Hierarchical organisation of Britain through percolation theory

Elsa Arcaute*[1], Carlos Molinero[1], Erez Hatna[2], Roberto Murcio[1], Camilo Vargas-Ruiz[1], Paolo Masucci[1], Jiaqiu Wang[1], and Michael Batty[1]

[1]Centre for Advanced Spatial Analysis (CASA), University College London, UK
[2]Center for Advanced Modeling, The Johns Hopkins University, USA

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Abbreviations: GB, Great Britain

Abstract

Urban systems present hierarchical structures at many different scales. These are observed as administrative regional delimitations, which are the outcome of geographical, political and historical constraints. Using percolation theory on the street intersections and on the road network of Britain, we obtain hierarchies at different scales that are independent of administrative arrangements. Natural boundaries, such as islands and National Parks, consistently emerge at the largest/regional scales. Cities are devised through recursive percolations on each of the emerging clusters, but the system does not undergo a phase transition at the distance threshold at which cities can be defined. This specific distance is obtained by computing the fractal dimension of the clusters extracted at each distance threshold. We observe that the fractal dimension presents a maximum over all the different distance thresholds. The clusters obtained at this maximum are in very good correspondence to the morphological definition of cities given by satellite images, and by other methods previously developed by the authors [3].

Introduction

Many different systems are intrinsically hierarchical. The different hierarchical levels can sometimes be identified by phase transitions, not necessarily of first or second order. For some of these cases, the system can be modelled in terms of percolation processes at different scales. Percolation theory [41, 13] studies how a piece of information (or a disease, or a fire, etc.) spreads in space, reaching a critical point at which a giant cluster appears. In its most general form, the process is defined in an infinite lattice and for a random occupation probability. Relaxing the constraints, the analysis can be extended to finite systems, where the clusters are the outcome of some thresholding process. Some of these systems present a multiplicity of percolation transitions, revealing a hierarchical organisation. This was observed for the brain [20], where the percolation process is considered in terms of the connectivity between voxels given by the different stimuli thresholds.

A crude analogy can be drawn between the structure of the brain and that of an urban system. Both consist of highly integrated modules which connect to each other at different scales, giving rise to a functional system. For the urban system, the modules correspond to its cities, and its different regional divisions are a manifestation of its inherent hierarchical structure [21, 11]. We hence implement a similar methodology to [20] on Great Britain, in order to unveil its hierarchical organisation independently of administrative provisions.

We take as the fundamental structure for the urbanised space the street network. This is one of the most pervasive structures whose evolution has been driven by strategic choices on communication between places, and community strength within settlements. It is thus not surprising that such a process could lead to an intrinsic hierarchy.

In this paper we investigate whether the spatial distribution of the street intersection points

*Corresponding author, email: e.arcaute@ucl.ac.uk

can reveal such a structure through percolation theory. Readers would have noticed to their surprise that the network is stripped away. Such a conscious choice is twofold: on the one hand, the street intersections correspond to the main points of convergence where the relevant interactions take place, as is the case of assembly places in Anglo-Saxon Britain [5]; on the other hand, we purge ourselves from the all time problem of the digitisation of the dataset, that is the problems with the topology of the network, the missing streets, the inaccuracy of streets almost meeting, etc. In any case, for the skeptical reader, we also provide a similar methodology developed directly on the entire street network, which has been carefully prepared and checked, and we show that the results are recovered.

The percolation process is hence applied to the street intersections, which correspond to the occupied sites in space, connected to each other through proximity only. The distribution of the street intersections in space relates very closely to the extension of the urbanised space and to spatial relations to other urban spaces [27, 12, 43]. Through a multiplicity of percolation transitions, the hierarchical structure of Britain emerges. Nevertheless, cities cannot be defined in terms of such transitions. Through the analysis of the fractal dimension of the emergent clusters, a threshold can be identified at which cities are well-defined. The morphological properties of cities and regions are notably different. These have been extensively analysed for street networks [42, 35, 36, 6, 31], nevertheless, the statistical properties previously found cannot be used to define the boundaries of a city, since there is no clear transition between urban and rural networks. Here we show that this specific morphological property observed over the whole system, gives a maximum over all the thresholds. At this maximum, the obtained clusters are in very good correspondence with other proxies for cities, such as satellite images of the urbanised space, and previous definitions of cities proposed by the authors [3].

