Lecture 9: Batty's Lecture 1-2

Basic Land Use Transportation Models

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Outline for Lecture 9: 1-2

- Entropy-Maximising Again and Related Measures
- Residential Location, Modal Split Models
- The London Tyndall Model: Applications
- Transportation Modelling: The Four Stage Process
- Modular Modelling: Coupled Spatial Interaction
- A Simple Example of Modularity: Lowry's Model
- DRAM-EMPAL Style Models
- Demand and Supply: Market Clearing
- Input-Output: The Echenique Models

Next Week's Lecture is again in two parts like this one

Entropy-Maximising Again and Related Measures

First we define entropy as Shannon information and we convert all our equations and constraints to probabilities. Shannon entropy is a measure of spread or compactness in spatial systems

$$H = -\sum_i \sum_j p_{ij} \log p_{ij}$$

We maximise this entropy subject to origin and destination constraints or some combination of these but noting now that we need another constraint on travel cost which is equivalent to energy so that we can derive a model

$$\sum_i \sum_j p_{ij} c_{ij} = \hat{C}$$

We thus set up the problem as

$$\max \quad H = -\sum_i \sum_j p_{ij} \log p_{ij}$$

subject to

$$\sum_j p_{ij} = p_i$$

$$\sum_i p_{ij} = p_j$$

$$\sum_i \sum_j p_{ij} c_{ij} = \hat{C}$$

But note that the probabilities always add to 1, that is

$$\sum_i \sum_j p_{ij} = \sum_i p_i = \sum_j p_j = 1$$

From this we get the Boltzmann-Gibbs distribution for the probabilities

By setting up a Lagrangian which is the method of maximisation, then we get

$$p_{ij} = \exp(-\lambda_i - \lambda_j - \lambda c_{ij})$$

or

$$T_{ij} = Tp_{ij} = \underbrace{A_i O_i}\,\underbrace{B_j D_j}\,\underbrace{\exp(-\lambda c_{ij})}$$

Now we can generate any model in the family of four models – unconstrained, singly-constrained (origin or destination) and doubly constrained by setting the redundant constraint parameters equal to zero and simplifying the model

To derive a residential location model which is origin constrained – we know the information at the origin but want to predict the flows to the destination and add up these flows to predict activity at the destination, we

We thus set up the problem as

$$\begin{cases} \max \quad H = -\sum_i \sum_j p_{ij} \log p_{ij} \\ \textit{subject to} \\ \sum_j p_{ij} = p_i \\ \sum_i p_{ij} = p_j \\ \sum_i \sum_j p_{ij} c_{ij} = \hat{C} \end{cases}$$

And we get

$$\begin{cases} p_{ij} = \exp(-\lambda_i - \lambda c_{ij}) \\ \textit{or} \\ T_{ij} = T p_{ij} = A_i O_i \exp(-\lambda c_{ij}) = O_i \dfrac{\exp(-\lambda c_{ij})}{\sum_j \exp(-\lambda c_{ij})} \\ \textit{where} \\ D_j'' = \sum_i T_{ij} \end{cases}$$

Several things to note:

There is no attractor value at the destination – we would need to put this in as a constraint – i.e. a piece of information to be incoporated by the model

This is a location model – we predict activity at the destination – in the case of a model that predicts how many people working in zone i O_i live in zone j, this is D'_j where the prime ' is the notation for predicted

Now let us put this model back into the entropy equation and see what we get – let us put the model back in in its exponential form

$$p_{ij} = \exp(-\lambda_i - \lambda c_{ij})$$

Then what we get is

$$H = -\sum_i \sum_j p_{ij} \log p_{ij} = -\sum_i \sum_j p_{ij} (-\lambda_i - \lambda c_{ij})$$

$$= \sum_i p_i \lambda_i + \lambda \hat{C} = \sum_i p_i \log \sum_j \exp(-\lambda c_{ij}) / p_i + \lambda \hat{C}$$

What we need to note is that entropy is partitioned into a fixed energy and free energy – the fixed is the second term and the free is the first – a series of weighted log-sums and it is often thought of a kind of accessibility.

In this case it is the sum of accessibilities, one for each origin zone. It has strong relations to utility in the random utility maximising version of this kind of model which is central to discrete choice theory

Residential Location, Modal Split

Let me illustrate in two ways how we can build models using this framework

If we say that residential location depends on not only travel cost but also on money available for housing we argue as before that

- The model is singly constrained – we know where people work and we want to find out where they live – so origins are workplaces and destinations are housing areas
- The model then lets us predict people in housing
- We argue that people will trade-off money for housing against transport cost

And we then set up the model as follows

This time using not the probability form but the trip activity-volume form, we get

$$\sum_j T_{ij} = O_i$$

$$\sum_i \sum_j T_{ij} c_{ij} = C$$

$$\sum_i \sum_j T_{ij} R_j = R$$

leads to

$$T_{ij} = A_i O_i \exp(\vartheta R_j) \exp(-\lambda c_{ij})$$

Note that we now add a constraint on money available for housing (like rent) R_j. We can of course find out from this location model how many people live in destination housing zones, so again it is a distribution as well as a location model

$$P_j = \sum_i T_{ij}$$

We can extend this model in lots of ways and we will show some of these later. We also can think about disaggregating the model into different transport modes – let us call each mode k and then set up the model so that we can predict T_{ij}^{k} as follows

The model is singly (origin) constrained because we want to predicts how many people travel from work to home. Given we know how many people work at origins, and we want to predict what mode of transport k they travel on. Then

