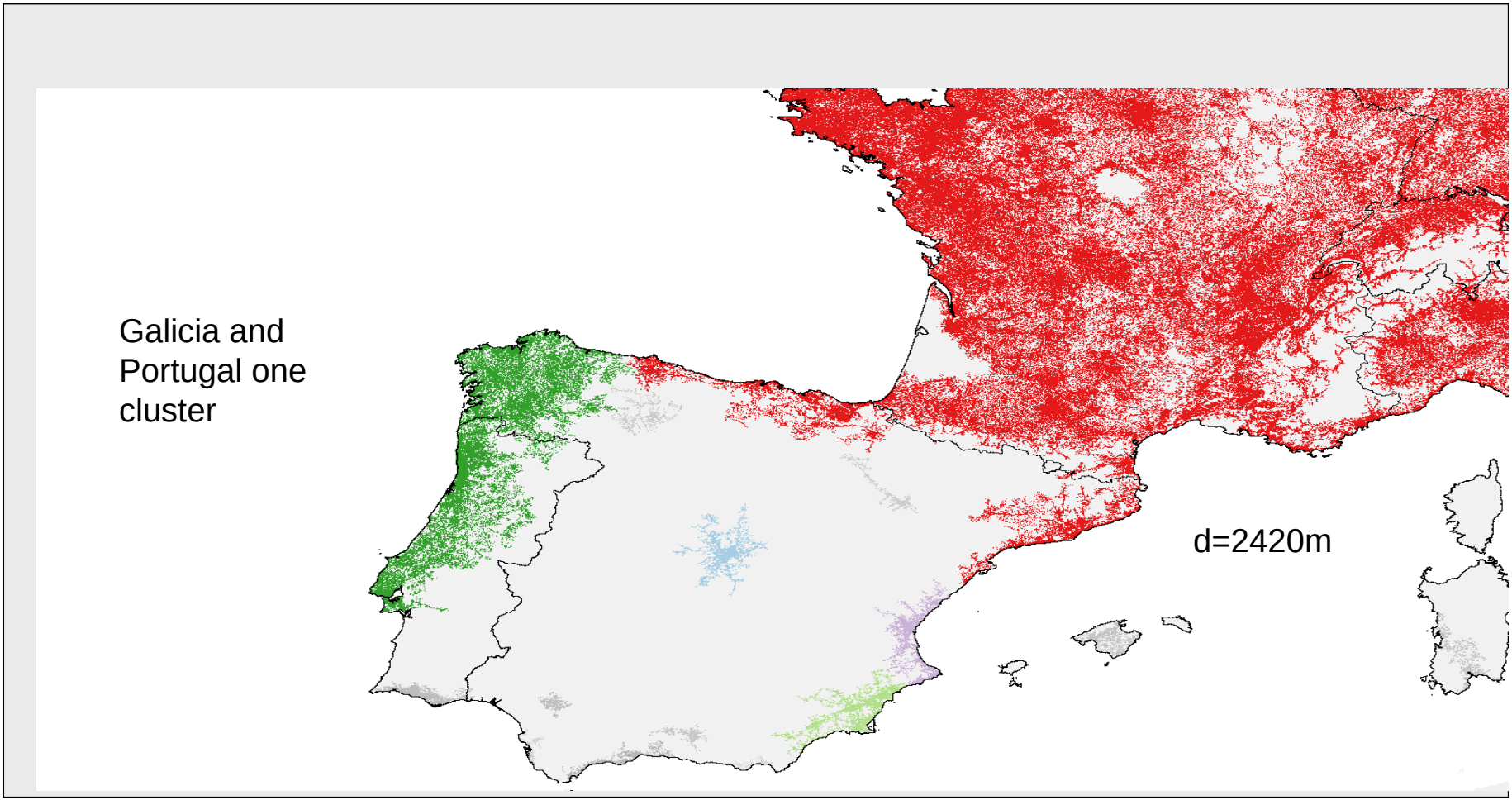

d=3980m
Kosovo

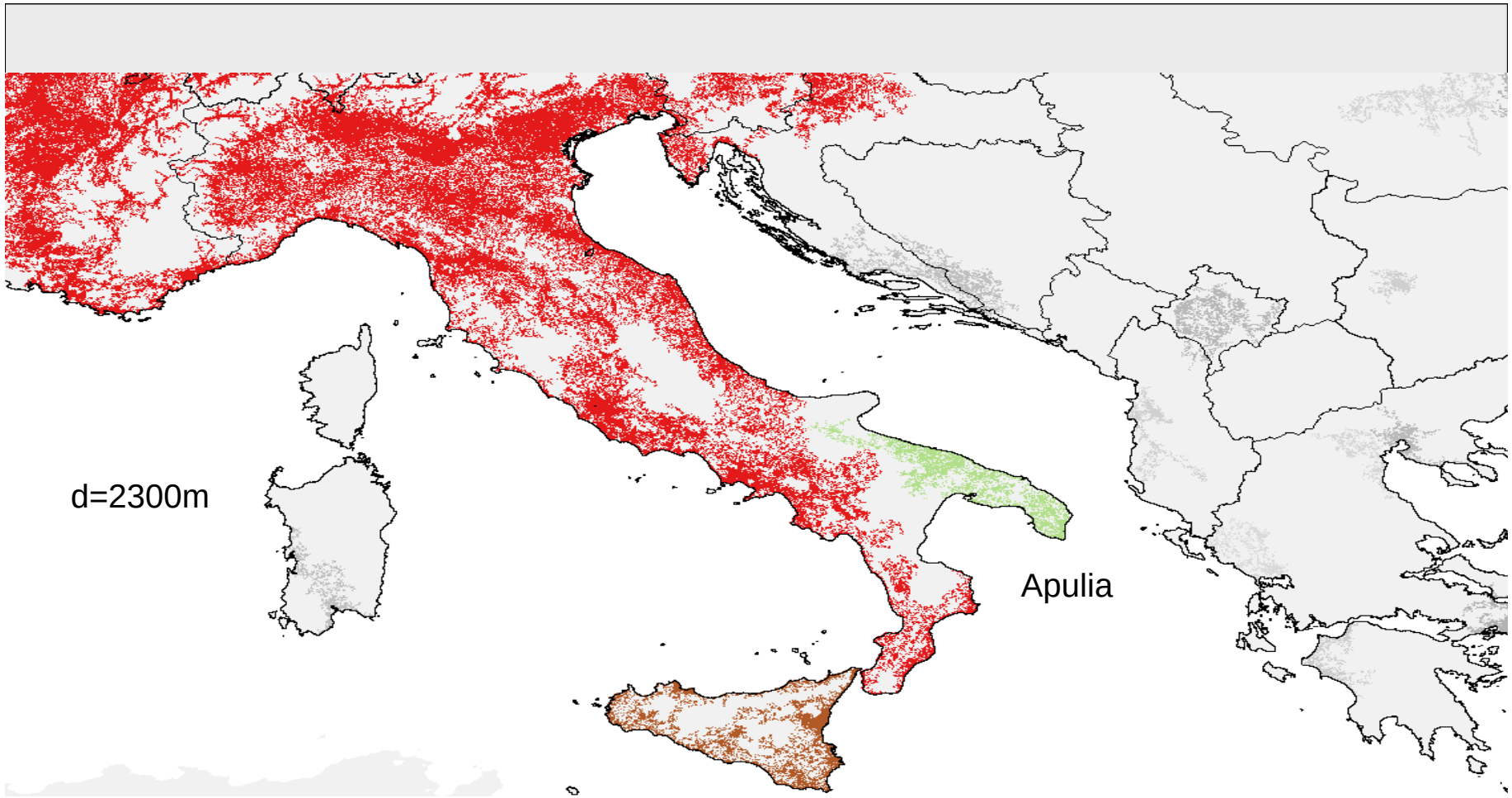