Methodology and dataset

Percolation theory is classically approached in terms of the probability of a site being occupied in a lattice. It can also be thought of in terms of bond percolation, in which the sites are all occupied, and the probability corresponds to a bond to be open and to connect sites. In our analysis, the sites will correspond to the intersection points.

In the following section we present two methodologies: 1) the percolation on the intersection points, and 2) the percolation on the street network. For both methods we use the most complete database for street networks in Britain: the OS MasterMap [1]. For computational purposes, we reduce the size of the dataset by introducing the following simplifications: 1) we remove the points that do not convey any morphological information, such as nodes of degree two, which for example correspond to streets changing name; 2) we replace roundabouts by a single intersection point, which is primarily relevant for the methodology on networks.

Percolation on the street intersections

For this method we take the dataset described above, and we remove all the street segments, leaving only the intersection points. We then apply a clustering algorithm that corresponds to a thresholding procedure parameterised by distance. This is simply defined as the Euclidean distance between points, whether they are connected or not. We observe different configurations of clusters appearing at different distances. This procedure can be interpreted in terms of bond percolation as follows: the probability of a bond to be open between sites, corresponds to the distance between the intersection points. In this sense, one can think of a fully connected network in which the distance between nodes gives the probability for the link to exist after a normalisation procedure.

In practical terms, the algorithm is similar to the CCA (City Clustering Algorithm) [37, 38] based on population distribution in space, and the *natural cities* definition given also in terms of road intersections [25]. In [17] this algorithm is also employed to understand the emergence of regions through percolation theory. It is important to note that most of these algorithms have been constructed in an effort to define cities in a consistent way, and considerable research is still undergoing in this direction [39, 19]. These algorithms differ from models of urban growth based on correlated percolation [28, 29, 32], and on correlations with urban sprawl [22].

In detail, our algorithm is defined in terms of a distance parameter that determines clusters of intersection points in which every point has a neighbour at a distance equal or smaller than the given threshold. The algorithm can be implemented on the continuous space, or for large datasets requiring computationally demanding calculations, on a grid covering the space of points. Please refer to the appendix for more detail of the implementation of the algorithm.

Percolation on the network

In this case, we are considering the 'real' network, where intersection points are connected if and only if there is a street connecting them. The clustering procedure is very similar to the procedure described above, but in this case the distance is given by the actual extent of the street. An open bond hence corresponds in this case to an existing street according to the different distance thresholds. And once again, the links can be re-interpreted in terms of probabilities if the distances are normalised.

Results

Urban hierarchies

In both cases the behaviour of the system is the same. If one thinks about the process starting from the maximum distance at which all the points in Britain are connected, then lowering the distance threshold leads to a series of percolation transitions that divide the space into clusters. The different transitions can be observed by looking at the evolution of the average cluster size removing the giant component [41]. We denote this by $< S >^*$. In order to avoid clusters that are given by single points, we impose a minimum cluster size $S_{min} = 600$. The choice is somewhat arbitrary; nevertheless since at this stage we are interested in the hierarchical structure of Britain and the natural emergence of regions, smaller clusters will not contribute to this. To put this number into context, the number of intersection points in large cities is of the order of 10^5 and of 10^4 for the 30 largest ones.

The evolution of $< S >^*$ from large d to smaller d, shown in Fig. 1 (its network counterpart can be found in the appendix Fig. 8), indicates points at which important transitions take place in the urban space, see Fig. 2. The first transition at $d = 1120m$ shows the split of Scotland from England and Wales. It is worth noticing that this transition is present with and without street segments, and that no natural barriers, such as mountains or rivers, are responsible for such a separation. See Fig. 9 for the transition on networks. A larger distance is naturally needed in the network, since this corresponds to the actual one that people would be travelling on to reach different places in the

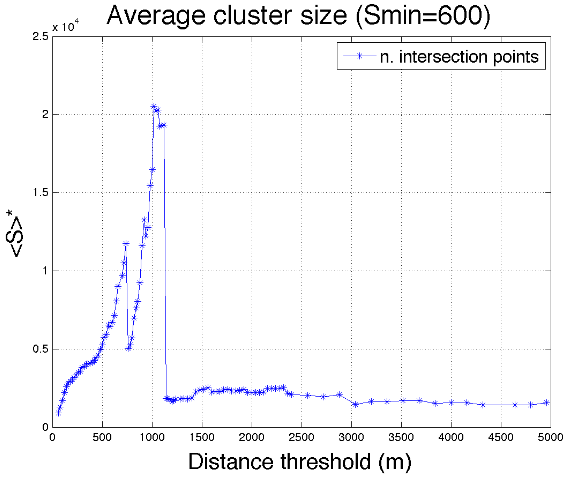

Figure 1: Evolution of average cluster size removing the largest cluster, for clusters with at least 600 intersections. Method: percolation on the intersection points.

