

Foundations of Urban Science Fall 2013: 19 November 2013

Lecture 10: Batty's Lecture 2-1

Large Scale Integrated Urban Models

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Outline

- The Assignment
- Demand and Supply: Market Clearing
- Input-Output: The Echenique Models
- Integrated Large Scale Model Structures
- Sketch for an Integrated Model
- Requirements for Large Scale Models:
 Computational Resources, Intelligibility,
 Accessibility, and Relations to Stakeholder
- Parallel Developments: CA and ABM: I will talk a little about these in the final lecture session





Demand and Supply: Market Clearing

So far most of these models have been articulated from the demand side – they are models of travel demand and locational demand – they say nothing about supply although we did introduce the notion that in simulating trips and assigning these to the network, we need to invoke supply. When demand and supply are in balance, then the usual signal of this is the price that is charged. In one sense the DRAM EMPAL model configures residential location as demand and employment location as supply but most models tend to treat supply as being relatively fixed, given, non-modellable





However several models that couple more than one activity together treat supply as being balanced with demand, often starting with demand, seeing if demand is met, if not changing the basis of demand and so on until equilibrium is ascertained. Sometimes prices determines the signal of this balance. If demand is too high, price rises and demand falls until supply is met and vice versa. Often this is done simply to ensure demand is not greater than supply

Most urban models do not attempt to model supply for supply side modelling is much harder and less subject to generalisable behaviour

A strategy for ensuring balance is as follows for a model with two sectors – like the one we illustrated earlier





In the following slide, we have two submodels – first residential location and second retail location

In each submodel, we first have interaction (trip distribution)

In each submodel, we first have interaction (trip distribution) and then location.

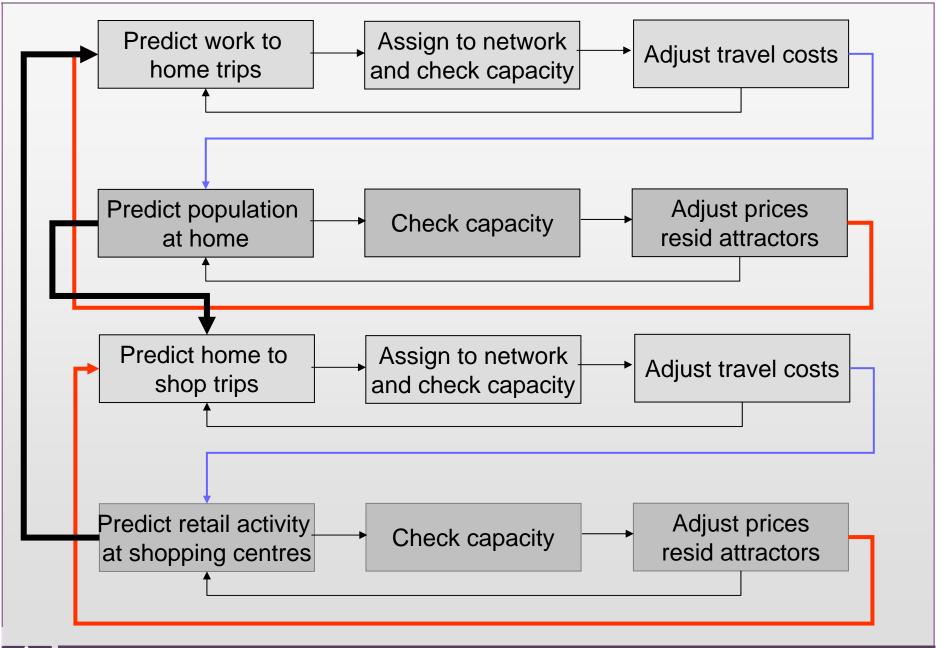
The first loops in terms of interaction are for capacity constraints on supply, the second are for capacity constraints on location ——

The second set of red loops involve reiterating the interaction and location so that we can get balance within the entire submodel

The thick black loop in the middle couples the residential to the retail mode, the thick black loop around the two models is used if retail predictions are to influence employment



