




Session 1: Lecture 1:
An Outline of Complexity Theory

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<http://www.complexcity.info/>
<http://www.spatialcomplexity.info/>



My Key Topics

- The Course
- Complexity in Spatial Systems with a Focus on Cities: A Brief Chronology and History
- Properties of Complex Systems: Decentralisation, Hierarchy, Emergence, Path Dependence
- Physical Demonstrations: Urban Form and Function
- Size and Scaling: Four (or more) Laws of Scaling
- Measuring Complexity: Spatial Complexity
- Two More Things : 1 Network Science 2: Dynamics
- How Do We Use This Perspective in Designing Cities?
- Conclusions: Scale and Size Again: The Next Lecture



The Course

10 lectures in five sessions

I will be adapting the lectures from the spatial complexity web site and from the handbook

I want you to log on and register with the **Complexity Explorer** and read watch the first set of videos

Next week in the workshop when I am away, we will arrange for the last three of the videos to be presented

So log onto www.complexityexplorer.org/ More about this below on the next page

Resources: the websites: www.complexcity.info and www.spatialcomplexity.info



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I want you to log onto

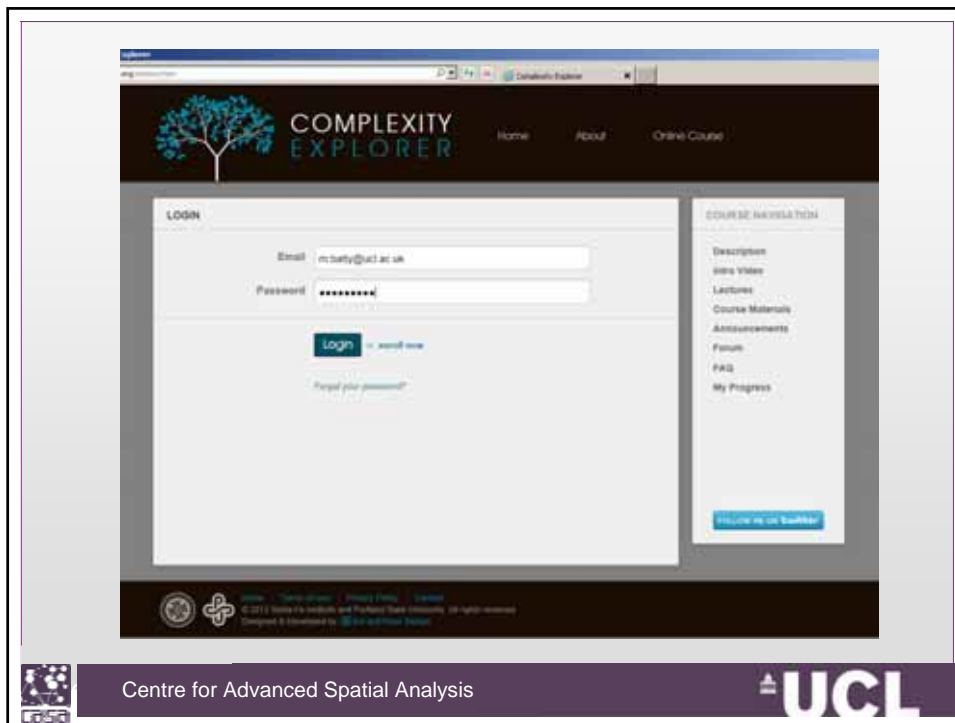
www.complexityexplorer.org/

And register – and use this as a background resource in complexity and spatial simulation --- and then