urban system. It is expected that many of the transitions will be given by natural geographical barriers, such as National Parks or even islands, as indicated by the map at $d = 1020m$, where Snowdonia and the Lake District can be identified. So the transitions of interest are the ones that are not the outcome of these geographical accidents. One such a transition can be observed at $d = 740m$ in Fig. 3. This transition is extremely surprising, since there are no natural divisions, such as mountains or rivers, that would create this split in terms of the infrastructure. In addition, it is not an artifice of taking only the intersection points, since this split is also present for the percolation directly on the network. Such a split is very well known to anybody living in the UK. It is a split that has been around since Roman times, and that corresponds not to a regional division of a geographical kind, but to a division of a social kind. The right hand map in Fig. 3 illustrates the split very clearly within the social context. The black boundaries correspond to administrative regional divisions called NUTS2 [33], and the heatmaps correspond to different levels of income per capita at the smallest geographical unit for urban measures, called a *ward*[1]. The country is clearly divided in terms of wealth, and such a split is referred to as the North-South divide.

[1]Scotland is excluded from this map, since the values are from the 2001 census, and Scotland has a different census to England and Wales.

Understanding the natural emergence of regions through connectivity in space dates back to locational analysis in the 60's [21]. These ideas broke the paradigm of focussing on the economic performance of regions in regional science, by conceiving a regional organisation in terms of the movement of individuals. In this respect, this work might be seen as an extension of these ideas. On the other hand, since the 30's there is the longstanding idea of subdividing the space in terms of a hierarchical structure using population density [15], in addition to some elaborated geometrical propositions [26]. In the present work, the regional breaks are a consequence of the density of the intersection points, which can be seen to be in very good correlation with population density. We will argue that this is the case in the next section. No particular geometries need to be introduced though in order to obtain the regions.

From the perspective of growth, the percolation on the road network can be interpreted in terms of road growth, if analysed from the bottom up. Road growth is closely linked to economic growth, and hence to regional development. In this context the division in Fig. 3 might not be as surprising, indicating that regional policy on infrastructure development is closely associated with wealth.

The role of distance in terms of the hierarchical structure of urban systems has been explicitly outlined in [27, 12, 43], where the space between settlements is shown to be highly correlated with the size of the settlements. Further relationships between cluster size and functions have also been explored with respect to a hierarchical perspective since the 60's [23, 10, 9]. In those earlier studies, the hierarchy outlined different sorts of relationships according to size. See for example early work [34] looking at the hierarchical structure between regions through its flow, in this case given by telephone calls.

Fractal properties

Given that we do not observe a transition at which cities could be defined, we investigate whether there are morphological properties of the emergent clusters that give an indication of more urbanised areas, corresponding to our cities. Within the extensive research that has been done in the area of the morphology of cities, the fractal dimension can be singled out as one of the most relevant [8, 7, 18].

Different sorts of clusters are obtained in the two different methodologies, hence different approaches need to be taken in order to compute the fractal dimension of its elements.

Clusters of intersection points

Until recently, a single fractal dimension was employed. It is well recognised now, that a spectrum of fractal dimensions needs to be employed to fully characterise systems that present different fractal properties at different scales and regions [40], as is the case of urban systems [2, 4, 24, 14]. These systems are called *multifractals.*

The traditional way to assign a fractal dimension α to a city is through the box counting algorithm. Nevertheless, this measure is extremely sensitive to the dataset and the implementation. In addition, it has been recognised that cities are actually multifractals. In this respect we compute the three well-known fractal dimensions denoted by D_0, D_1 and D_2, where D_0 is the capacity dimension, and in practical terms it corresponds to the box-counting measure; D_1 is the information dimension and it can be seen as Shannon's entropy; and D_2 is the correlation dimension, which is considered to be the most accurate one. For the specific system at hand, we need to extract the characteristic fractal dimensions at each distance threshold. We proceed by taking the average of the 100 larger clusters of the above mentioned measures[2], but we relax the minimum cluster size to $S_{min} = 50$ intersection points, so that we ensure that we have at least 100 clusters at each distance threshold. We set a maximum distance threshold of $d = 660m$, since the percolation method clearly returns regions beyond this distance threshold, moving further and further away from a configuration of cities. The results can be seen in Fig. 4. We notice that all three dimensions show a maximum at $d = 160m$. The urban system defined in terms of the clusters at this maximum is in excellent correspondence with the identified urbanised space given by the Corine dataset [16]. In Fig. 2, the contours correspond to the classified urbanised areas.