d=3620m
Ukraine

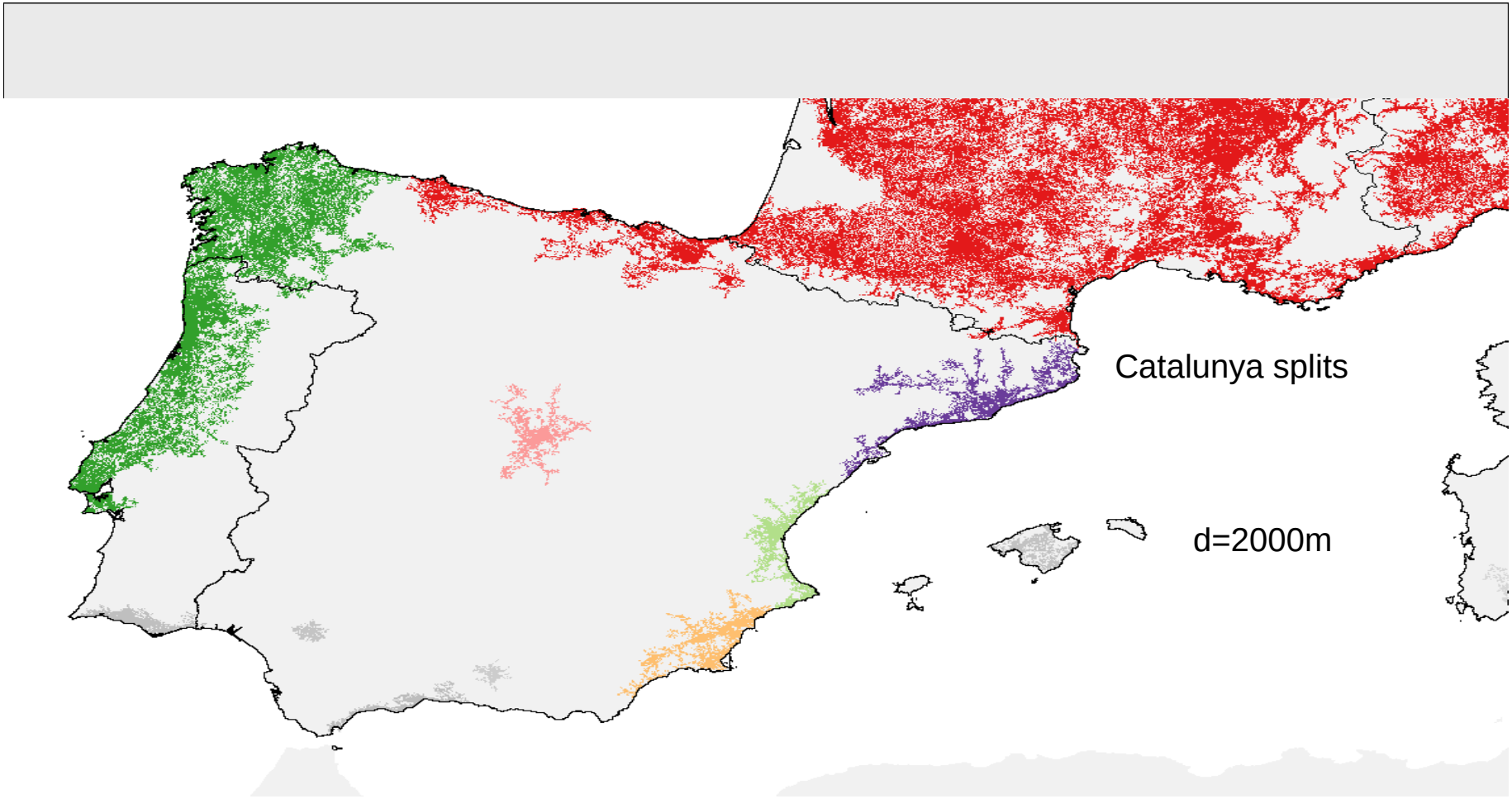

Stockholm
splits
natural
barriers
d=3200m
Valencia
and Murcia
split
Romania
Sicily