$$\sum_{j}\sum_{k} T_{ij}^{k} = O_i$$

$$\sum_{i}\sum_{j}\sum_{k} T_{ij}^{k} F_j = F$$

$$\sum_{i}\sum_{j} T_{ij}^{k} c_{ij}^{k} = C^k$$

And the model can be specified as

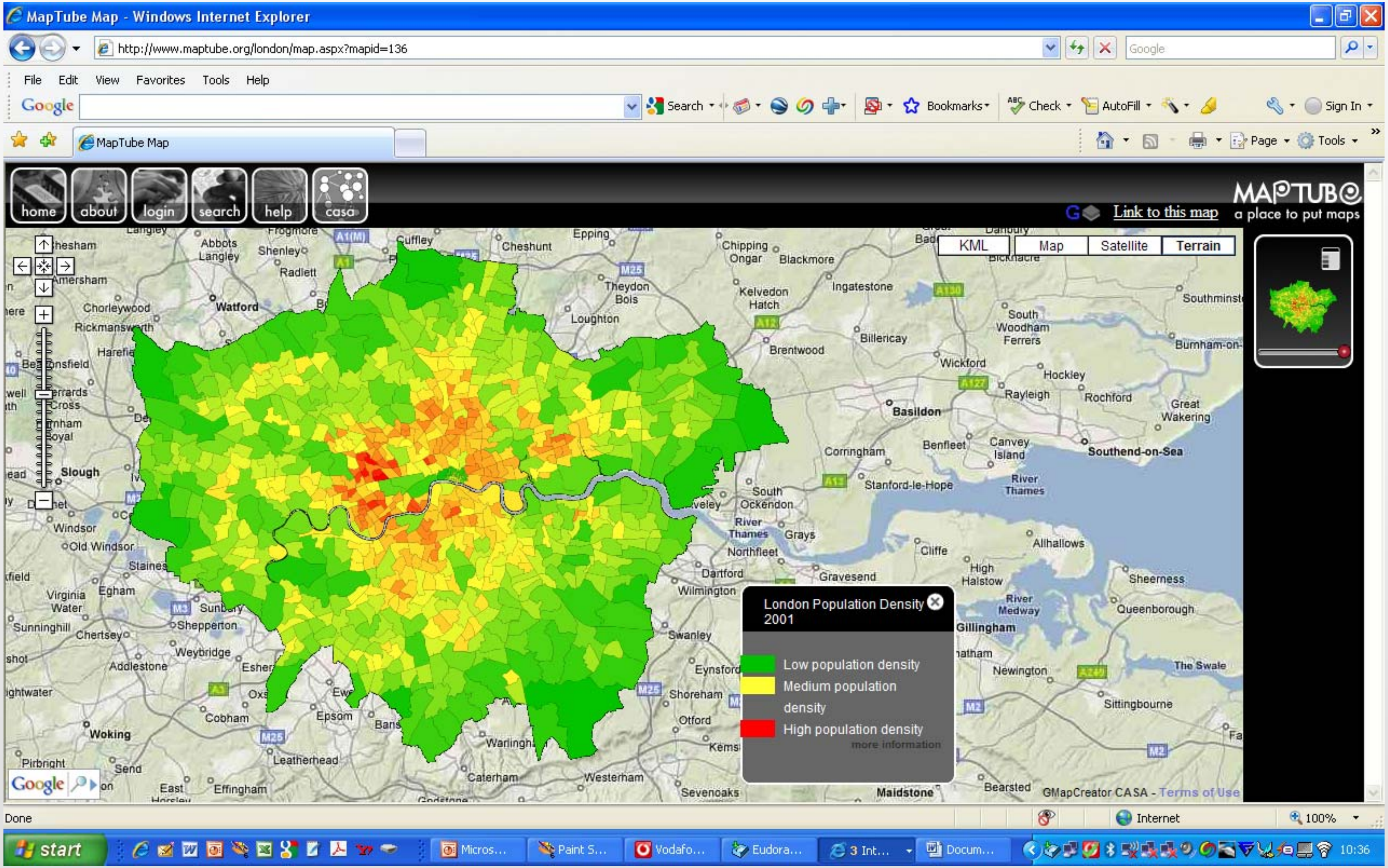
$$T_{ij}^{k} = O_i \frac{F_j \exp(-\lambda^k c_{ij}^k)}{\sum_j \sum_k F_j \exp(-\lambda^k c_{ij}^k)} = O_i \frac{F_j \exp(-\lambda^k c_{ij}^k)}{\sum_j F_j \sum_k \exp(-\lambda^k c_{ij}^k)}$$

Note that the mode split is a ratio of the competitive effects of each travel cost, that is

$$\frac{T_{ij}^{k}}{T_{ij}^{\ell}} = \frac{\exp(-\lambda^k c_{ij}^k)}{\exp(-\lambda^\ell c_{ij}^\ell)}$$

In short the model is not only distributing trips so that locations compete but also that modes compete BUT modes do not compete per se with locations

Now let us see how we can build this model for real



Go to www.maptube.org to see many maps of Greater London

Visual Analytics and Modelling Processes

London and the Thames Gateway Land Use Transportation Model

Cities Research Programme

Tyndall°Centre for Climate Change Research

CASA UCL · Newcastle

This program is a rudimentary land-use transportation model built along classical lines which allocates population and employment to small zones of the urban system. It uses spatial interaction principles which bind the population sector (residential or housing) to employment sector (work or industrial and commercial) through the journey to work (work trips) and the demand from services (which loosely translate into trips made to the retail and commercial sector).

The model is being built for Greater London and the Thames Gateway at ward level - 633 in all - so that it can be used in a wider process of integrated assessment focussed on assessing the impact of climate change on small areas in this metropolitan region. In particular rises in sea level and pollution are key issues, and as such the model sits between aggregate assessments of environmental changes associated with global and regional climate change models and environmental input output models, and much more disaggregate models related to the detailed hydrological implication of long term climate change.

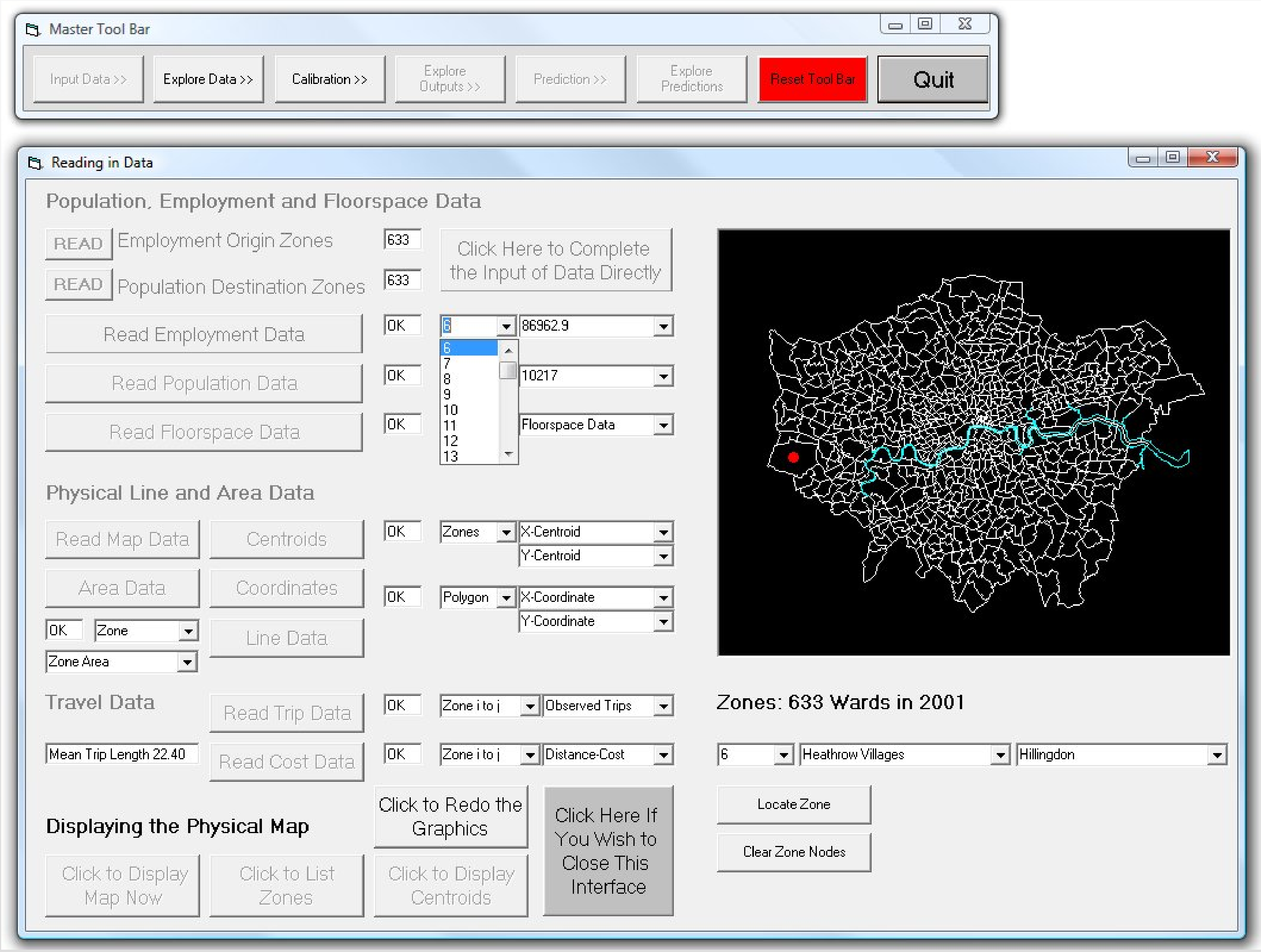
The programme enables the user to read in the data and explore it spatially, to calibrate the parameters of the model and explore its outputs spatially and to engage in various predictions ranging from the typical' business as usual scenarios' to much more radical changes posed limits on spatial behaviour which either result from climate change and, or mandated by government. The predictions and scenarios are intended to go out to 2100 and thus the model is largely designed as a sketch planning tool.

