CENTER FOR URBAN SCIENCE+PROGRESS

Foundations of Urban Science
Fall 2013:

Lecture 9: Batty's Lecture 1-1

Networks and Flows in the Science of Cities:

Social Physics, Accounting, Spatial Interaction

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Outline for Lecture 9: 1-1

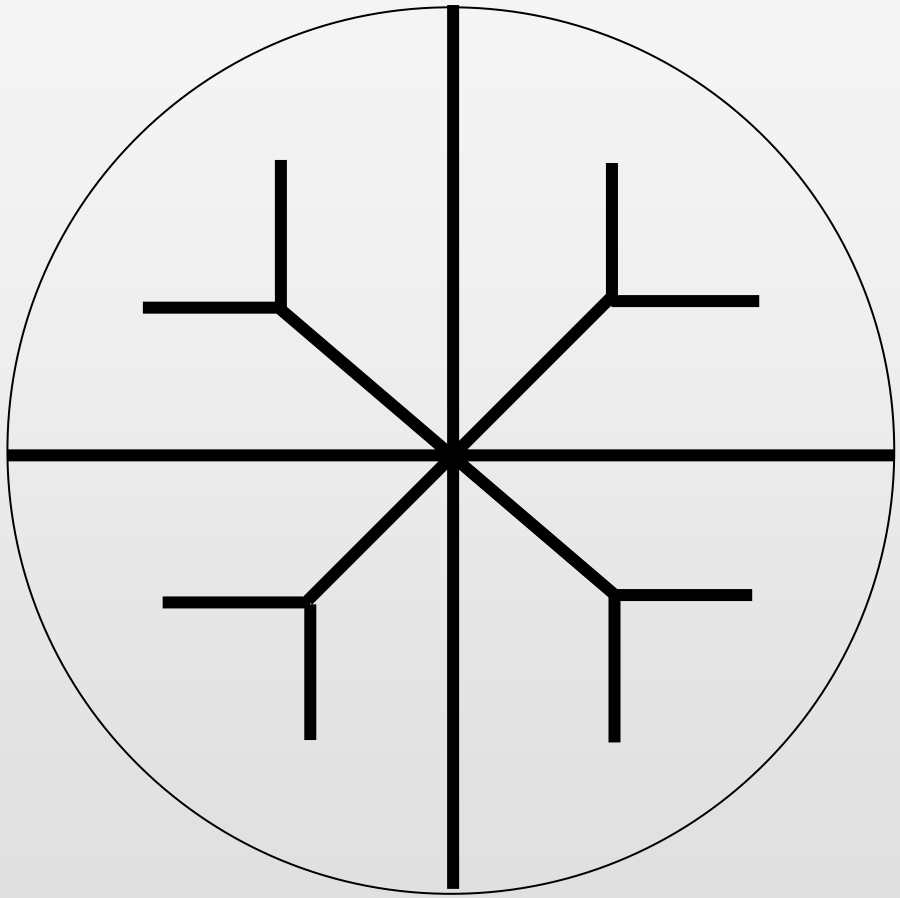
- Ideas about Networks and Flows
- Spatial Structure: Profit and Cost, Distance, Agglomeration, Accessibility
- Two Examples: Von Thunen and DLA
- Gravitation: The Basic Models
- Trip Distribution: Constraints on Volume & Location
- Derivation Methods: Entropy-Maximising
- Residential Location, Modal Split
- Transportation Modelling: The Four Stage Process
- Modular Modelling: Coupled Spatial Interaction