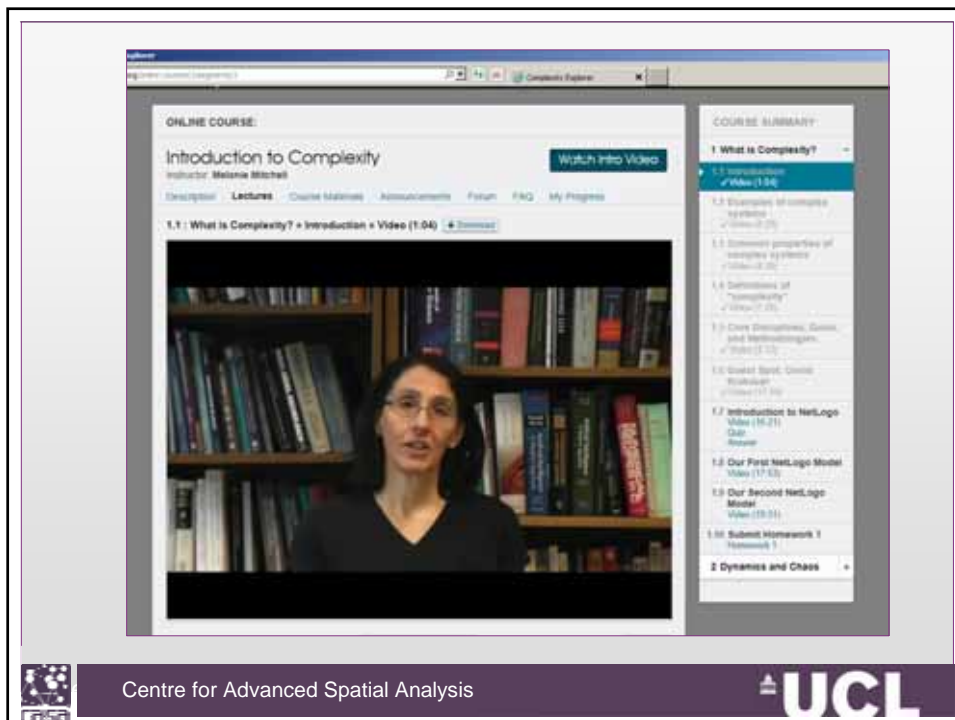
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The screenshot shows the homepage of the 'A Science of Cities' website. At the top, there is a navigation menu with links for Home, About, Publications, Events, Contact, and a search bar. Below the navigation is a header section titled 'A Science of Cities' with five small images representing different aspects of urban science. The main content area features three article teasers: 'Towards a Science of Cities', 'The Dangerous Lecture', and 'Urban Transformations and Urban Structure'. Each teaser includes a small image and a brief text snippet. On the right side, there is a 'Recent Posts' section with a list of article titles and dates.

<http://www.complexcity.info/>

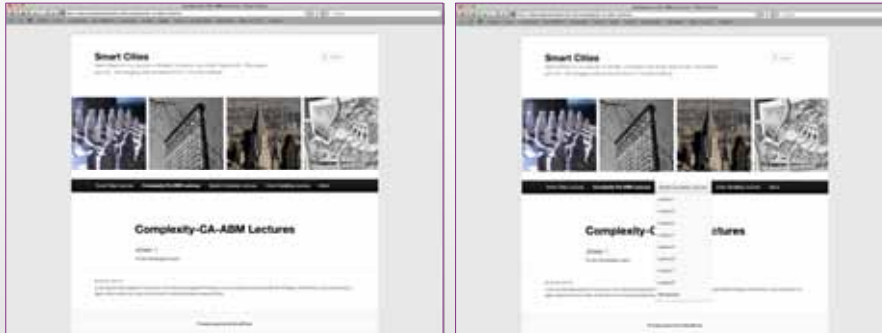
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The screenshot shows the homepage of the 'Smart Cities' website. At the top, there is a navigation menu with links for Home, About, Publications, Events, Contact, and a search bar. Below the navigation is a header section titled 'Smart Cities' with four small images representing different aspects of smart cities. The main content area features two article teasers: 'Geodesign' and another article with a book cover image. Each teaser includes a small image and a brief text snippet. On the right side, there is a 'Recent Posts' section with a list of article titles and dates.

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<http://www.spatialcomplexity.info/>

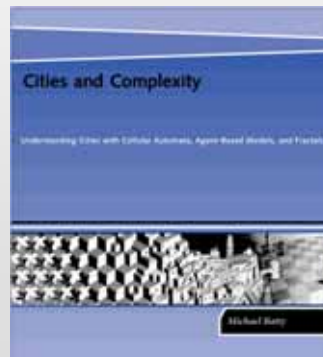


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Two references that I will refer to – one online, one in the library I hope,

My paper: <http://www.complexcity.info/flows/> This is background a bit technical *and then* My book again technical *but dip into it*



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Complexity in Spatial Systems with a Focus on Cities:

Ideas about complex systems go back to prehistory, but it was not until the 1920s that these ideas began to develop coherently in the systems approach

The convergence of biological systems thinking with engineering and control in cybernetics and Operations Research led to formal statements after WW2.