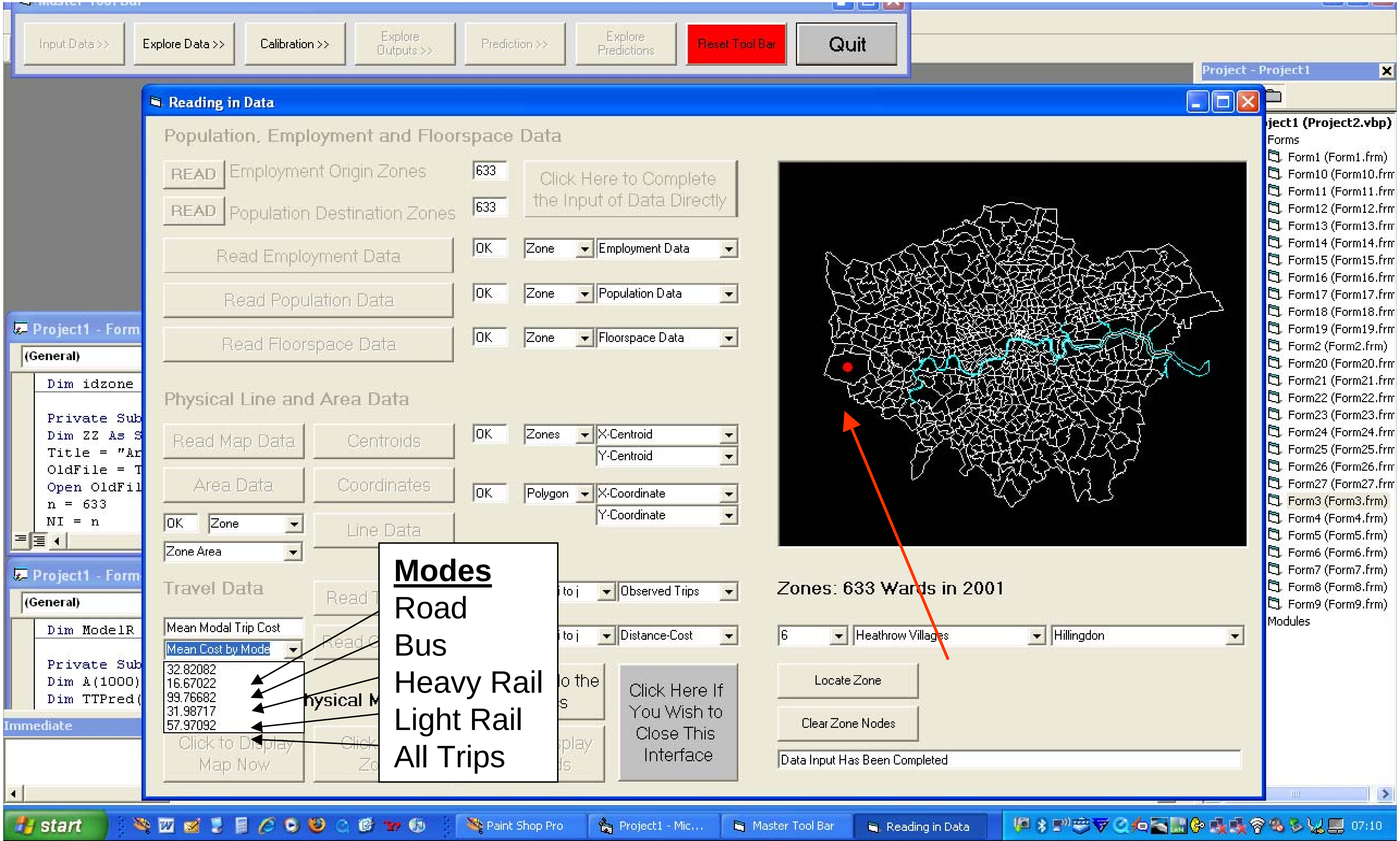
These various stages of the model contained in a master tool bar which is activated when the GO! button is pressed on this screen. The master tool bar enables the users to proceed through the various stages indicated and to display outputs in map and statistical form at any stage.