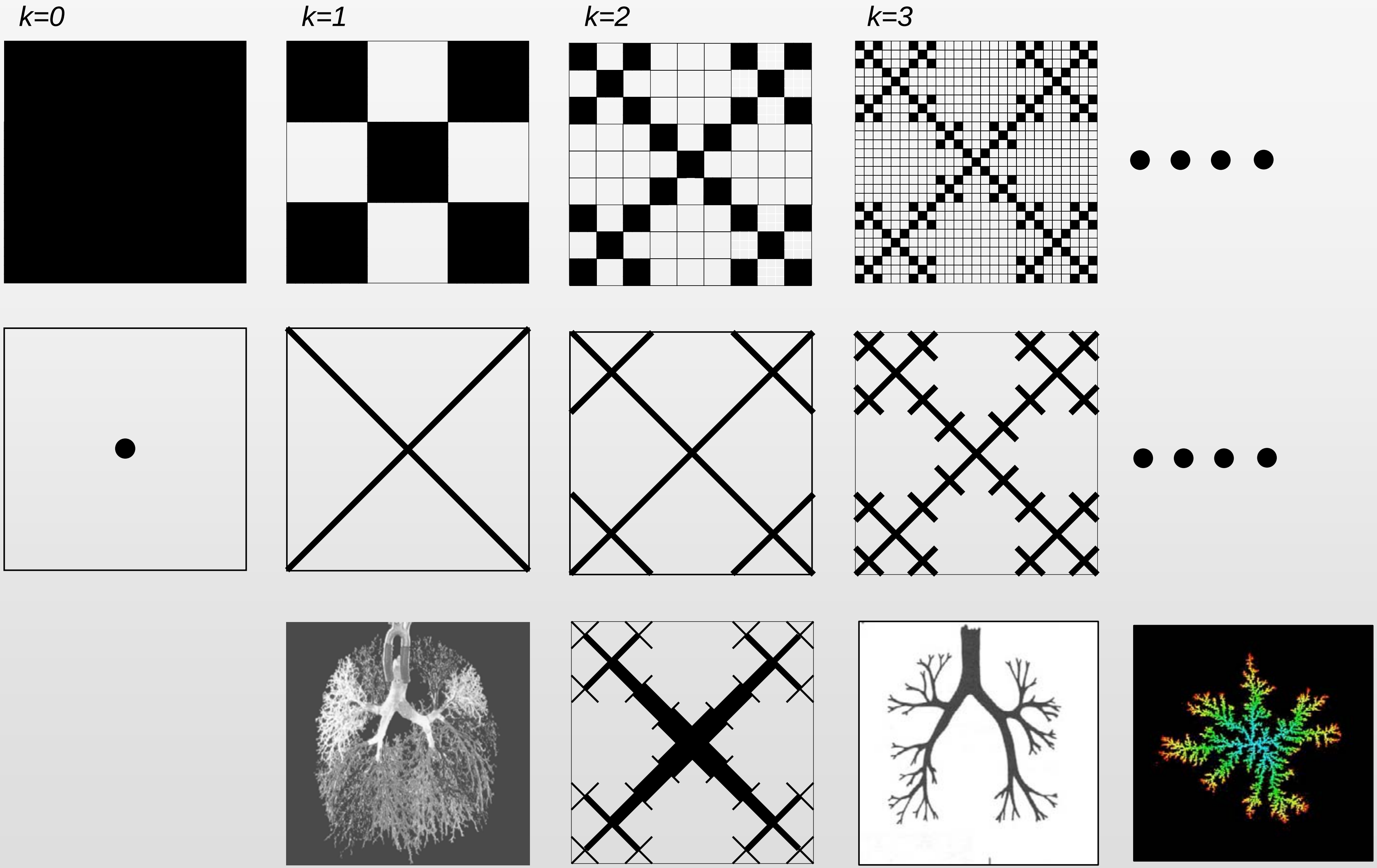
Ideas about Networks and Flows

We can think of cities as being network structures where the idea of the network is to deliver energy and information to the bits that make up the city in the most efficient way

Networks cannot serve everywhere – they are linear structures for canalising energy and information to distribute it to all the points or nodes that define the city.

Geoff West talked to you about allometry and scale and implicit in his discussion is that networks which branch like trees are the most efficient ways of distributing energy to the city. As the city grows around a market center or CBD, the most efficient way of distribution is the tree