The key notion was that systems were ordered and the idea was to express this order in generic terms that could be applied to any ordered collections of ideas, objects

The Systems Approach which developed in the 1950s and 1960s found its clearest expression in areas where theory and practice was inchoate, poorly developed, somewhat arcane.



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Systems were conceived as being centrally ordered, composed of subsystems organised hierarchically, dominated by negative feedback which suggested they were in equilibrium for the most part, and subject to explicit control.

By and large, good candidates for such applications were things like cities and regions, that looked as though they might be in equilibrium, largely because their physical form was relatively inert and seemed unchanging for the most part.

But nothing could be further from the reality. We made the mistake of assuming that 'what we get is what we see'. Not so. Cities are never in equilibrium, they are constantly changing, they are dominated not by negative but by positive feedback. They are the crucibles of innovation. Their behaviour can be surprising, unpredictable. They are much more volatile than their appearance suggests.



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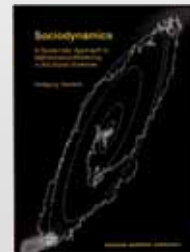
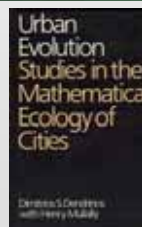
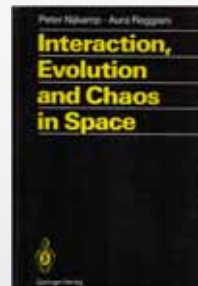
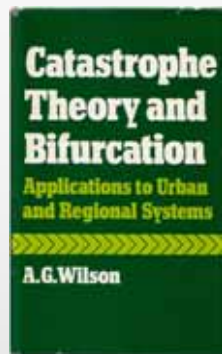
Properties of Complex Systems: Decentralisation, Hierarchy, Emergence, Path Dependence

In essence, this movement gathered pace informally but then more formally in computer science where the idea of control from the bottom up came onto the agenda, in economics to an extent and of course in physics where notions of dynamics were being explored.

The notion of smooth change was quickly abandoned as ideas concerning catastrophe theory, bifurcation, chaos came onto the agenda. The notions of systems being far-from-equilibrium took hold. The notion of positive feedback is essential to these dynamics. Here are contributions in the 80s, early 90s from people, some of whom are still very active



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Much of the formal development of the idea that systems evolve from the bottom up has come from the Santa Fe Institute but the superstructure of ideas in this area is now much, much larger.

Essential to the notion of systems that are organised from the bottom up is that they evolve from their constituent parts, they grow and change, but most of all order and pattern emerge from the basic soup.

Emergence is key: it suggests no overall control, a limit on predictability, the notion that where we start matters – history matters – and that what ultimately comes about is dependent on the path we take – path dependence

Many of the old ideas in systems – hierarchy, interaction, subsystem structure – are still important to these concepts.



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Physical Demonstrations: Urban Form and Function

Ok. Let me try to demonstrate some of these ideas visually. One of the reasons why complexity theory emerged is because when we look at physical forms – patterns in man-made artefacts -- cities, and in natural -- rivers – we see order and organisation and hierarchy that emerges from the bottom up.

We see this best in nature really but we see it in cities rather clearly.