with GLA ECONOMICS LONDON

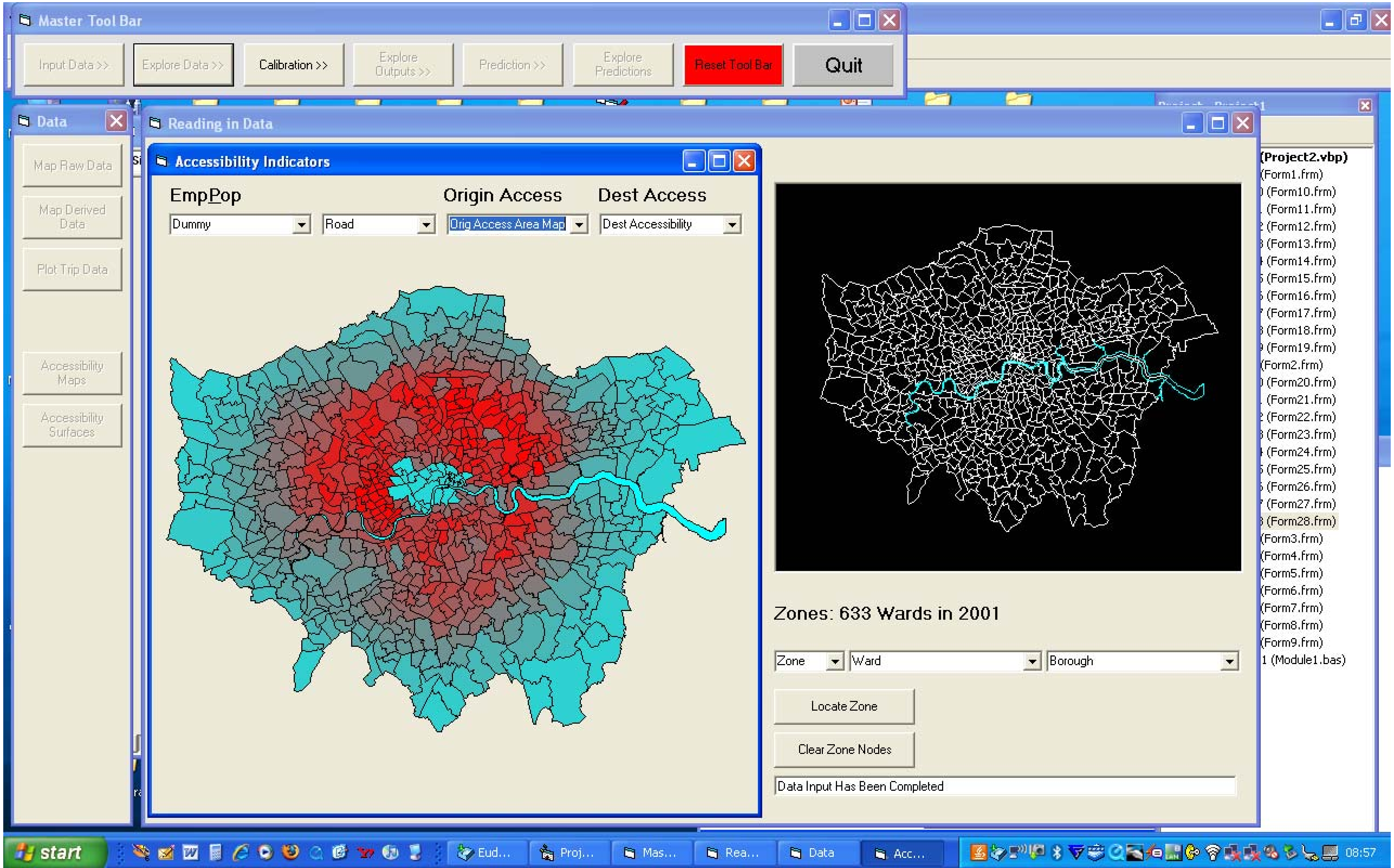
GO!

Program Manual


Master Tool Bar
Input Data >>
Explore Data >>
Calibration >>
Explore Outputs >>
Prediction >>
Explore Predictions
Reset Tool Bar
Quit
Reading in Data
Population, Employment and Floorspace Data
READ Employment Origin Zones 633
READ Population Destination Zones 633
Click Here to Complete the Input of Data Directly
Read Employment Data OK 6 86962.9
Read Population Data OK 10217
Read Floorspace Data OK Floorspace Data
6 7 8 9 10 11 12 13
Physical Line and Area Data
Read Map Data
Centroids
OK Zones X-Centroid
Y-Centroid
Area Data
Coordinates
OK Polygon X-Coordinate
Y-Coordinate
OK Zone
Zone Area
Line Data
Travel Data
Read Trip Data
OK Zone i to j Observed Trips
Mean Trip Length 22.40
Read Cost Data
OK Zone i to j Distance-Cost
Displaying the Physical Map
Click to Redo the Graphics
Click Here If You Wish to Close This Interface
Click to Display Map Now
Click to List Zones
Click to Display Centroids
Zones: 633 Wards in 2001
6 Heathrow Villages Hillingdon
Locate Zone
Clear Zone Nodes



Road: 38%; Bus: 12%: Heavy Rail: 12%: Light Rail 19%; Other (Walk, Bike, Fly): 19%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Data
Map Raw Data
Map Derived Data
Plot Trip Data
Accessibility Maps
Accessibility Surfaces
Reading in Data
Accessibility Indicators
EmpPop
Origin Access
Dest Access
Dummy
Road
Orig Access Area Map
Dest Accessibility
Zones: 633 Wards in 2001
Zone
Ward
Borough
Locate Zone
Clear Zone Nodes
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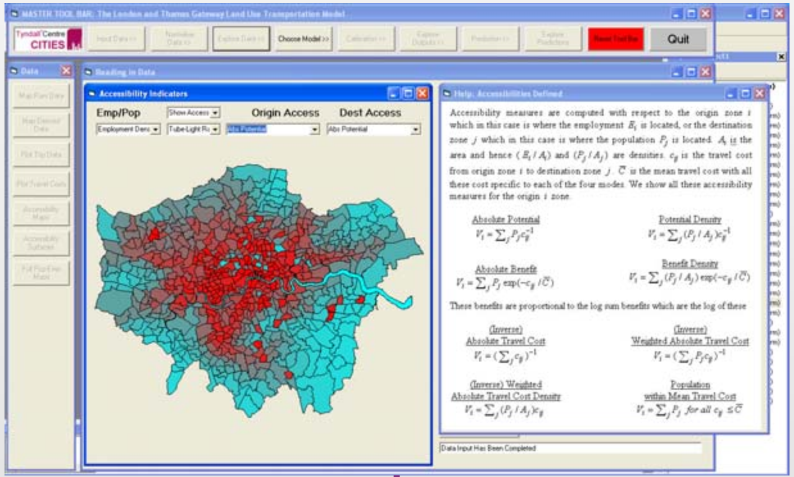
Accessibility from the LUTM model

Many different accessibility measures, 8 in all



Help: Accessibilities Defined

Accessibility measures are computed with respect to the origin zone i which in this case is where the employment E_i is located, or the destination zone j which in this case is where the population P_j is located. A_i is the area and hence (E_i / A_i) and (P_j / A_j) are densities. c_{ij} is the travel cost from origin zone i to destination zone j. $\overline{C}$ is the mean travel cost with all these cost specific to each of the four modes. We show all these accessibility measures for the origin i zone.