Let also compute the fractal dimension α of the system in terms of the scaling relationship between the mass and the radius of gyration of the clusters. The mass is given by the number of intersection points N and the radius is denoted by r_{max}, see Eq.(1).

$$N \sim r_{max}^{\alpha} \tag{1}$$

[2] We ensure that the measures follow a normal distribution through a Shapiro-Wilk test.

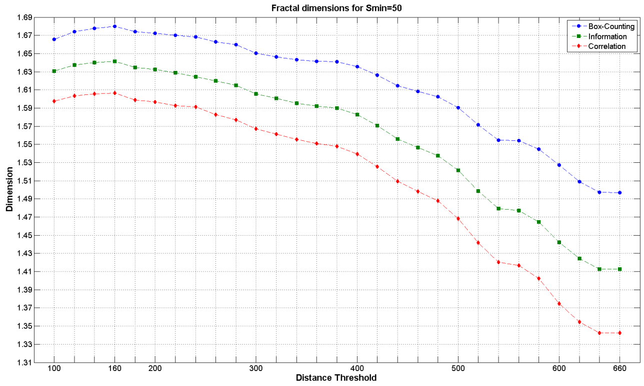

Figure 4: Fractal spectrum of the 100 largest clusters obtained from the percolation on the intersection points.

We obtain the result given in Fig. 5. Oncde again we observe that although the behaviour is less smooth in this case, it still has a maximum at $d = 160m$.

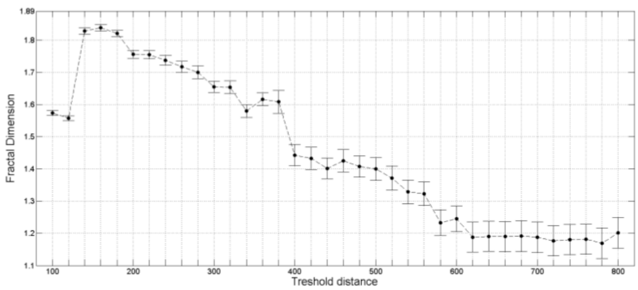

Figure 5: Fractal dimension computed according to eq.(1) of the 100 largest clusters obtained from the percolation on the intersection points.

Clusters of networks

For these clusters, we also compute the fractal dimension in terms of eq.(1), where this time r_{max} corresponds to the diameter of the network. This is the same methodology that was implemented in [20]. Note that for this system we need to take a slightly larger maximum distance threshold $d = 800m$ to ensure we are well within the cities definition.

For this case the results show a maximum around $d = 300m$, see Fig. 6. And once again we see that the urban system defined at this maximum is in excellent correspondence with the definition of cities. See Fig. 9, where the contours correspond to the urbanised areas.

In order to obtain a measure of how good the correlation is, we take only the clusters such that $S_{min} = 600$, since for this configuration

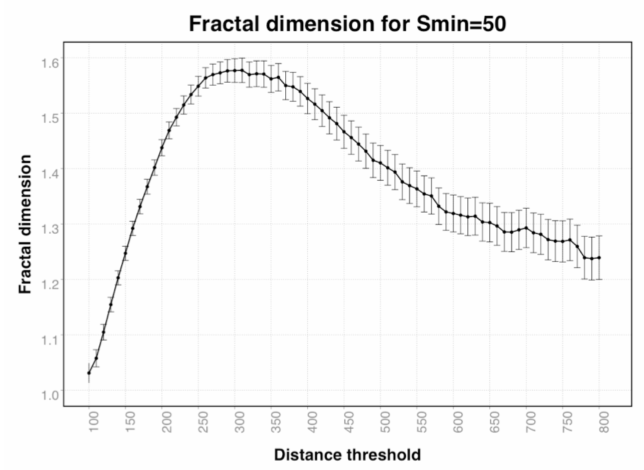

Figure 6: Fractal dimension of the whole urban system defined through the scaling relationship between mass and diameter for the percolation on the network.

we have 584 clusters, and perform a correlation. Fig. 7 indicates that the maximum of this correlation is also at $d = 300m$.