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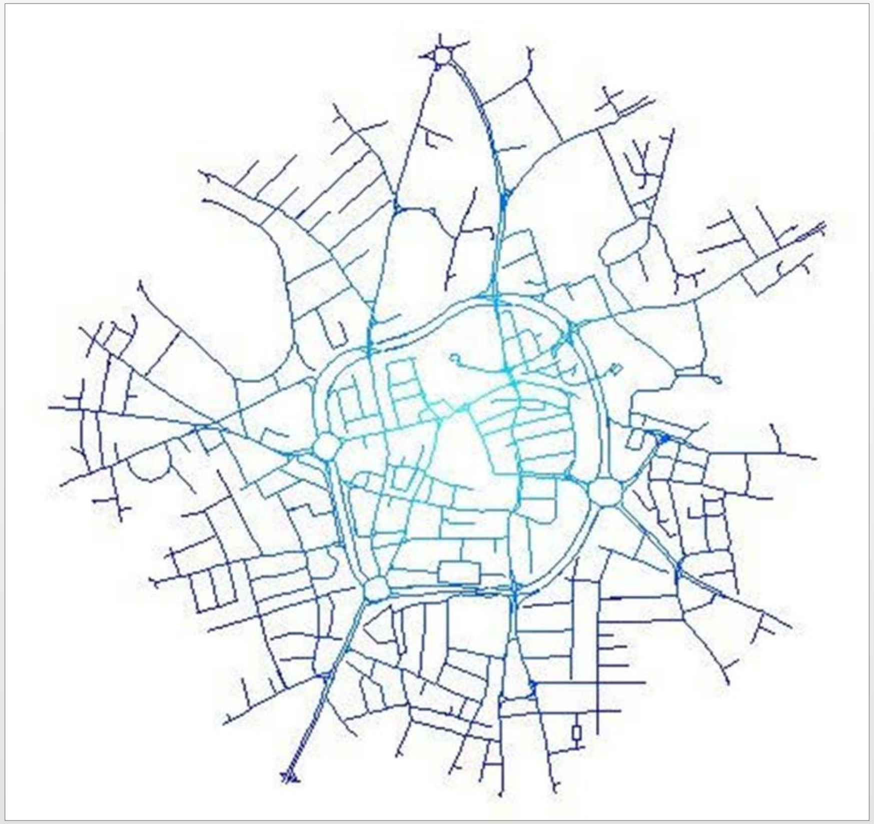
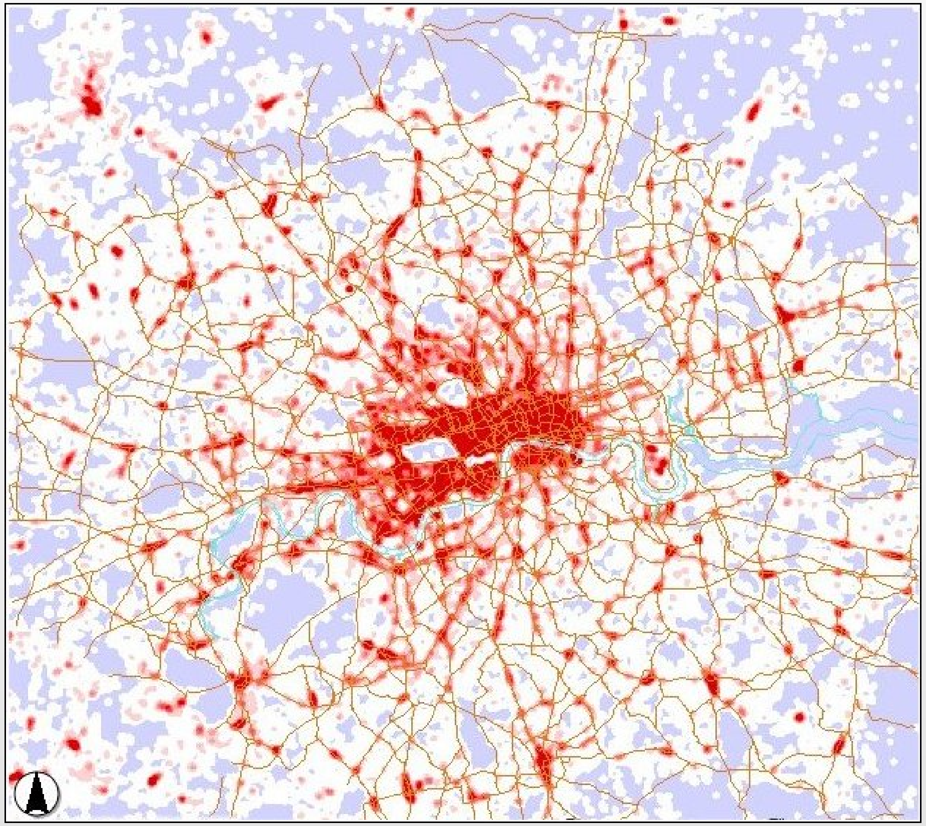
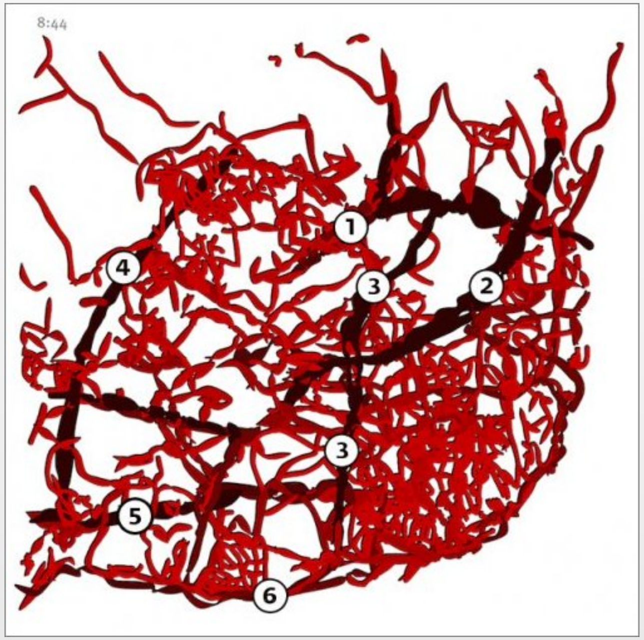