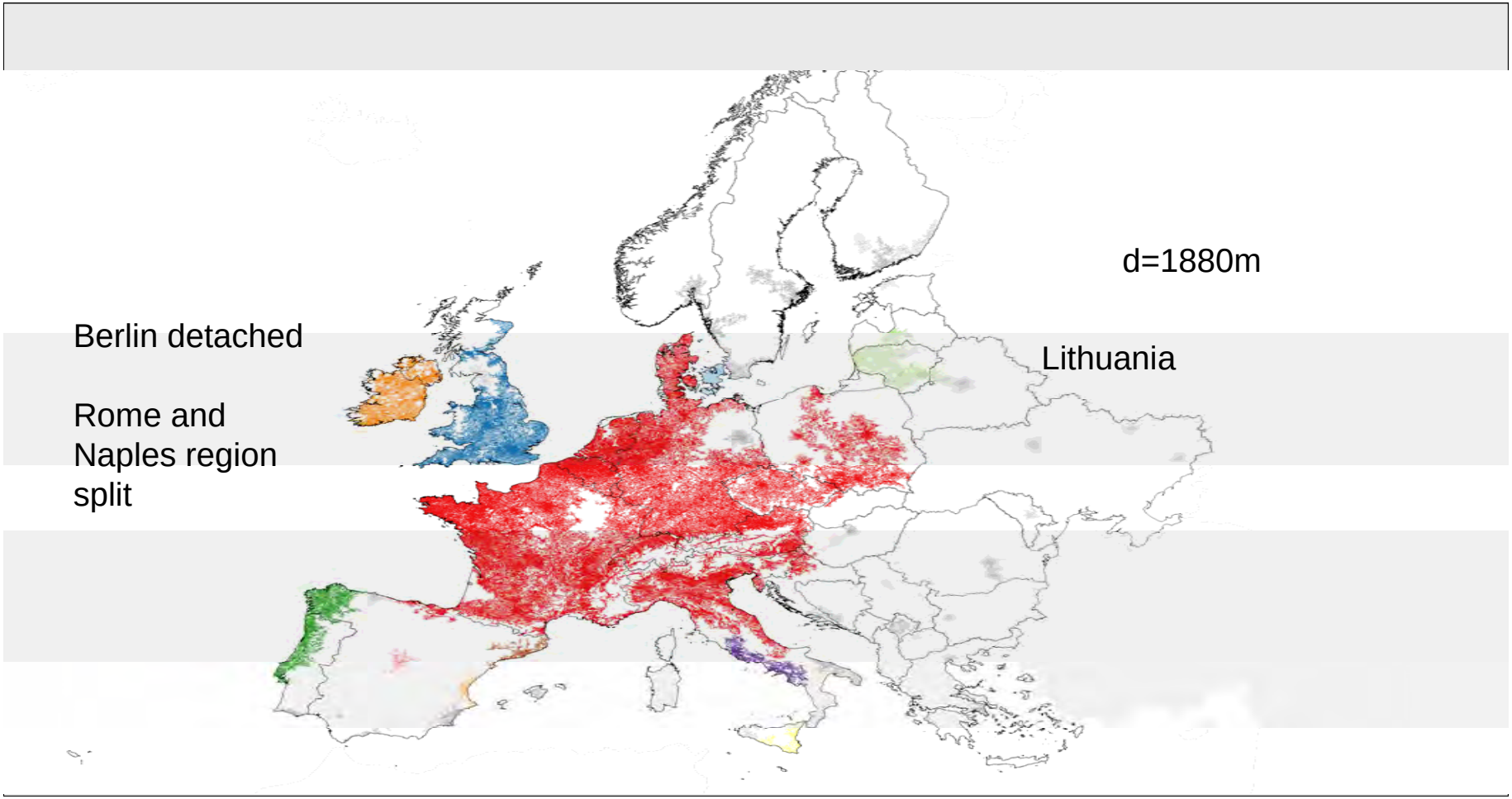

d=2960m
Madrid splits

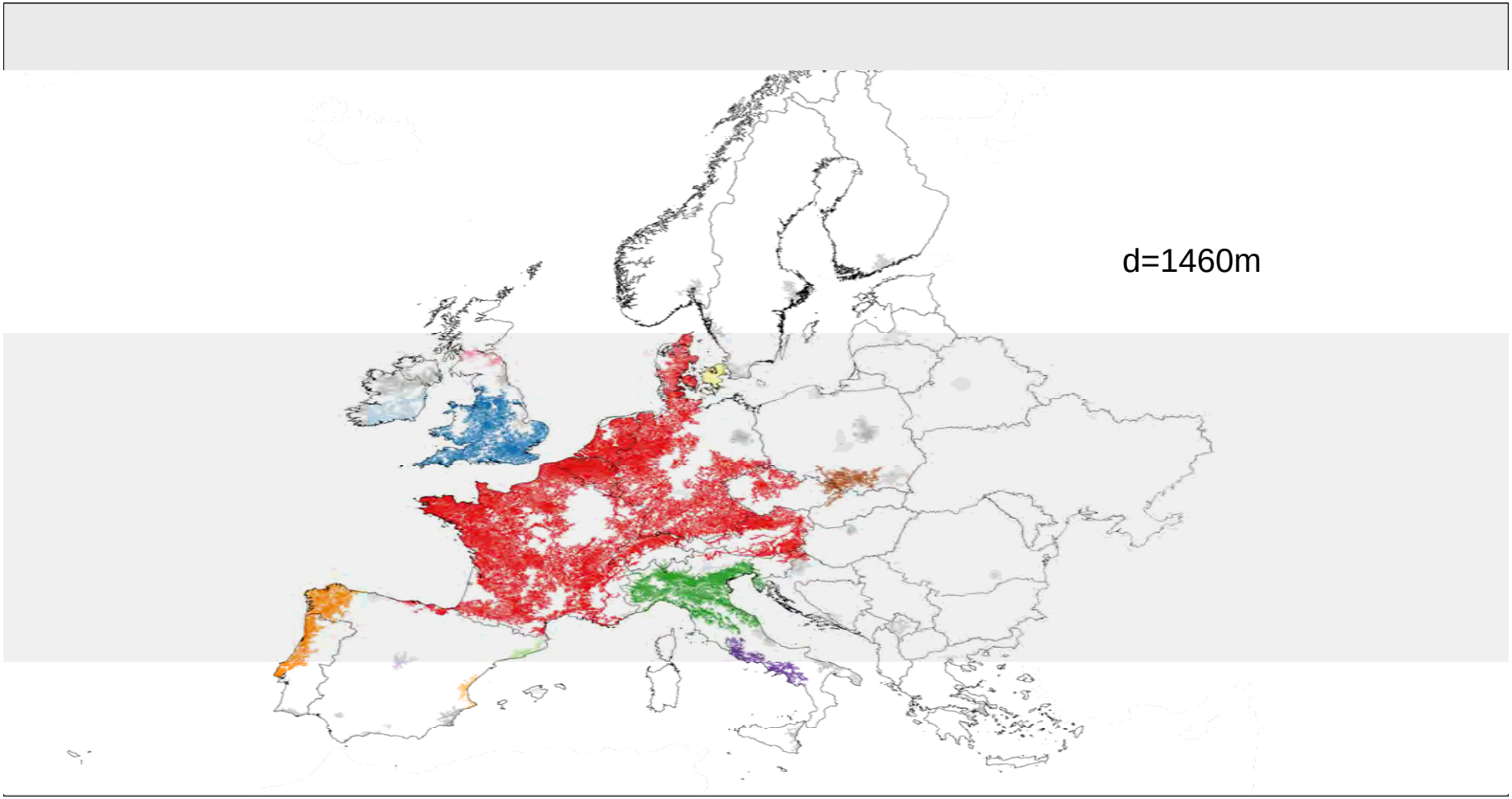

d=2840m
Belarus,
Lithuania, Latvia
and Estonia split
from Poland

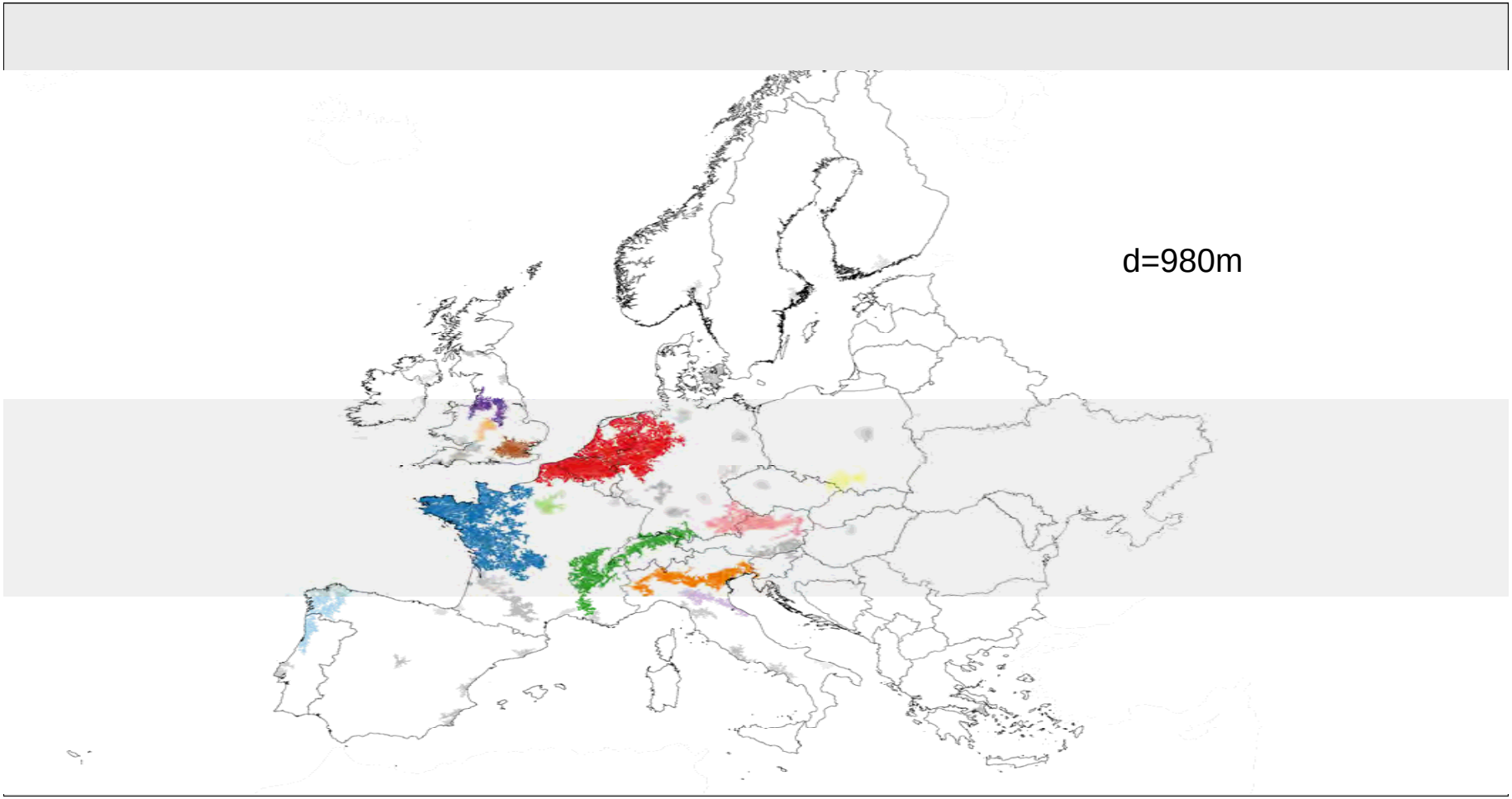

Galicia and
Portugal one