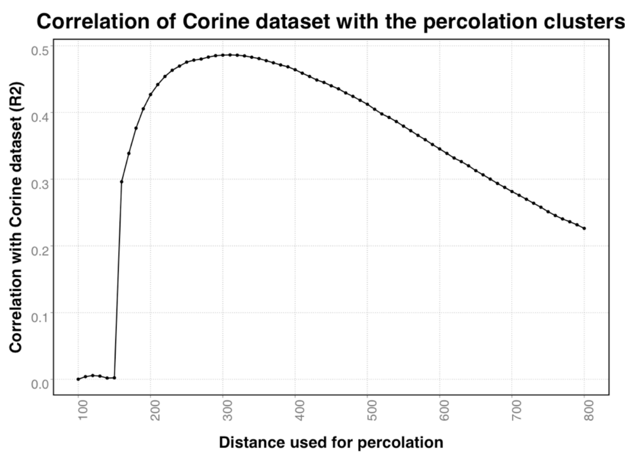

Figure 7: Correlation of the clusters from the network percolation with the boundaries of the Corine dataset.

It is important to note that the distance is not universal nor uniquely characterised. It is not universal, because it depends on the nature of the dataset. Hence a distance of $d = 300m$ might suit this specific dataset for Britain, but might not suit another dataset, nor another European country. It is not uniquely defined, because the maximum corresponds to some sort of plateau. Hence any definition in the vicinity of $d = 300m$ would be as accurate or as inaccurate as the one for $d = 300m$.

Conclusions

We have shown that applying percolation theory to the road intersections or the whole road network allows us to obtain in a very simple way the hierarchical structure of the urban system of Britain. This formalism can be implemented in incomplete datasets, and the level of detail that can be extracted will depend on the granularity of the data, nevertheless some information with respect to the organisation of the urban system can be recovered. This method can be extrapolated to other spatial distributions, where data is sparse. Some of the authors are implementing this for archaeological data.

Comparing this method to the previous method developed to consistently define cities [3], this one allows us to define cities in a more accurate way, since the threshold can be tuned locally, contrary to the previous one, in which a global population density was applied throughout the space, and the level of precision was constrained by the geographical unit of a ward. A further refinement of the percolation approach can be found in [30], where each city is adjusted to its condensation threshold. In this case we observe that the global distance gives a representation of cities that fits very well at least for the bigger cities. We need to explore further the level of accuracy.

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Appendix

Algorithm for percolation on intersection points

The algorithm is based on the geographical location of intersections. We consider a pair of intersections as connected if they are no more than d meters apart. In order to reduce the computational complexity of the procedure, the actual analysis is performed using a grid of squared cells ($10x10$ meters each). A cell has one of two values: 1 if at least one intersection is within its area or null if it contains no intersections. As the percolation analysis is based on distance, we calculate a distance grid where each cell is assigned the distance to the closest cell that contains an intersection. We use this grid in the percolation procedure.

The percolation procedure for a distance d consists of the following steps:

1. Each cell of the distance grid that has a distance value of d meters or below is marked as 1 otherwise, it is marked as null.

2. A unique identifier is assigned to each continuous set of marked cells. A cell is considered adjacent to its four nearest neighbours (i.e., its von Neumann neighbour).

3. Each intersection is assigned the unique identifier of its containing cell.

The method is implemented in ESRI ArcMap 10.1 using the following tools:

- The intersection grid is created using the Points to Raster tool.

- The distance grid is created using the Euclidian distance tool.

- The marked cells grid is created using the Raster Calculator tool.

- The unique identifiers grid is created using the Region Group tool.

- The unique identifiers are copied to the intersection points using the Extract Values to Points tool.

Algorithm for the network based percolation

Given a graph of the road network, where nodes represent intersections and the weight for each edge is the length of the street that connects them and a certain metric threshold (e.g. 5000m) we produce a network percolation by:

1. Selecting the transition of the graph with the smallest weight (distance), generating a new cluster and inserting both its nodes into the cluster.

2. We will keep a first-in first-out queue of *nodes to expand*, from which we will extract a node to continue the process. We add both nodes of the transition selected in step 1 to this queue. Nodes are only added to this queue if they are not already included.

3. Extract a node from the queue of *nodes to explore* and if a transition departing from that node (not yet included in the cluster) is smaller than the threshold, include the transition in the cluster and the end node of the transition in the queue of nodes to explore.

4. Repeat step 3 until no further node can be expanded (the queue is empty) and if there are transitions left in the graph that do not belong to any cluster, generate a new cluster by choosing the smallest available transition and repeat from step 1.