This is a small English town Wolverhampton population about 400,000

This is a large city: Greater London population about 8 million

Some examples of networks in cities, mainly London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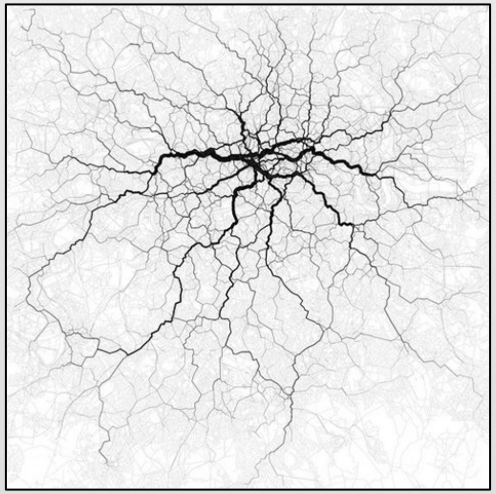


Spatial Structure: Profit and Cost, Distance, Agglomeration, Accessibility

We don't have time for a detailed development of theories of spatial structure but benefits and costs – profits and losses - are key to location. These define the pull and push factors of selecting any location defining agglomeration and dis-agglomeration economies, & accessibility and inaccessibility

We will define two very different approaches that lead to the same sort of structures

First from urban economics, and Second the same phenomena from physical movement

Von Thunen and then DLA

Essentially benefits (or profits) depend on nearness to market which under normal profits we measure as rent payable

This is balanced against the cost of transport to the market. This might relate to space that one might get – as one moves further away from a point, more space is accessible

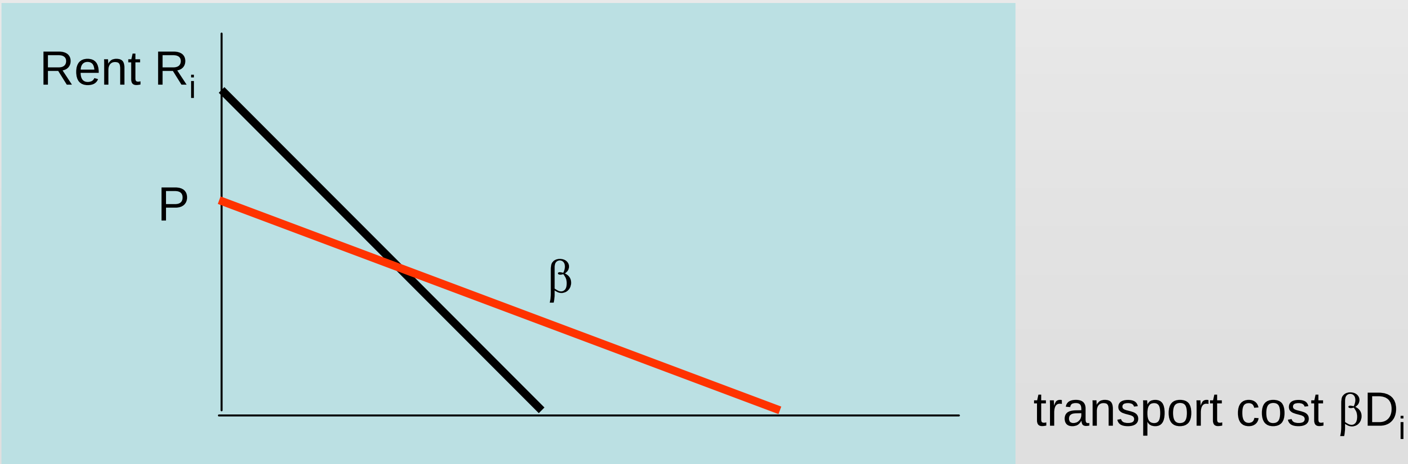
Who locates where depends on trade off of benefits versus transport cost

We define profit or yield at the market P, distance D_i at location i from the market or centre, transport cost per unit of distance β and then rent payable at i which is R_i

We thus define the key equation for the costs and benefits of location as

$$R_i = P - \beta D_i$$

This is a linear equation where we can think of profit or yield as the intercept and slope as the transport cost per unit of distance. As we vary these the slope of the line will vary as