cluster
d=2420m

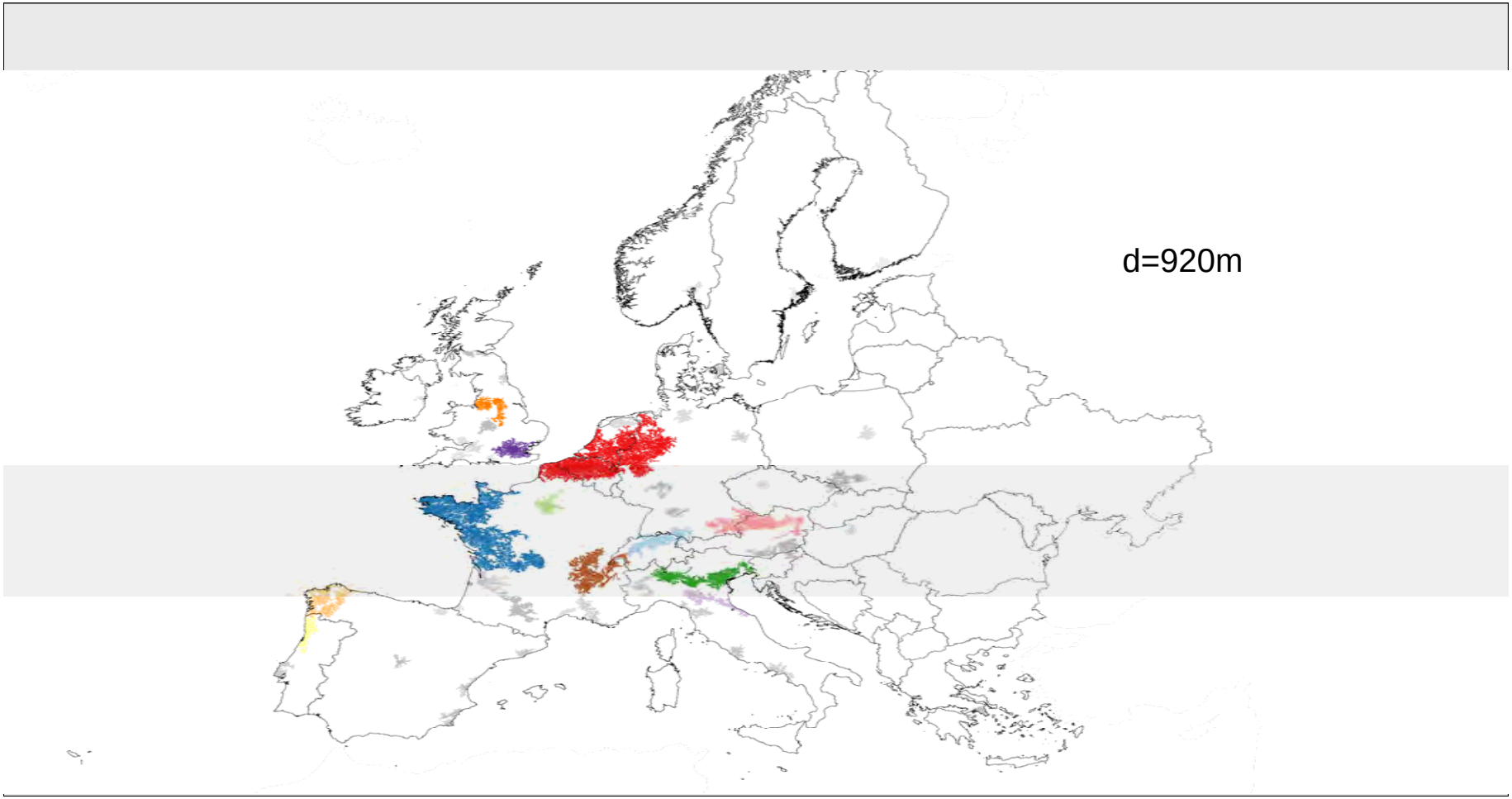

d=2300m
Apulia


Catalunya splits
d=2000m


d=1880m
Berlin detached
Lithuania
Rome and
Naples region
split


d=1460m


d=980m


d=920m


1
2
3
4
5
6
7
8
9
10
11
12
13
50