Results for the percolation on the network

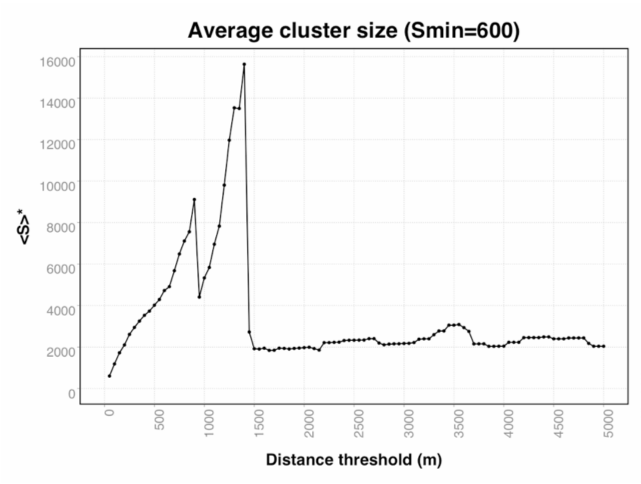

Figure 8: Evolution of the average cluster size removing the largest cluster, for clusters with at least 600 intersections. Method: percolation on the network.

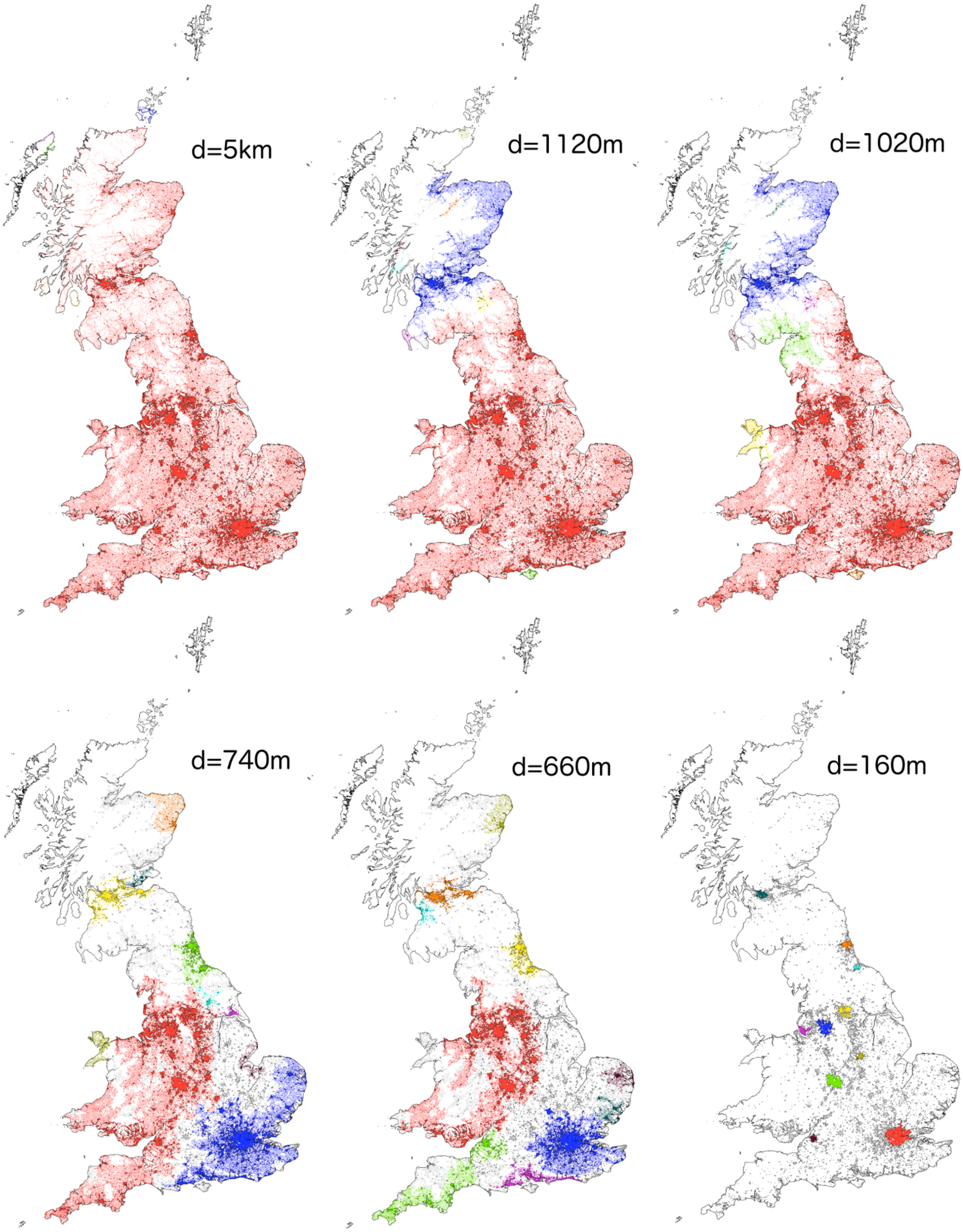