Absolute Potential

$$V_i = \sum_j P_j c_{ij}^{-1}$$

Potential Density

$$V_i = \sum_j (P_j / A_j) c_{ij}^{-1}$$

Absolute Benefit

$$V_i = \sum_j P_j \exp(-c_{ij} / \overline{C})$$

Benefit Density

$$V_i = \sum_j (P_j / A_j) \exp(-c_{ij} / \overline{C})$$

These benefits are proportional to the log sum benefits which are the log of these

(Inverse) Absolute Travel Cost

$$V_i = (\sum_j c_{ij})^{-1}$$

(Inverse) Weighted Absolute Travel Cost

$$V_i = (\sum_j P_j c_{ij})^{-1}$$

(Inverse) Weighted Absolute Travel Cost Density

$$V_i = \sum_j (P_j / A_j) c_{ij}$$

Population within Mean Travel Cost

$$V_i = \sum_j P_j \ \ for\ all\ c_{ij} \le \overline{C}$$

Master Tool Bar

Input Data >> | Explore Data >> | Calibration >> | Explore Outputs >> | Prediction >> | Explore Predictions | Reset Tool Bar | Quit

Predict

Input Scenario Data

- Scenario from File
- Employment Changes
- Floorspace Changes
- Distance Changes

Run Scenario Model

- Run Model

More Scenario Runs ...

- Expansion .....
- Expansion .....
- Expansion .....

Reading in Data

Prediction Routines

Long Term Scenarios Based on the Impact of Changes in Employment, Residential Floorspace, and Transport Costs

Predictions with the model involve forecasting the location of small area populations and the trip patterns associated with the four modes used to distribute employment as population to these small (residential) areas. This involves changing the input variables - employment and residential floorspace by small area, and the travel costs associated with each mode of transport, which in turn imply changes to the transport infrastructure. The user also has control over the parameter values on the friction of Travel Cost or travel cost associated with each mode. This can be changed in value to reflect changes in the average Travel Cost or cost travelled on each mode.

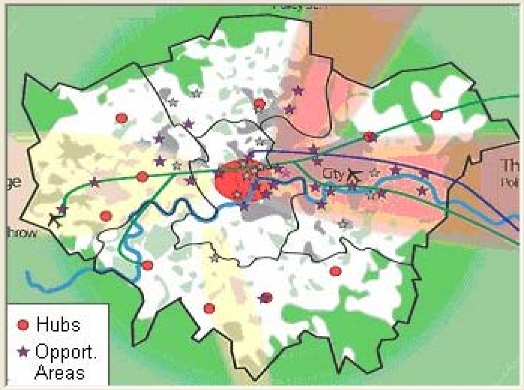
Users have a choice of inputting a preset scenario in which all these variables are changed exogenously or a process of changing these variables interactively, on screen. The interactive process can involve many thousands of changes and is probably best used to input data which reflects 'what-if' scenarios which require a small number of rather simple changes in the inputs reflecting substantial or radical change.

By clicking the 'Scenario from File' button in the toolbar to the left, a preset scenario is loaded and the user is then taken to the point where the model must be run. Alternatively if the user clicks the Employment Changes button, the user activates a screen where each employment zone can be identified by pointing the mouse at it and clicking. Then the user can use a slider bar to increase the value of employment in that zone by up to 100 percent or decrease it by up to 100 percent. As many zones as required can be changed using this method. When the user is satisfied with the employment scenario which has been developed, a button accepting these changes can be clicked. The same can then be done for floorspace activated by clicking the relevant button from the toolbar to the left.

Finally the travel cost on any link by any mode from one zone to another can be changed using the same method. An origin and then a destination zone need to be clicked and then reduced or increased travel cost (by up to 100 percent) made using the slider bar. The user must choose the mode each time and the program then recomputes all the shortest routes implied by these changes once the changes are accepted.

The user then proceeds to run the model as for the 'Scenario from File' option and once this is done, the outputs can be visualised using the same system for exploring the data and calibration results.

Key Elements of the London Plan to 2025 Shown Below.