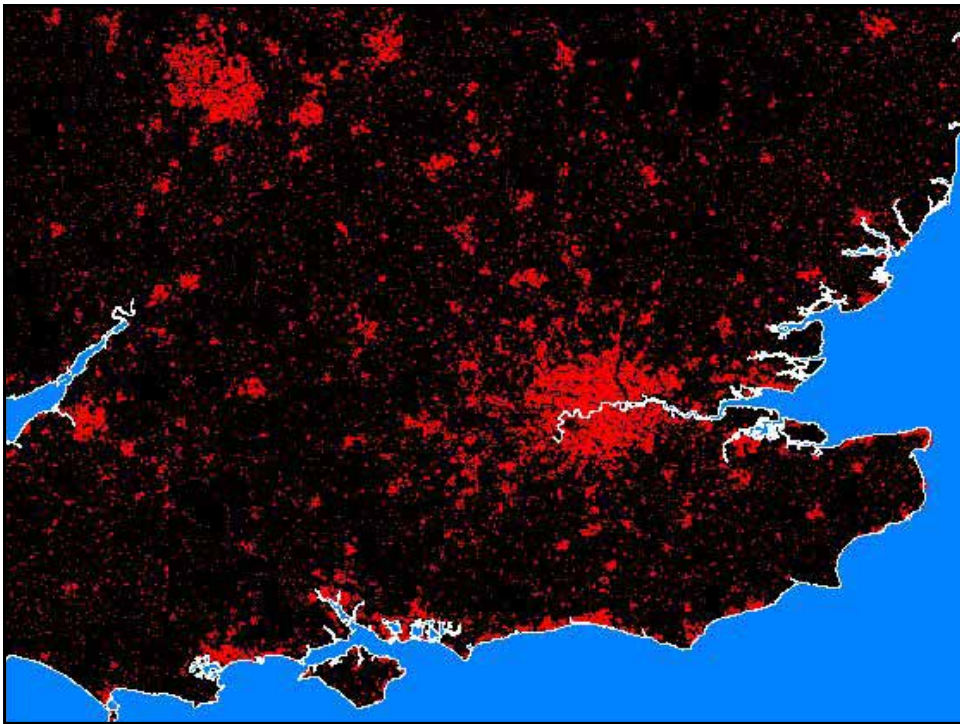
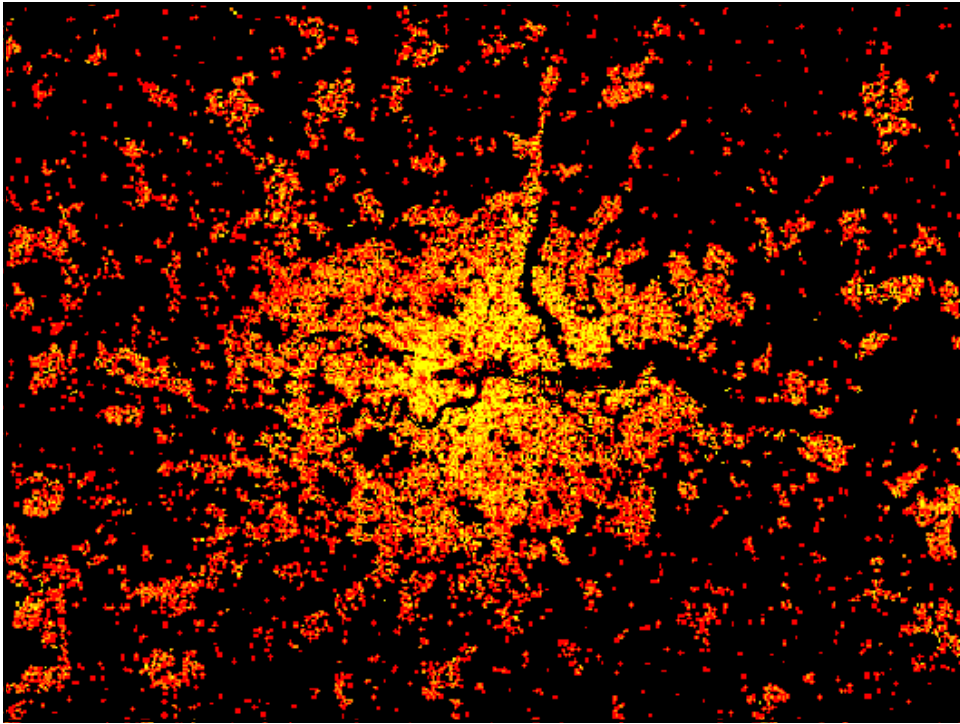
And yet the problem we have is that the dynamics of cities lies beneath these patterns and gives us a false sense of security that we can explain them