Figure 2: Maps of clusters at some distance thresholds, for the percolation on the intersection points. Only the 10 largest clusters are depicted.

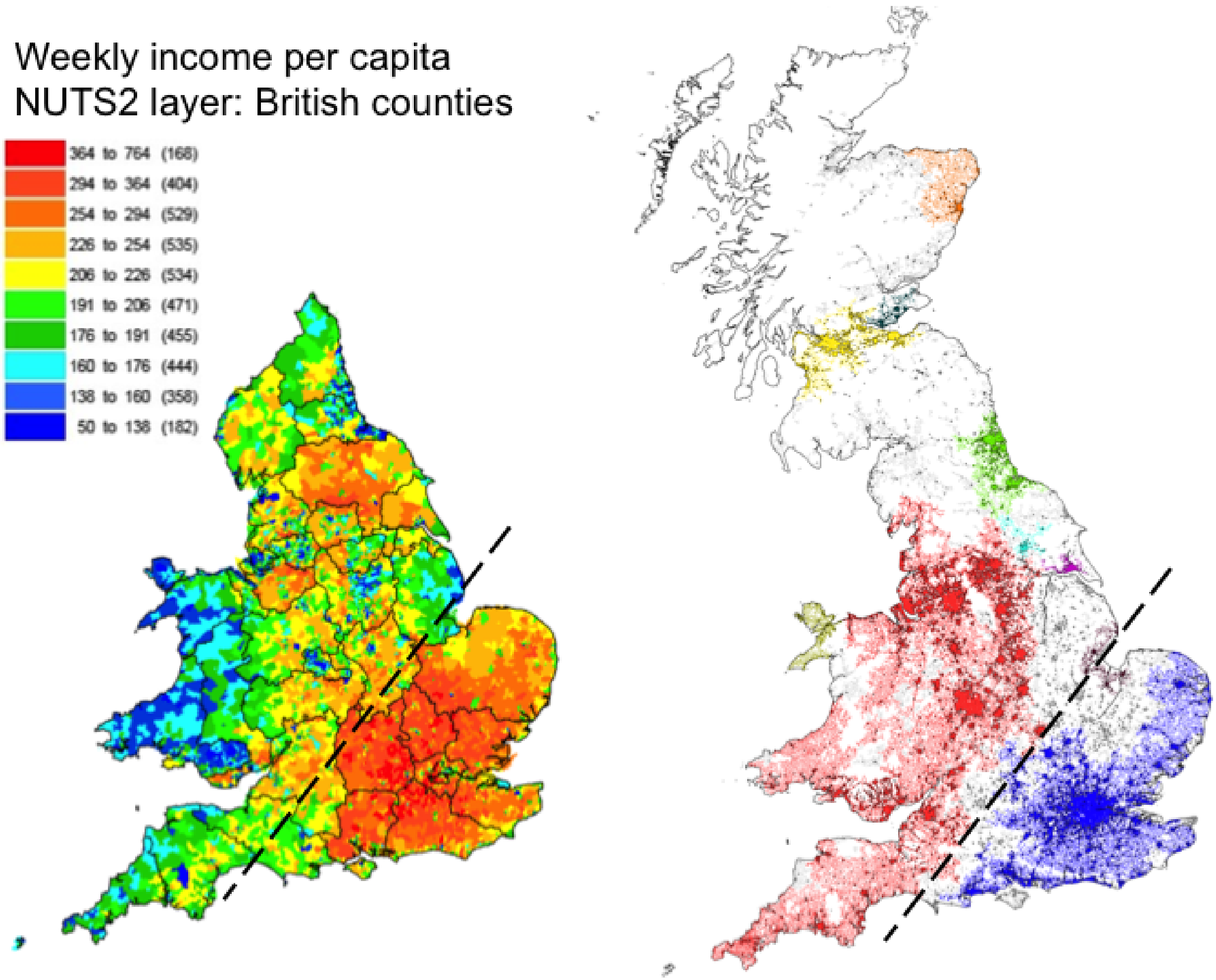

Figure 3: Right: map of Britain at $d = 740m$; left: map of England and Wales with regional divisions given by NUTS2, and heatmap of income at the ward level.