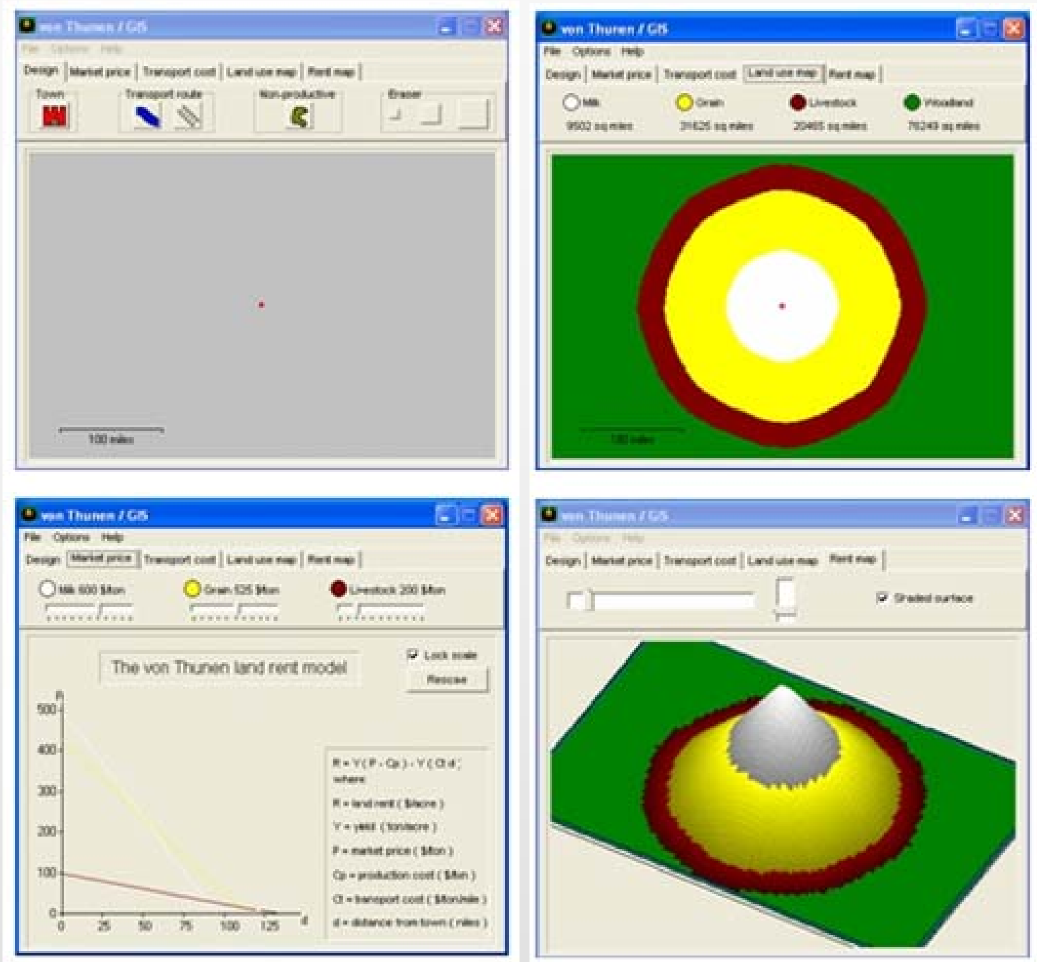


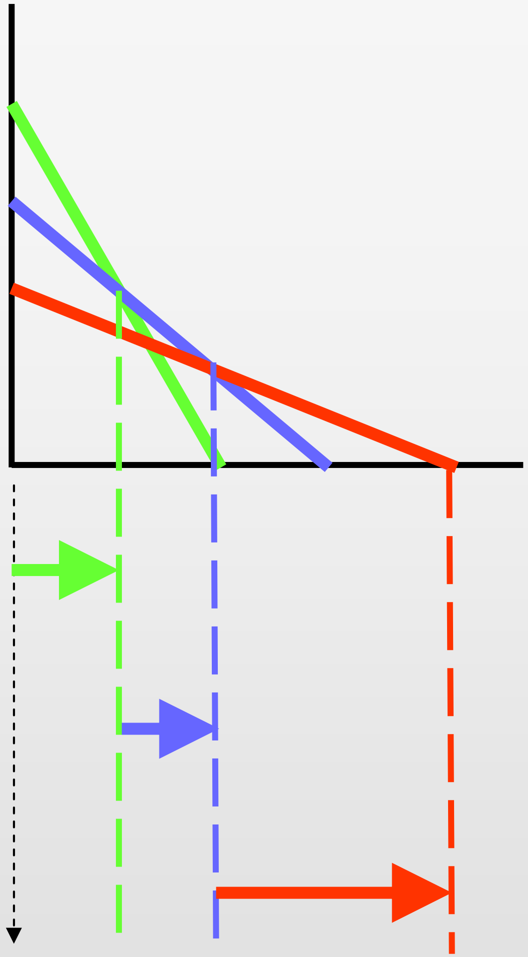
Let me load the von Thunen model and show you how this works

Go to our web site and download to save or run the model from http://www.casa.ucl.ac.uk/software/vonthunen.asp

What you see is the following which I will show before we run it: a simply canvas on which you plant a market, you can adjust the benefits and cost equation for a series of different land uses – all agricultural which imply different types of production and transport of goods to market

And market clearing takes place where one land use outbids in terms of rent any other: this assumes normal profits





Land use are determined by the bid rent curves in terms of their dominance and the circular pattern of land use falls out from this – I will run the model but you should do it too

The Diffusion-Limited Aggregation Model

My second model is quite different – it is one that I will introduce next week as part of Cellular Automata (CA). This is a physical model in which an agent wants to locate at a town to get economies of scale but also wants to be as far from the town as possible.