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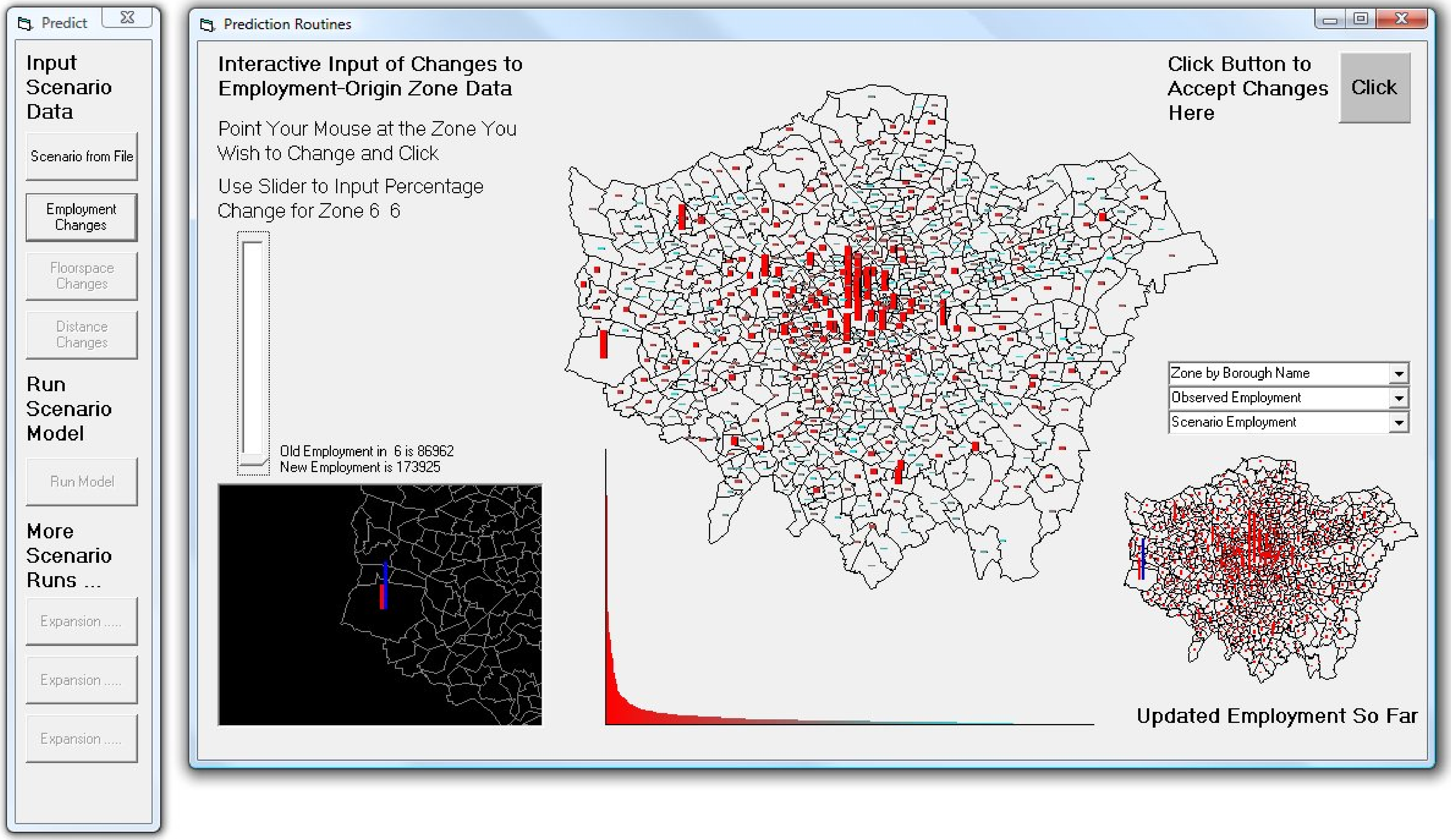
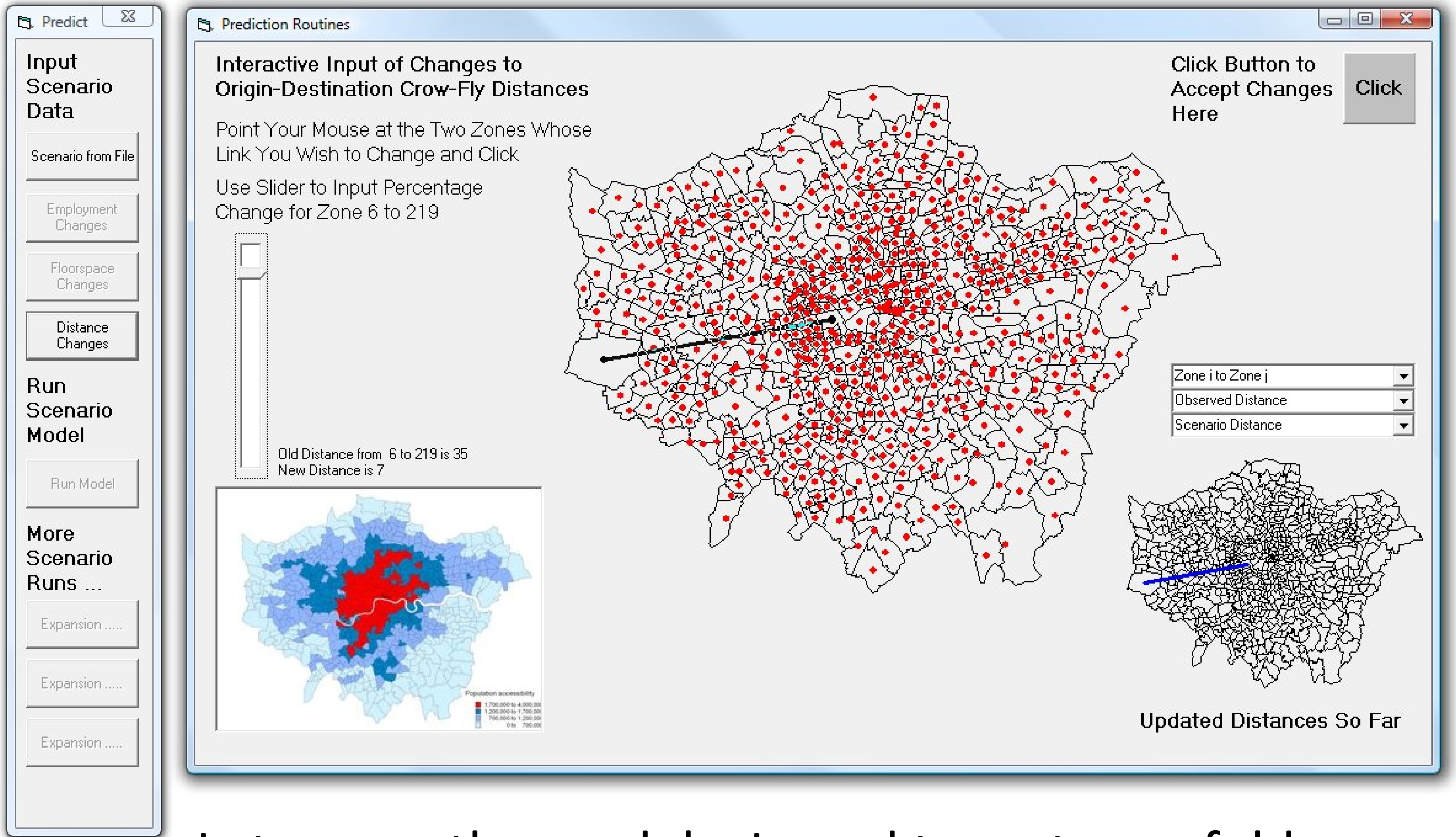

Predict
Input Scenario Data
Scenario from File
Employment Changes
Floorspace Changes
Distance Changes
Run Scenario Model
Run Model
More Scenario Runs ...
Expansion .....
Expansion .....
Expansion .....
Prediction Routines
Interactive Input of Changes to Employment-Origin Zone Data
Point Your Mouse at the Zone You Wish to Change and Click
Use Slider to Input Percentage Change for Zone 6 6
Old Employment in 6 is 86962
New Employment is 173925
Click Button to Accept Changes Here
Click
Zone by Borough Name
Observed Employment
Scenario Employment
Updated Employment So Far



Let us ***run*** the model… I need to go to my folder…>>

Run

For a very old movie of all this go to our web site
http://www.casa.ucl.ac.uk/transportmodel/transportmodel.asp

We need to re-track and say something more about these kinds of spatial interaction models and how they can be extended

Transportation Modelling: The Four Stage Process

I should make a brief point about transport modelling – we have included transport and location together here but traditionally the transport model is based on a four stage process that involves generation, distribution, modal split and assignment

The other issue is that in the standard transport modelling process, once trips are assigned to the network, then one can assess whether the network can take the load – this is matching travel demand against supply and if not then the model is iterated to match demand to supply. This is another generic issue in urban modelling – demand and supply and the way the market resolves this.