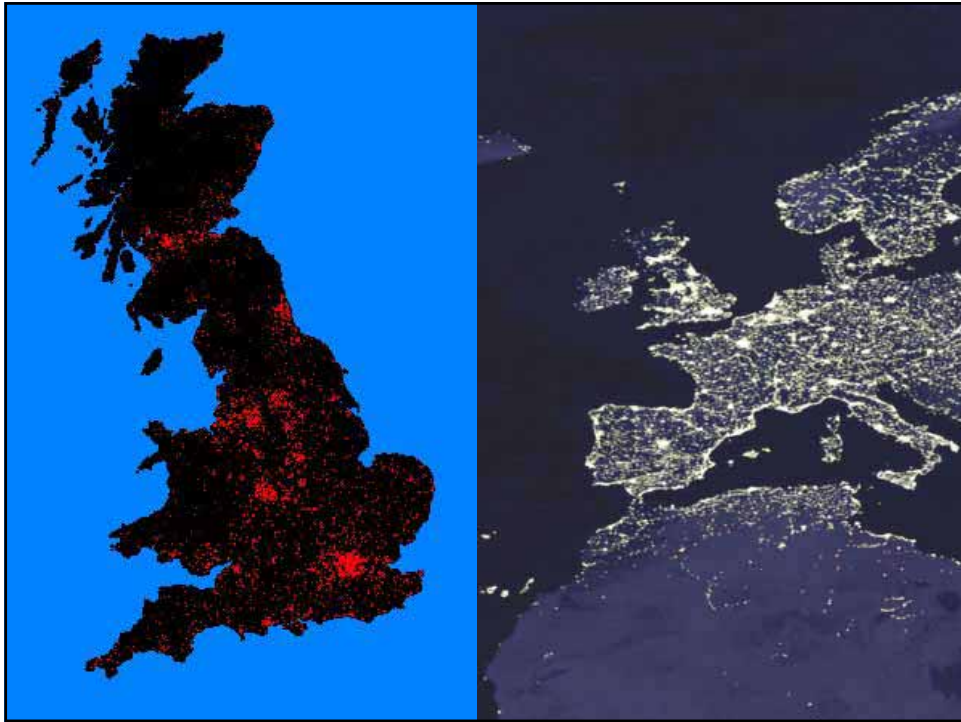
Here are some examples from nature and cities



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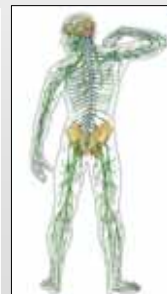
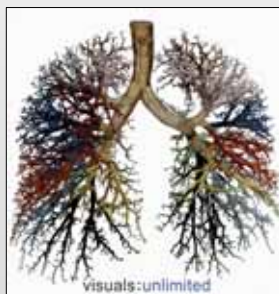
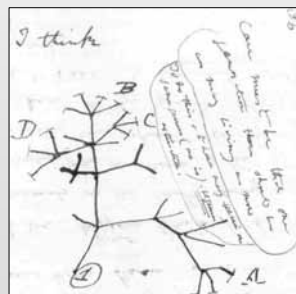






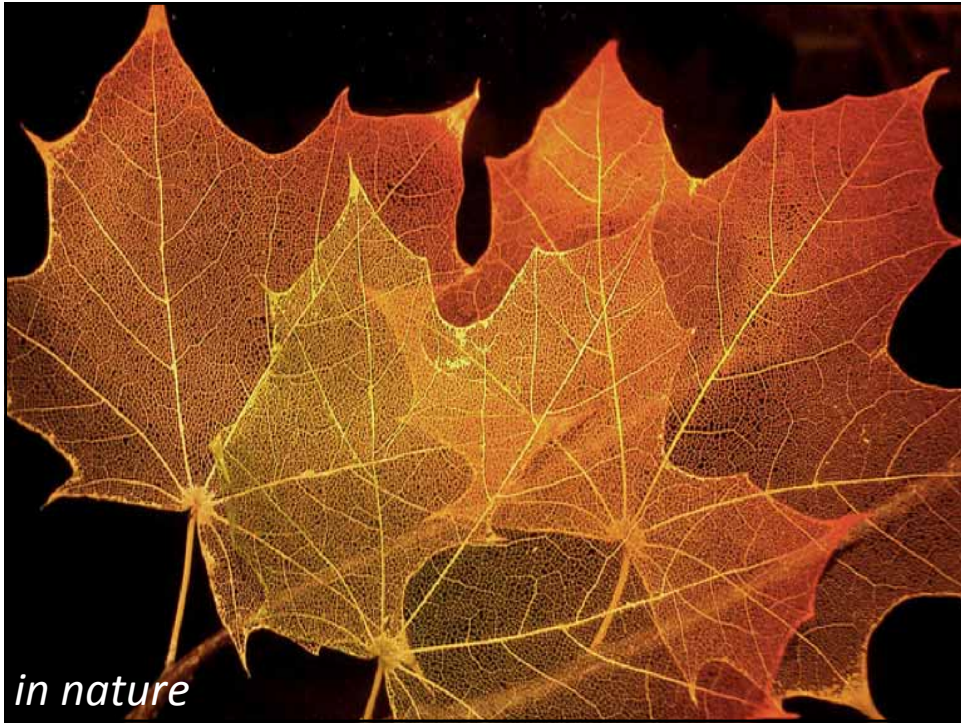
You can see size, and hierarchy, and scale in all of these – in fact these pictures are rather a good demonstration of scaling in city size – Zipf's Law but we are getting ahead of ourselves.