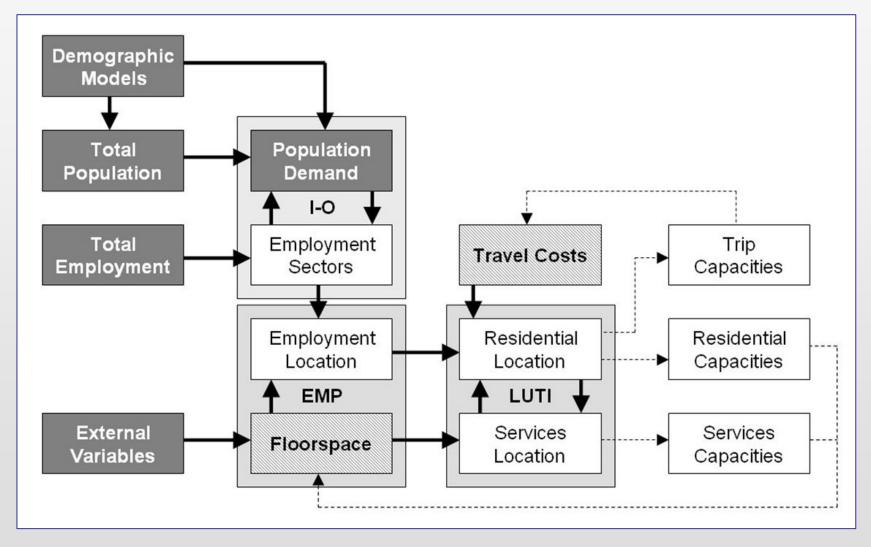








This is the integrated structure that we introduced briefly last time for the London model







- The decision to nest what loop inside what other loop is a big issue that makes these models non-unique
- If the supply side is modelled separately then the way this is incorporated further complicates the sequence of model operations.
- In large scale integrated models, that we will deal with next, these are crucial issues
- There is one further structural issue we will deal and this involves extending the models sectorally and the Echenique input-output formulation is a good example of this extension





Input-Output: The Echenique Models

So far we have only developed couplings between models that are added together in ordered sequences that string sectors together apart from reference last time to the Lowry model which organised this sequence around the basic-non-basic employment multiplier.

We can extend this to a series of linked causal multipliers between different sectors by extending this chain to an input-output model framework. In essence we define many different sectors involving households, labour, industries, services and so on and build the model so that there are consistent economic relations between each





Echenique's MEPLAN models are structured in this fashion. We can introduce these as follows.

Essentially the system is divided into production and consumption based on activities m that are produced in zone I, X_i^m , and consumed as activities n in zone j, Y_j^n

These are organised as in an input output table but noting that they are spatially specific

$$X_i^m = \sum_j \sum_n T_{ij}^{mn}$$

$$Y_j^n = \sum_i \sum_m T_{ij}^{mn}$$

Here is the typical I-O table





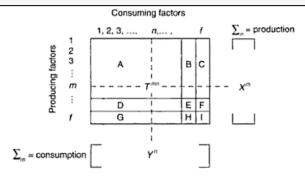


Figure 2. Transaction matrix T.

- Section A of the matrix T^m represents the transactions between factors.
 This area is normally included in standard input-output models (Leontief, 1951). It represents the sales from sector m to sector n.
- Section B of the matrix T^{mn} represents the transactions between factor
 m and the household group n, in other words the consumption by the
 households of products or services m.
- Section C of the matrix T^{mn} represents the transactions between factors m
 to be exported to outside the area in consideration. Normally, both sections
 B and C are considered the final demand in standard input—output models
 that also includes investments and government consumption. It is described
 as the exogenous sector, that is to say, it is determined outside the model.
- Section D represents the sale of labor or other income received by socioeconomic groups m from the factor n (e.g. dividends).
- Section E represents the sale of labor from socio-economic groups m to households in socio-economic groups n (e.g. domestic labor).
- Section F represents the sale of labor or other income received from the exogenous factor, such as pensions and other payments from government, etc.
- Section G represents the imports from outside the area and payments to the exogenous factor such as taxes to the government. In this sector, rental of property or land is sometimes included.
- Section H represents the payments by the households factor such as taxes, rental, etc.
- Section I represents payments by the exogenous factor to itself, such as imports for the government or for investments.