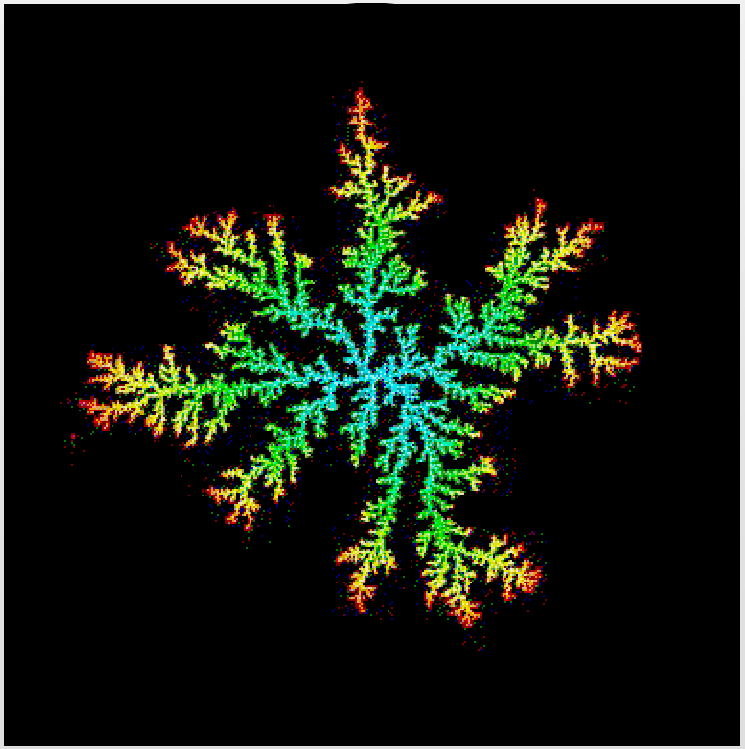
Thus the agent wants to realise agglomeration, clustering but also economies of getting as much space as possible

Thus the issues are to balance centripetal forces with centrifugal.

We can set up a model to show how this occurs. Plant a seed and let many agents wander randomly in a circular region around the seed. The rule for fixing location are dead simple

When an agent touches another agent that is already fixed then that agent sticks and doesn’t move any more

The first agent of course to find the seeds sticks at the seed – the market centre and all agents wander randomly in space.



What we get is the following dendritic pattern: this is a model called diffusion limited aggregation, DLA where the diffusion is limited or constrained

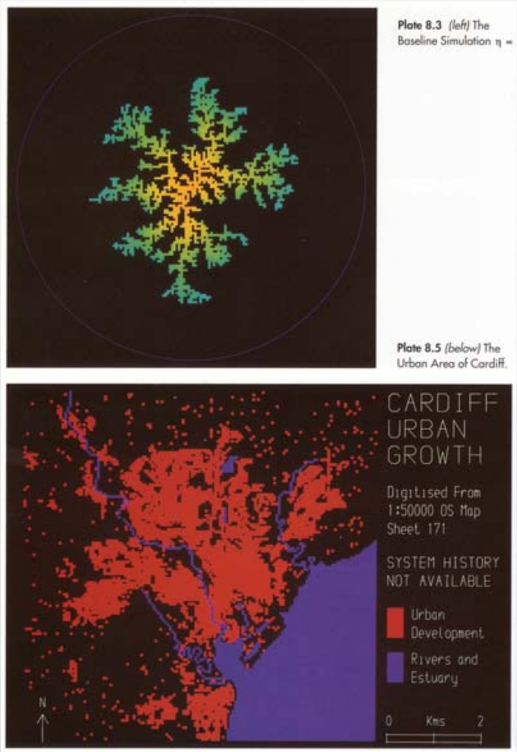


Plate 8.3 *(left)* The Baseline Simulation η =

Plate 8.5 *(below)* The Urban Area of Cardiff.