These patterns imply network structures that lie beneath and these are closely related to interactions. Here we can see the classic dendritic form that is key to the way nature delivers energy in an efficient manner to sustain its living systems



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in cities

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I could talk for hours about these sorts of patterns but let me say one more thing before I try and give you some sense of how these can all be tied together

Essentially the geometry that describes all these patterns is fractal geometry, first developed 40 years ago by Benoit Mandelbrot, and best seen in his wonderful book *The Fractal Geometry of Nature* (1977, 1982)

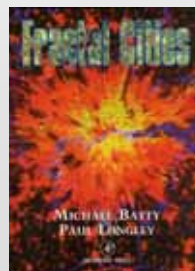
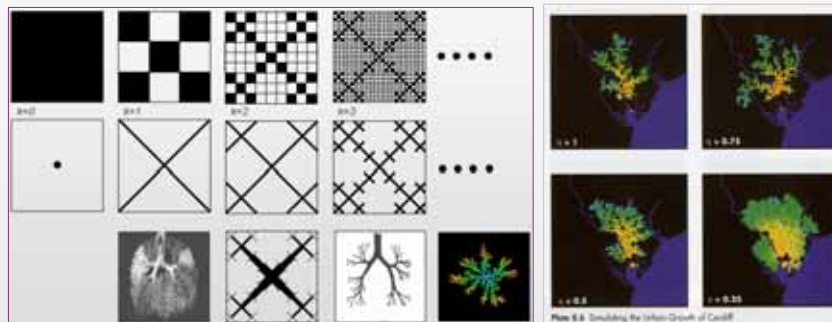
Fractal geometry throws up its own complexity – coastlines are infinite in length but the area they enclose is finite

The dimension of a fractal object is a real number not an integer and most of our world is in fact fractal – integer dimensions are the special case

But most important is that fractal objects scale – as you zoom in or out they look the same. They are self-similar. For example



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And this was the book that preceded my *Cities and Complexity* book that Paul Longley and myself wrote nearly 20 years ago

<http://www.fractalcities.org/>



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Idea along these lines in geography go back a long way. Certainly to the quantitative geographers who began work in the late 1950s early 1960s such as Waldo Tobler and his colleagues at Seattle, then Michigan amongst other places. By people at the Harvard Lab for Computer graphics which set the world on fire in terms of GIS such as Mike Woldenberg

Hierarchy was key to these ideas and widely exploited as a way of indexing self-similarity as in central place theory.

Formal ideas about fractals were explored by people here such as Mike Goodchild and David Mark in the “The Fractal Nature of Geographic Phenomena” *Annals of the Association of American Geographers*. Volume 77, Issue 2, pages 265–278, June 1987

And this all marked the bridge from top down to bottom up – from systems theory to complexity theory



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Size and Scaling: Four (more or less) Laws of Scaling

Central to all this are ideas of scaling and these in the first instance are physically rooted.

The classic signature of scaling is a power law because it is the only algebraic function that has the same form when its scale is changed – change the scale from x to $2x$ and then the function changes from $f(x)$ to $2f(x) = f(2x)$

The only function with this property is a power law such as $f(x) = 1/x$

Now let me digress a little and say something about the growth of cities and what we know from complexity theory and scaling and fractal geometry so far. When cities grow we have observed several types of scaling – let me list these



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As cities grow, they cluster more and more people together and the number of potential interactions grows more than proportionately

A city of P persons has '*potentially*' P^2 interactions, only a fraction of these can be realised of course but there is pressure to increase the number of interactions more than proportionately to population size

Cities change in scale and size as they grow – first there are less of them in terms of size.

Then their functions scale with size in terms of attributes of scale. This is what biologists call 'allometry' and economists call 'economies of scale' or 'agglomeration' economies

I won't make the links particularly here but there is a major push at present for thinking of cities in these terms by the Santa Fe cities group.