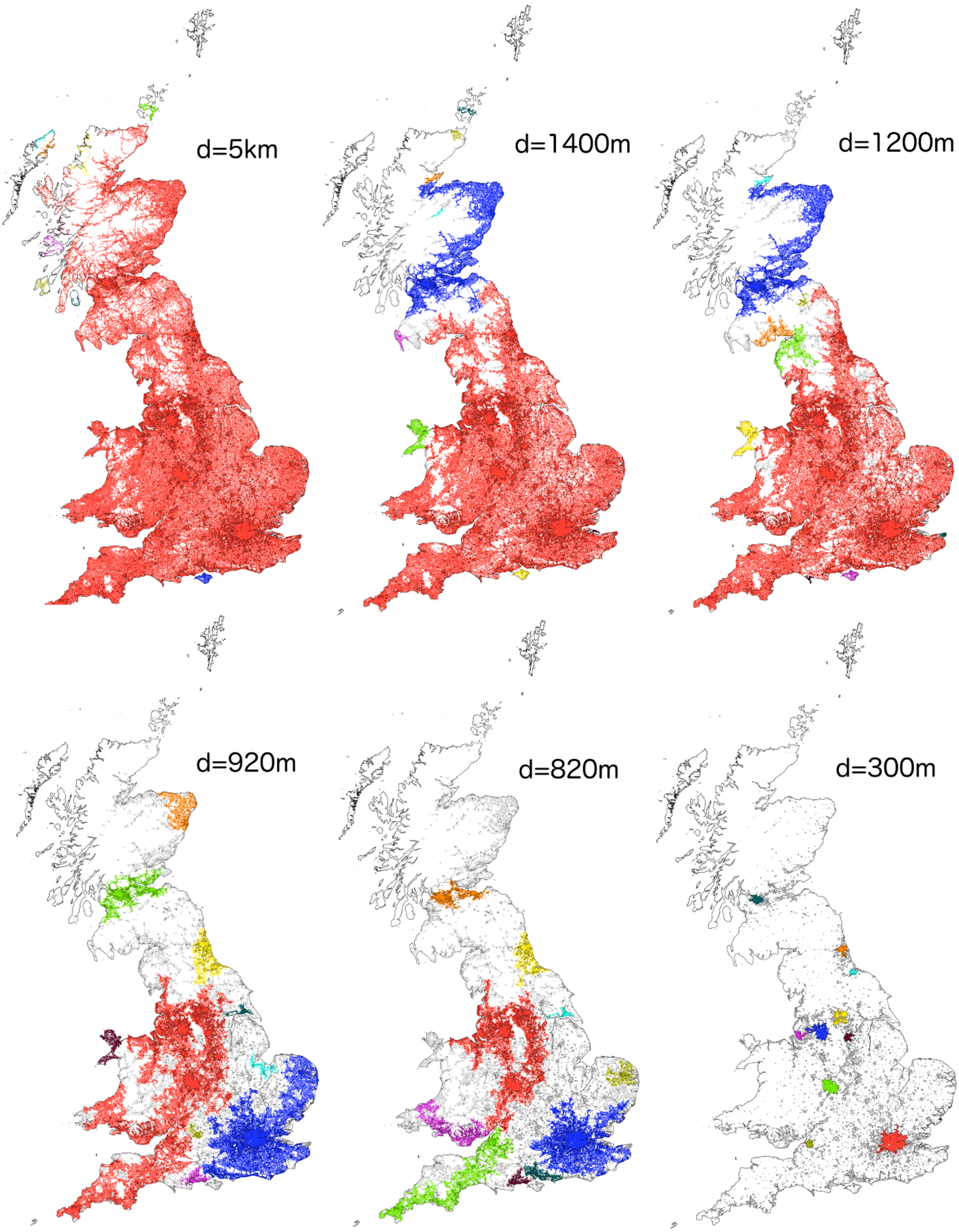

Figure 9: Maps of clusters at some distance thresholds for the network percolation. Only the 10 largest clusters are depicted.