London
Birmingham
Manchester
Leeds
Glasgow
Paris
Milan
Rome
Madrid
Athens
At d=200m

London: Unpicking Its Structure

Let us generate clusters this time from the bottom up, from a very local threshold up to the radius of the entire city. We are looking here at London within the M25 – the GLA area for purely data availability reasons – we should go wider – include Oxford?

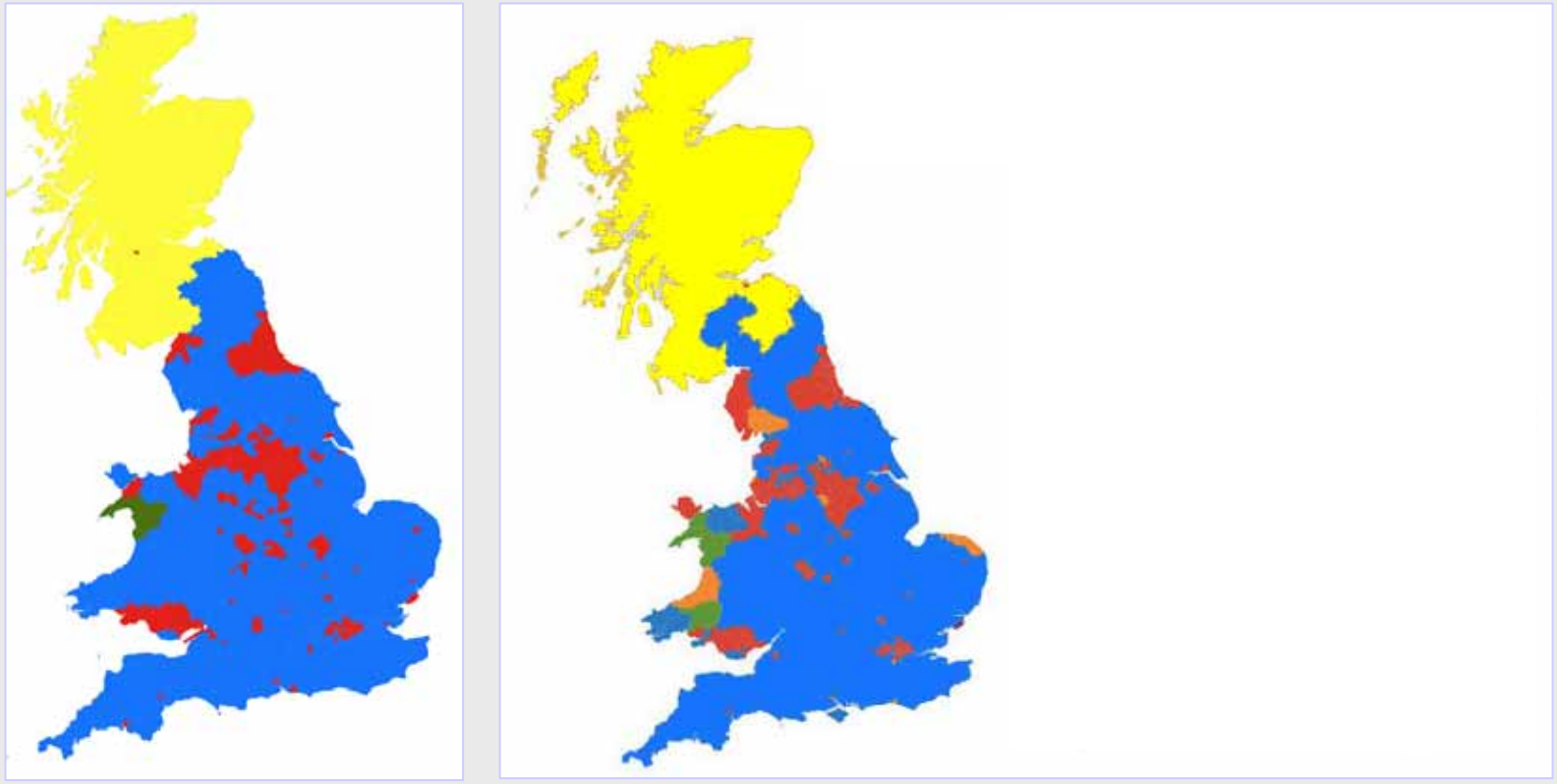
Before I conclude, let me qualify heavily the subtext