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All others things being equal, ceteris paribus.....we can state the following about cities

- As they grow, the number of 'potential connections' increases as the square of the population (Metcalfe's Law, the network equivalent of Moore's Law, & Gilder's Law, bandwidth)
- As they grow, the average time of travel inside them increases
- As they grow, the 'density' in their central cores tends to increase and in their peripheries to fall
- As they grow, more people travel by public transport
- As they get bigger, their average real income (and wealth) increases (West's Law) – *this is allometry*
- As they get bigger, they get 'greener' (Brand's Law)
- As they get bigger, there are less of them (Zipf's Law) – *this is city size – rank size – tomorrow's talk!*



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In one sense, all of these are scaling laws – whether there are four or more is disputable but let me look briefly at the third of these observations: that is, as cities grow, the density in their central cores tends to increase and in their peripheries to fall

One of the key conclusions of urban economics as developed from von Thunen to Alonso-Beckman-Muth-Mills et al. for monocentric cities is the notion that rents scale inversely with distance (or travel cost) – as a power law – of course this is central to spatial interaction, so we really need to define a fifth law of scaling that says that *densities and rents decline as a power law with distance from their cores*

Provisionally I will call this Alonso's Law, but it appears everywhere from Clark 1951 to Wilson 1967 to Barabasi's 2012 article from his group in a recent issue of **Nature**



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In one sense, these are all underpinned by Tobler's law:

The screenshot shows the Wikipedia article for 'Tobler's first law of geography'. The article text includes: 'The first law of geography according to Walter Tobler is "Everything is related to everything else, but near things are more related than distant things."^[1] This observation is embedded in the gravity model of big distribution. It is also related to the law of demand, in that interactions between places are inversely proportional to the cost of travel between them, which is much like the probability of purchasing a good is inversely proportional to the cost. It is also related to the ideas of Isaac Newton's Law of universal gravitation and is essentially synonymous with the concept of spatial dependence that forms the foundation of spatial analysis. The link structure of Wikipedia's collection of geolocated articles has been demonstrated to be consistent with Tobler's first law of geography.^[2]

References

- ^[1] Tobler, W. (1973) "A computer model simulating urban growth in the Detroit region". *Economic Geography*, 49(2): 234-240
- ^[2] Healey, B., Steiner, E. "Evolution of Tobler's first law in a dynamic, bottom-up model representation of urban expansion". In Healey, K.B., Clewley, C., Denis, M., Lopez, G., eds. *Spatial Information Theory: 9th International Conference, COGIT 2009, Aberystwyth, Wales, September 21-25, 2009. Proceedings*. Volume 5759 of *Lecture Notes in Computer Science*. Springer (2009) 88-102.

Scale and Size are functions of distance and of each other and these lie at the heart of the new science of spatial complexity, or rather than not so new science ...



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My point is of course that there are several scaling laws that pertain to cities and regions, and that these tend to be power laws – ranging from positive and negative allometry to inverse powers and so on

Whether they are negative exponential or inverse power can generate furious debate and there is substantial effort being put into ways in which power laws can be generated using simple models –

The heritage in this area is long and distinguished from Pareto, Yule, Lotka, Simon to Gabaix et al. so on fusing urban growth theory with random stochastic models in the Gibrat tradition, and these deal with city size (and firm size and so on)

And in terms of allometry from Huxley, Haldane and so on though to the Santa Fe group.



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In spatial interaction from von Thunen to Alonso to Wilson and so on

And of course from the urban economic tradition.

Last but not least, much of this work in fractals and self-similarity came out of the quantitative revolution in geography from Garrison and Berry to Tobler, Getis, Nysteu, to Woldenberg and many others.

Although we talk of complexity and cities now using complexity sciences, the heritage is deep, the gestation period long.

What I mean to show here is that despite the entangled nature of these scaling relationships, they do represent the signatures of complex systems, and there is much work of synthesis to do on a science of complexity in our field which makes it clear how we can reconcile them, one with another.



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Measuring Complexity: Spatial Complexity

Any talk on complexity would not be complete without some sense of how we might measure it and in our field we have already flirted extensively with such measures in the early days when we used entropy and information to generate, yes – scaling models for spatial interaction.

Entropy is a good candidate for such a measure because it trades off size for distribution.