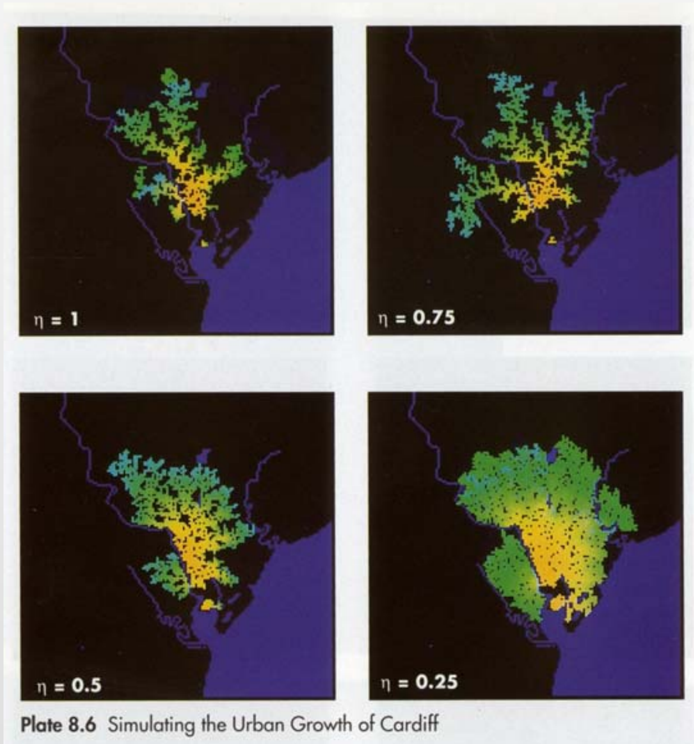


Plate 8.6 Simulating the Urban Growth of Cardiff

There are many examples on the internet that you can search for yourselves: one is from the Boston group and I will run it

Gravitation: The Basic Models

Ok, we can incorporate these ideas in the basic model of forces which was first articulated by Newton as his second law of motion – force is proportional to mass times acceleration

In more conventional terms we might write the force between two bodies as

$$F_{12} \sim M_1 \, M_2 \, / \, (d^2)_{12}$$

There is a very long history of analogies between force and social interaction going back to Newton himself. There are many references and I will add some to the web page

But let me immediately generalise this and say that we need to define many interactions – we break our system in to areas or points which we define as origins and destinations i and j

And then we measure the distance as in von Thunen not as distance per se but as travel cost or rather generalised cost c_{ij}

We also define the mass at the origins and destinations as O_i and D_j and we then write the conventional spatial interaction of gravity model as

$$T_{ij} \sim \frac{P_i P_j}{c_{ij}^2} = K \frac{P_i P_j}{c_{ij}^2}$$

Where K is the gravitational constant

- This is the model that has been used for years but in the 1960s and 1970s various researchers cast it in a wider framework – deriving the model by setting up a series of constraints on its form which showed how it might be solved and produced various generating mechanisms that could generate consistent models
- The *constraints logic* led to **consistent accounting**
- The *generative logic* lead to analogies between **utility and entropy maximising** and opened a door that has not been much exploited to date between entropy, energy, urban forms physical morphology and economic structure
- In particular the economic logic rather than the energy entropy logic was called choice theory, specifically discrete choice

Now to introduce all this, we need to define some more terms

We will refer to the size of volume of origins and destinations not as population P but as O_i and D_j assuming they are different from one another. We will also assume that the inverse square law on distance or travel cost does not apply and that whenever c_{ij} appears it will be parameterised with a value that varies which we call λ

We will assume trips are as we have defined them as T_{ij} but we will also normalise trips by their total volume T to produce probabilities

$$p_{ij} = \frac{T_{ij}}{T} = \frac{T_{ij}}{\sum_{ij} T_{ij}}$$

Note that we use summation extensively in what follows

Trip Distribution: Constraints on Volume & Location

We must move quite quickly now so let me introduce the basic constraints on spatial interaction and then state various models

The constraints are usually specified as origin constraints and destination constraints as

$$O_i = \sum_j T_{ij}$$

$$D_j = \sum_i T_{ij}$$

And we can take our basic gravity model and make it subject to either or both of these constraints or not at all

So what we get are four possible models

Unconstrained $T_{ij} = KO_i D_j c_{ij}^{-\lambda}$

Singly (Origin) Constrained so that the volume of trips at the origins is conserved $T_{ij} = A_i O_i D_j c_{ij}^{-\lambda}$

Singly (Destination) Constrained so that the volume of trips at the destinations is conserved $T_{ij} = B_j O_i D_j c_{ij}^{-\lambda}$

Doubly Constrained trip volumes at origins + destinations are conserved $T_{ij} = A_i B_j O_i D_j c_{ij}^{-\lambda}$

The first three are location models, the last is a traffic model

Ok, so what are these parameters that enable the constraints to me met – well they can be very easily produced by summing each model over the relevant subscripts – ie origins or destinations and then simply substituting and rearranging

I will do this but I will leave you to work through the algebra in your own time and many of you will know this anyway. Here are the factors which are sometimes called balancing factors

Unconstrained $K = T \Big/ \sum_i \sum_j O_i D_j c_{ij}^{-\lambda}$

Origin Constrained $A_i = 1 \Big/ \sum_j D_j c_{ij}^{-\lambda}$

Destination Constrained $B_j = 1 \Big/ \sum_i O_i c_{ij}^{-\lambda}$

Doubly Constrained $A_i = 1 \Big/ \sum_j B_j D_j c_{ij}^{-\lambda}$ $B_j = 1 \Big/ \sum_i A_i O_i c_{ij}^{-\lambda}$