- To an extent, all this is hardly new but it focuses on something we have been very bad at doing - looking at the properties of cities. What we don't know about cities and their science, one could write a book about
- We now have a lot of work going on about these properties – partly it is addressed by looking at elements and how they vary with size and shape – how they scale – scaling – rank size, power laws and so on
- But it is also about how cities vary in their patterns of interaction. How we travel differently as cities get bigger etc

- What we need is a well worked out theory – a wider framework – and there are elements of this in the making – in this talk allometry of course and also if hierarchy and central place theory and so on
- It would be very nice to connect all this up and for someone, not me, to try and pull many things together
- I think much of this is consistent with quantitative and theoretical human geography of the last 50 years, and of urban economic theory too
- But it has to be cast in terms that we know which is size and shape of cities, and it has to be cast in how cities change through time and how technology changes

Let me finish by returning to the General Election

- We did make some predictions. What we did was to cluster the nodes in the street network as you have seen.
- Then we produced a hierarchy of clusters and from the overlaps with constituencies we allocated the 2010 votes.
- As a constituency overlapped a cluster we then produced new voting patterns from the averages of how much of the clusters overlapped the constituencies.
- This gave us a new voting profile. Then we made some assumptions about what else from socio-economic data affected voting and this gave us a predictive model

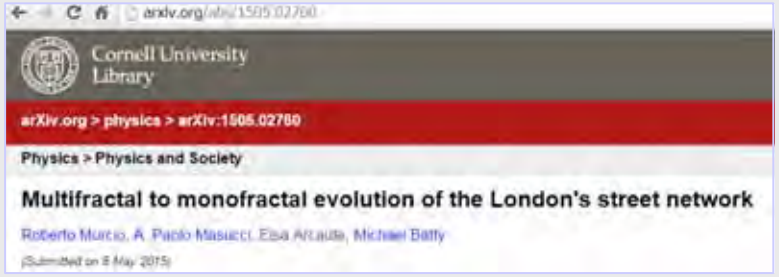


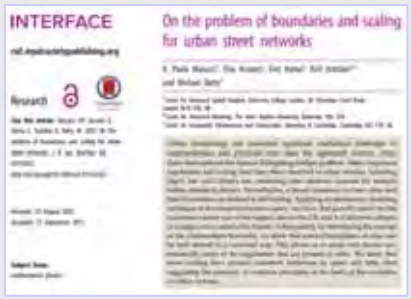
supply your own caption

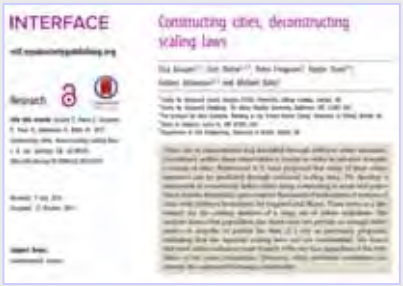
I have to thank a lot of my colleagues for this work. I also need to thank the ERC and EPSRC

Drs. Elsa Arcaute, Carlos Molinero, Erez Hatna (Johns Hopkins), Anders Johansson (Bristol/Crowd Vision), Pete Ferguson, Camilo Vargas-Ruiz, Roberto Murcio, Jaiqiu Wang, Paolo Masucci & Clementine Cottineau.

We have a bunch of unpublished papers
in the Arxiv, two published ones, one submitted







Cities and regions in Britain through hierarchical percolation

Elsa Arcaute, Carlos Molinero, Erez Hatna, Roberto Murcio, Camilo Vargas-Ruiz, Paolo Masucci, and Michael Batty

Thank You

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