The flows are based on spatial interaction models of the form

$$T_{ij}^{mn} = Y_j^n \frac{\exp(-\beta^m c_{ij}^m)}{\sum_{i} \exp(-\beta^m c_{ij}^m)}$$

Where the generalised interaction costs also include other costs such as prices of good m at I

$$c_{ij}^m = p_i^m + t_{ij}^m + w_{ij}^m$$

The order in which these equations are solved and linked together is given in the following flow chart

Note that prices are determined from spatial interactions as

$$p_i^m = \frac{1}{\beta} \log \sum_i \exp(-\beta^m c_{ij}^m)$$





And then linked back to the prices of goods produced as

$$p_j^n = \sum_m a^{mn} p_j^m$$

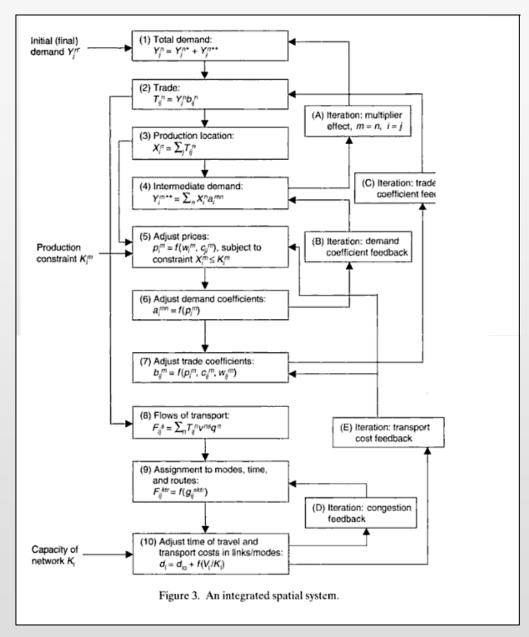
$$a^{mn} = \frac{\sum_{i} \sum_{j} T_{ij}^{mn}}{\sum_{j} Y_{j}^{n}}$$

The precise details of how the model works are extremely hard to figure out from the papers but the following flow chart goes some way to showing how the various elements are configured.

This is a general point. In models that are coupled in this fashion – integrated, then it is often hard to figure out the precise ordering or the structure





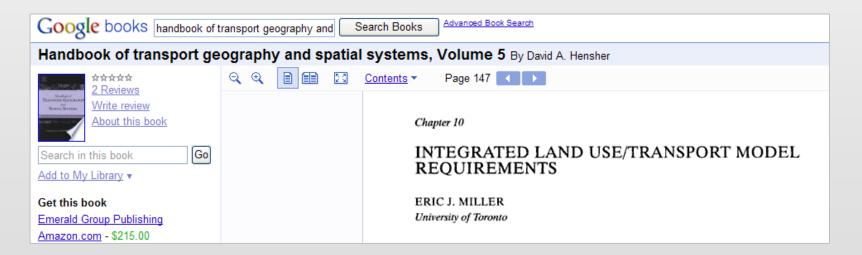






Integrated Large-Scale Model Structures

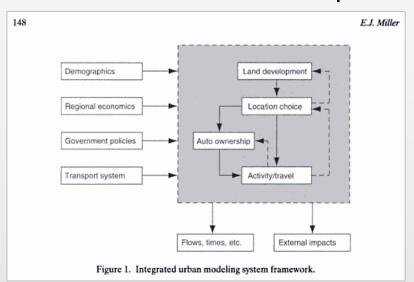
I will simply point you in the right directions here – the Handbook I referred you to in the last lecture contains several very good papers on these issues and I will briefly present some notes from Miller's article

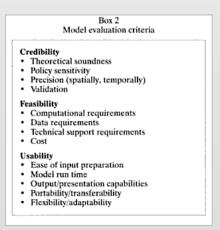






Here is a summary from his article of the key structure of such models and also their requirements





Box 1 Integrated urban model design issues Physical system representation Time Space (land) Building stock Transportation networks Services Representation of processes Land development Location choices Job market Demographics Regional economics Automobile holdings · Activity/travel demand Network performance Representation of decision-makers Persons Households Private firms · Public authorities "Generic issues" Level of aggregation/disaggregation · Endogenous versus exogenous treatment · Level of "process type" Model specification Implementation issues Data requirements