Modular Modelling: Coupled Spatial Interaction

Now we have a module for one kind of interaction – consider stringing these together as more than one kind of spatial interaction

Classically we might model flows from home to work and home to shop but there are many more and in this sense, we can use these as building blocks for wider models. This is for next time too

What we will now do is illustrate how we might build such a structure taking a journey to work model from Employment to Population and then to Shopping which we structure as --

First we have the journey from work to home model as

$$T_{ij} = E_i \frac{F_j \exp(-\lambda c_{ij})}{\sum_j F_j \exp(-\lambda c_{ij})} \quad , \quad \sum_j T_{ij} = E_i$$

$$P_j = \alpha \sum_i T_{ij}$$

And then the demand from home to shop

$$S_{jm} = P_j \frac{W_m \exp(-\beta c_{jm})}{\sum_m W_m \exp(-\beta c_{jm})} \quad , \quad \sum_m S_{jm} = P_j$$

$$S_m = \sum_j S_{jm}$$

And there is a potential link back to employment from the retail sector $E_m = f(S_m)$

A Simple Example of Modularity: Lowry's Model

Lowry's (1964) model of Pittsburgh was a model of this nature but it also incorporated in it – or rather its derivatives did more formally – a generative sequence of starting with only a portion of employment – basic – and then generating the non-basic that came from this. This non-basic set up demand for more non-basic and so on until all the non-basic employment was generated, and this sequence followed the classic multiplier effect that is central to input-output models.

A block diagram of the model follows

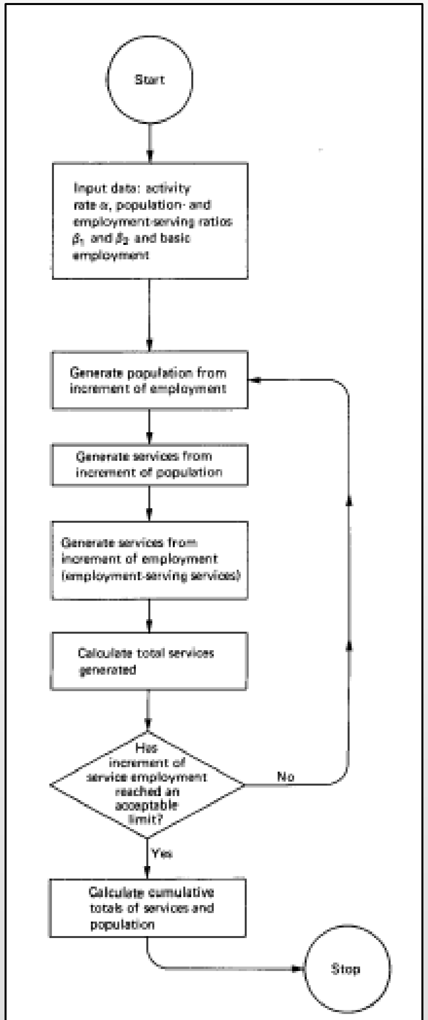


Fig. 2.1. The sequence of operations in generating population and services from basic employment.

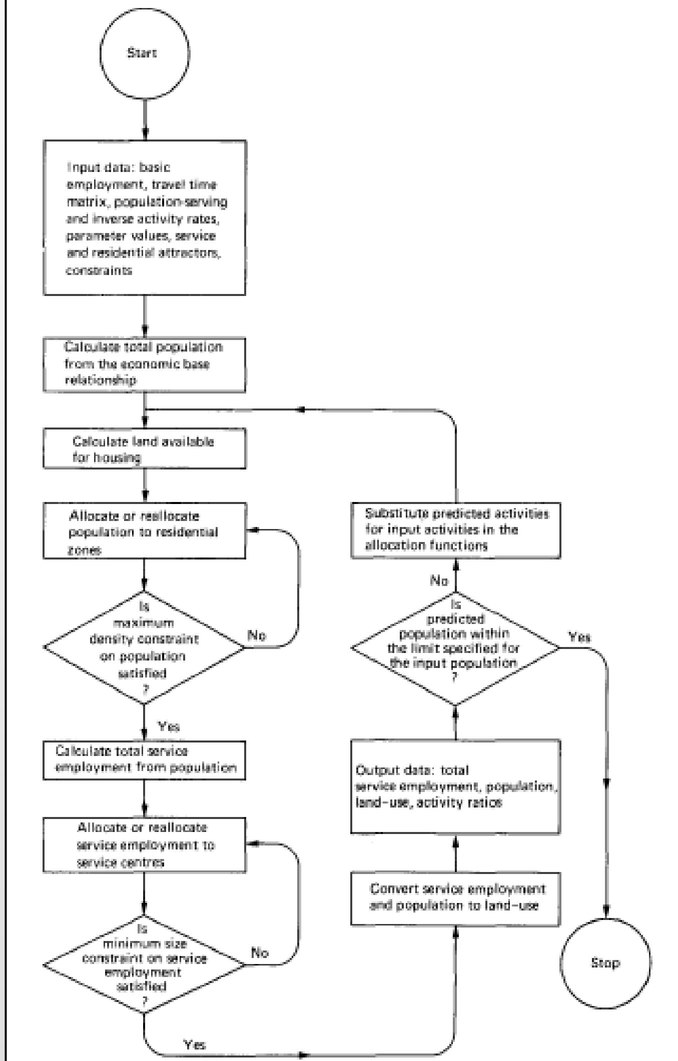


Fig. 3.2. Generalised flow chart of the Lowry model.

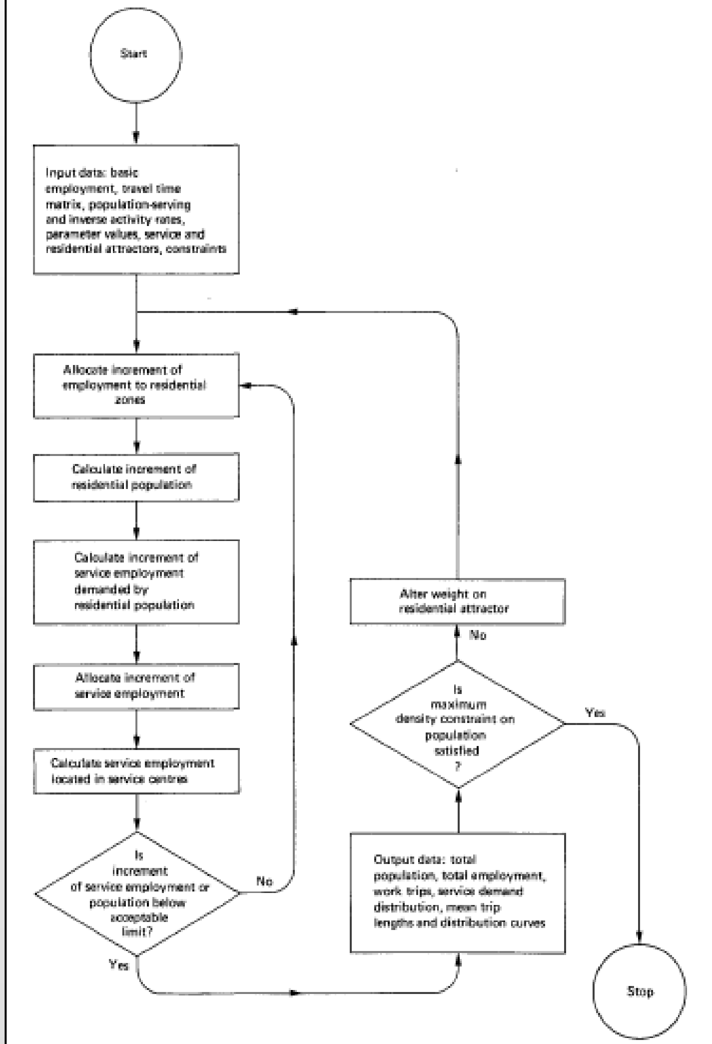


Fig. 3.3. Generalised flow chart of the Activity Allocation model.