Derivation Methods: Entropy-Maximising

Now we have only dealt with constraints through consistent accounting – we now need to deal with generative methods that lead to the same sort of accounting– entropy maximising, information minimising, utility maximising and random utility maximising, and also various forms of nonlinear optimisation – in fact all these methods may be seen as a kind of optimisation of an objective function – entropy utility and so on – subject to constraints

We will define entropy maximising. First we define entropy as Shannon information and we convert all our equations and constraints to probabilities. Shannon entropy is

$$H = -\sum_i \sum_j p_{ij} \log p_{ij}$$

We maximise this entropy subject to the previous constraints – dependent on what kind of model we seek but noting now that we need another constraint on travel cost which is equivalent to energy so that we can derive a model

We thus set up the problem as

$$\max \quad H = -\sum_i \sum_j p_{ij} \log p_{ij}$$

subject to

$$\sum_j p_{ij} = p_i$$

$$\sum_i p_{ij} = p_j$$

$$\sum_i \sum_j p_{ij} c_{ij} = \hat{C}$$

But note that the probabilities always add to 1, that is

$$\sum_i \sum_j p_{ij} = \sum_i p_i = \sum_j p_j = 1$$

I am not going to work this through by setting up a Lagrangian and differentiating it and then getting the result. There is a lot of basic algebra involved and all I want to show is the result

For this optimisation the model that we get can be written as

$$p_{ij} = \exp(-\lambda_i - \lambda_j - c_{ij}^{-\lambda})$$

or

$$T_{ij} = Tp_{ij} = A_i O_i B_j D_j \exp(-\lambda c_{ij})$$

Let us note many things

1. This is the doubly constrained model but with an exponential of travel cost replacing the inverse power
2. We can get any of the other constrained models in the family by dropping constraints and we can do this directly

3. We can begin to explore what entropy means by substituting the probability model into the entropy equation – I will reserve this for an ad hoc seminar on entropy if you are interested.
4. We can think of this method as one in which the most likely model is generated given the information which is in the constraints
5. In terms of statistical physics, this model is essentially the Boltzmann-Gibbs distribution
6. Entropy can be seen as utility under certain circumstances
7. We solve the model – ie find its parameters by solving the entropy program which is equivalent to solving the maximum likelihood equations
8. We can then use this scheme to develop many different kinds of model – where we add more and more constraints and also disaggregate the equations to deal with groups

Residential Location, Modal Split

Let me illustrate in two ways how we can build models using this framework

First if we say that residential location depends on not only travel cost but also on money available for housing we can argue that

1. The model is singly constrained – we know where people work and we want to find out where they live – so origins are workplaces and destinations are housing areas
2. The model then lets us predict people in housing
3. We argue that people will trade off money for housing against transport cost

And we then set up the model as follows

It is

$$\sum_j T_{ij} = O_i$$

$$\sum_i \sum_j T_{ij} c_{ij} = C$$

$$\sum_i \sum_j T_{ij} R_j = R$$

leads to

$$T_{ij} = A_i O_i \exp(\vartheta R_j) \exp(-\lambda c_{ij})$$

We can of course find out from this location model how many people live in destination housing zones, so it is a distribution as well as a location model

$$P_j = \sum_i T_{ij}$$

In terms of modal split we break the trips into different modes and then let the modes compete with locations for travellers

In this way we produce a combined modal split location model.

Sometimes we may want the modes to be constrained and in generating specific constraints on total travellers by mode, this is equivalent to adding parameters that distort the travel costs – in fact the generic equation can be seen as one where the travel cost or energy is modified by the volume constraints

That is

$$p_{ij} = \exp(-\lambda_i - \lambda_j - c_{ij}^{-\lambda})$$

or

$$T_{ij} = Tp_{ij} = A_i O_i B_j D_j \exp(-\lambda c_{ij})$$

Transportation Modelling: The Four Stage Process

I should conclude with saying that the transport model is part of a four stage process that involves generation, distribution, modal split and assignment and that the spatial interaction approach can be seen as either applying solely to distribution and modal split or in more integrated ways

So far all our models have been demand model but the transport system is capacitated and in transport modelling we need to assessing trips to the network and then figure out if it is possible to meet theses assignments in terms of supply – if not we iterate to clear the transport market

The same is true of the residential market in terms of supply and we will develop all these ideas in the next talk

Modular Modelling: Coupled Spatial Interaction

Now we have a module for one kind of interaction – consider stringing these together as more than one kind of spatial interaction

Classically we might model flows from home to work and home to shop but there are many more and in this sense, we can use these as building blocks for wider models. This is for next part too

Finally if we have time I will demo our London model but we will look at this in the next part of the lecture.

Reading for the first part of this lecture

I will put all the up on the web site today as well as references

Any Questions?

www.spatialcomplexity.info/CUSP

Here is an unashamed plug for my new book

Chapters 2, 3 & 9 deal with some of the material in these lectures

Look at the blog to get details