Computational requirements

· Technical support requirements





Table 1. General facts

Developer	Operational history	Platform	Commercial availability	Support
S. H. Putman	Developed over the last 25 years; operationally applied in many US cities plus selected overseas (40 plus calibrations)	Originated in FORTRAN for mainframe/work-station. PC version (METROPILUS) in ArcView shell, which provides linkage to ArcView GIS (Windows compatible)	Yes	Consulting firm, with commercial documentation and technical support (user's manual, newsletter, user group)
M. Echenique	Much shared history over 25-year development. Operational applications throughout the world, including the USA (Sacramento for both; Washington State for MEPLAN; Oregon State and Baltimore for TRANUS)	MEPLAN originated in FORTRAN for mainframe; now PC based	Yes	Consulting firm, with commercial documentation and technical support (user's manual, newsletter)
T. de la Barra		TRANUS developed directly for PC (Windows orientation)	Yes	Consulting firm, with commercial documentation and technical support (user's manual)
F. Martinez	Operational in Santiago, Chile. Developed over last 8–10 years	PC based; runs under Windows. Interfaces with a relational database management system (Access). GUI and GIS	Yes	University-based research team in collaboration with the Government of Chile
A. Anas	Currently being implemented in New York City. Based upon previous models (CATLAS, CPHMM, NYSIM) developed in Chicago and New York over the last 20 years	PC or workstation. FORTRAN program	Yes	Alex Anas & Associates (a small firm). Limited documentation
P. Waddell	Currently being implemented in Honolulu, Eugene/Springfield and Salt Lake City. Historical validation performed in Oregon	Platform independent, written in Java. Viewer currently implemented in MapObjects GIS on Windows 95/NT	Yes; public domain via website (www.urba nsim.org)	University of Washington. Limited documentation currently. Reference manual, user guide, software available at website (www.urbansim.org)
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Sketch for an Integrated Model

I am very quickly going to sketch an integrated model which builds on the ideas of the last lecture – I will not disaggregate the model into m employment types and n housing types but we can assume that this is a complicating feature that simple makes the presentation trickier – so we will simply deal with the aggregate version

The model has three sectors – employment, retailing and residential location with a link from retailing into part employment. Three different models are built for each sector – spatial interaction for residential and retailing and a linear model of land development for employment





$$T_{ij}^{k} = E_{i} \frac{R_{j} \exp(-\lambda^{k} c_{ij}^{k})}{\sum_{jk} \exp(-\lambda^{k} c_{ij}^{k})}$$

$$P_{j} = \sum_{ik} T_{ij}^{k} \qquad \text{Residential location}$$

$$if \ P_{j} > P_{j}^{\max} \rightarrow R_{j}^{*} = R_{j} \frac{P_{j}}{P_{j}^{\max}}$$

$$if \ S_{z} > S_{z}^{\max} \rightarrow W_{z}^{*} = W_{z} \frac{S_{z}}{S_{z}^{\max}}$$

In the next slides, we show the loops which need to be invoked to balance demand and supply and to couple the submodels

$$E_{i} = X_{i} + \phi S_{i}$$

$$\sum_{q} \alpha_{q} x_{qi}$$

$$X_{i} = X \frac{1}{\sum_{q} \sum_{i} \alpha_{q} x_{qi}}$$

$$Cation$$

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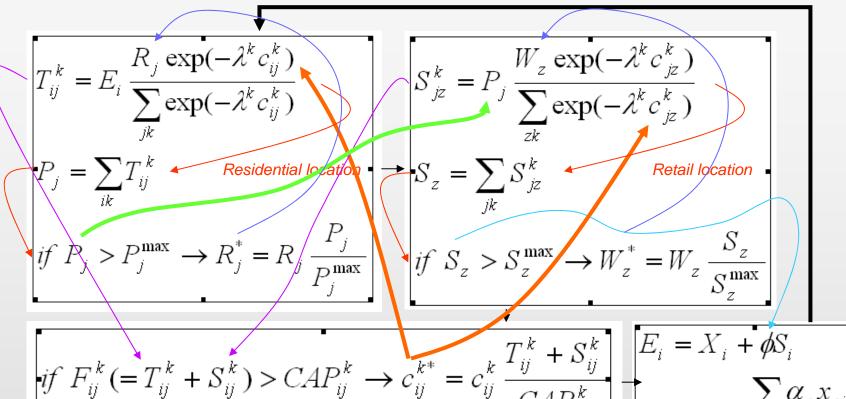
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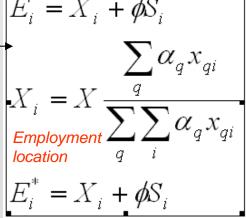






Capacitated Transport Constraints

Here are all the loops







Reading about integrated models is more tricky as these models are convoluted – involved – that clear statements are hard to find. Two papers are relevant and I will put these up later today.



Iacono, M., Levinson, D., and El-Geneidy, A. (2008) Models of Transportation and Land Use Change: A Guide to the Territory, **Journal of Planning Literature**, **22**, 323-340, and



Hunt, J. D., Kriger, D. S. and Miller, E. J.(2005) Current Operational Urban Land-Use-Transport Modelling Frameworks: A Review, **Transport Reviews**, **25**, 329 — 376