From http://www.casa.ucl.ac.uk/urbanmodelling/

DRAM-EMPAL Style Models

Essentially what we have here is the notion of simultaneous dependence – ie one activity generates another but that other activity generates the first one – what came first – the chicken or the egg?

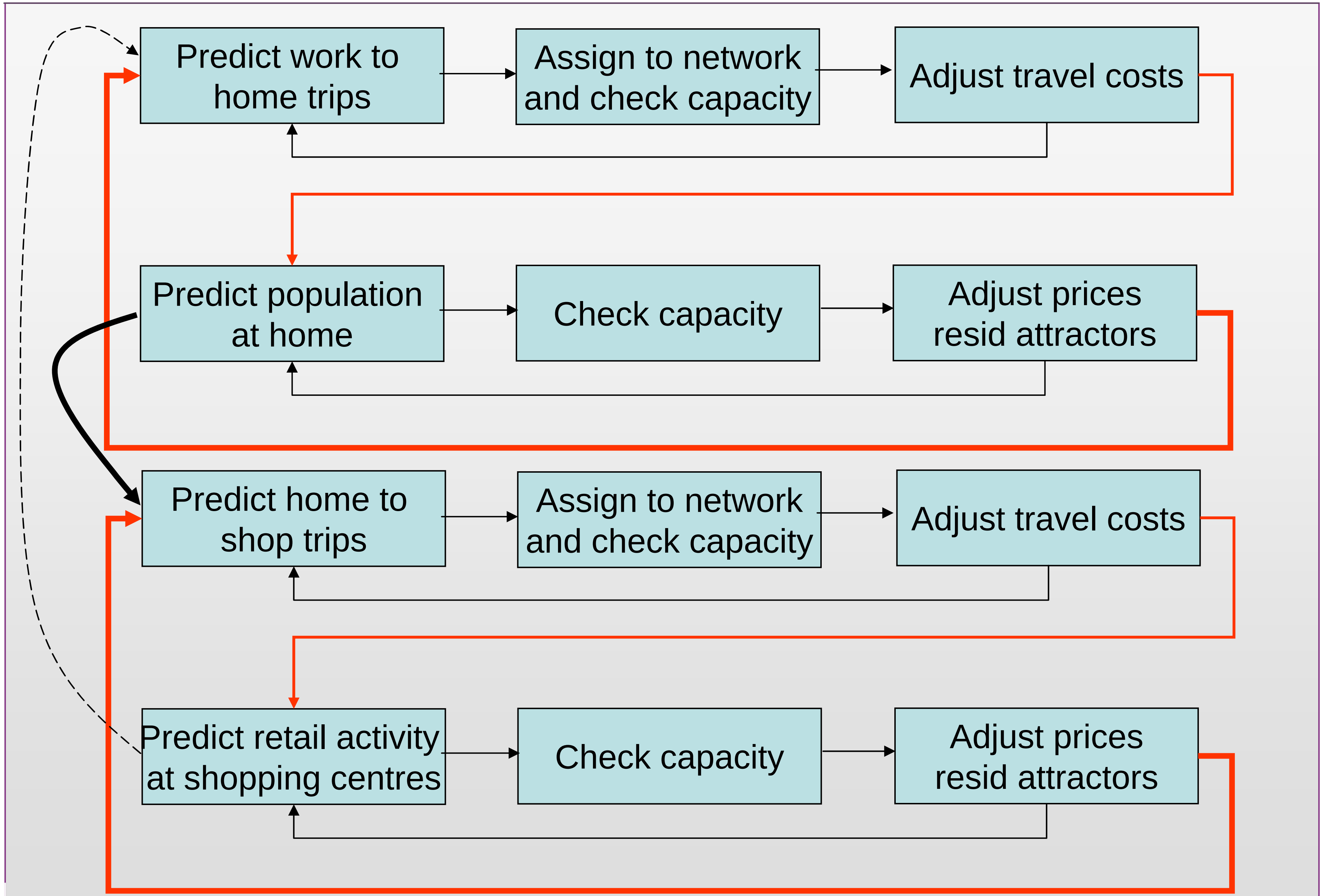
Stephen Putman developed an integrated model to predict residential location DRAM and another to predict employment location EMPAL. In essence different models are used to do each – the employment model tends to be based on very different factors – it is a regression like model of key location factors not a flow model

Demand and Supply: Market Clearing

So far most of these models have been articulated from the demand side – they are models of travel demand and locational demand – they say nothing about supply although we did introduce the notion that in simulating trips and assigning these to the network, we need to invoke supply.

When demand and supply are in balance, then the usual signal of this is the price that is charged. In one sense the DRAM EMPAL model configures residential location as demand and employment location as supply but most models tend to treat supply as being relatively fixed, given, non-modellable

- However several models that couple more than one activity together treat supply as being balanced with demand, often starting with demand, seeing if demand is met, if not changing the basis of demand and so on until equilibrium is ascertained. Sometimes prices determine the signal of this balance. If demand is too high, price rises and demand falls until supply is met and vice versa
- Most urban models do not attempt to model supply for supply side modelling is much harder and less subject to generalisable behaviour
- A strategy for ensuring balance is as follows for a model with two sectors – like the one we illustrated earlier


Predict work to home trips
Assign to network and check capacity
Adjust travel costs
Predict population at home
Check capacity
Adjust prices resid attractors
Predict home to shop trips
Assign to network and check capacity
Adjust travel costs
Predict retail activity at shopping centres
Check capacity
Adjust prices resid attractors

The decision to nest what loop inside what other loop is a big issue that makes these models non-unique

If the supply side is modelled separately then the way this is incorporated further complicates the sequence of model operations.

In large scale integrated models, that we will deal with next time these are crucial issues

In fact we don’t have time but there is one further structural issue we will deal with when we meet next time and this is

Input-Output: The Echenique Models

Reading for this lecture

up on the web today

http://www.spatialcomplexity.info/CUSP/

Here is an unashamed plug for my new book

Chapters 2, 3 & 9 deal with some of the material in these lectures

Look at the blog to get details