Essentially Shannon's formula which can be stated as

$$p_i = \frac{P_i}{\sum_i P_i} \quad \text{with entropy defined as } H = -\sum_i p_i \log p_i$$



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As the number of objects n in the systems goes up, the entropy increases so this is a good property

But entropy also measures dis-order or dis-organisation with the assumption that a highly peaked probability distribution – ie where all the population live in the same place – shows that the system has little disorder whereas a uniform distribution shows considerable disorder

Ok, I can't give you a primer on entropy but there is a lot going on in this area linking entropy to power laws – not simply in terms of deriving spatial distributions using entropy-maximising but actually measuring entropy per se

I had a go at this many years ago and more recently too and there are many extensions. Let me state my own version of spatial entropy where we add space explicitly



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We now define the entropy of the probability density which is

$$\rho_i = \frac{p_i}{\Delta x_i}$$

We can simply take the expected value of the log of the inverse of this, that is the expected value of

$$\log \frac{1}{\rho_i} = -\log \rho_i$$

So the spatial entropy formula becomes

$$S = -\sum_i p_i \log \rho_i = -\sum_i p_i \log \frac{p_i}{\Delta x_i}$$

The spatial entropy formula has some very nice properties as it is really composed the Shannon measure and an expected size term as follows



$$S = -\sum_i p_i \log \rho_i = -\sum_i p_i \log \frac{p_i}{\Delta x_i}$$

$$= -\sum_i p_i \log p_i + \sum_i p_i \log \Delta x_i$$

This is the distribution and the number size effect in terms of n in entropy

This is the area size effect

Now I don't have time to go expand all this but suffice it to say that I believe we need to develop real measures of system size and complexity and measure real places using these ideas. In this way we will get a better handle on the intrinsic structure of the systems that concern us.

And we may be able to show quite unambiguously that cities are getting more complex.



Two More Things Very Briefly: First Network Science

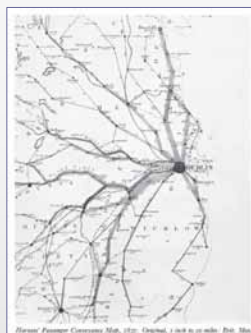
A talk on complexity would also not be complete without noting the remarkable development of network science and the kinds of scaling structures that dominate that field.

Barabasi is credited for noting that networks as graphs tend to be scaling in the size of their hubs or nodes and thus these are said to be 'scale-free'

There are other structures too such as small worlds and there is now a clear correspondence between how we might treat location in terms of the size of activities – summations of flows and summations of links. But there are many qualifications when it comes to spatial graphs – planar graphs Work is beginning on the dynamics. Let me show some pictures.



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Harriet's Passenger Concourse Map, 1827. Original, 1:100,000 scale. Date: 1827.

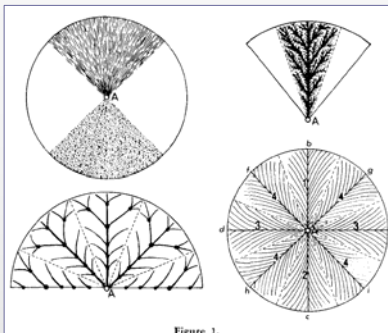
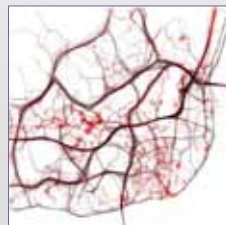


Figure 1.



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Second: Dynamics: New Styles of Model

I mentioned dynamics before & a lot of new models of cities and regions have built on this – but some of the earlier ones have been rather aggregate. The quest to disaggreate has been pursued ruthlessly and now there are new classes of model

ABM or agent-based models, microsimulation, cellular automata –focussing on very simple, often local dynamics generating global patterns emerging from micro decisions. Many models have tracked down scale where agents are individuals, traffic objects etc but some deal with groups and institutions.

Indeed the stream called geo-computation in this meeting is representative of these kinds of models. In a sense I showed some of these briefly before for our fractal models of cities.



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How Do We Use This Perspective in Designing Spatial Systems with my example again of Cities?

Let me return to Jane Jacobs in her 1961 book *The Life and Death of Great American Cities* which anticipated much of our concern for complexity and how to handle it. Much of her book is about the destruction of the old city of face to face contexts but her context of understanding draws from complexity.

Her message is that cities should be thought of as 'organisms' not 'machine' self-organising systems that evolve, not top down mechanical systems that are rigorously controlled.

There are many quotes: for example



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“Cities happen to be problems in organized complexity, like the life sciences. They present ‘situations in which a half-dozen or even several dozen quantities are all varying simultaneously and in subtly interconnected ways.’

“Why have cities not, long since, been identified, understood and treated as problems of organized complexity? If the people concerned with the life sciences were able to identify their difficult problems as problems of organized complexity, why have people professionally concerned with cities not identified the kind of problem they had?”

One of the hallmarks of complexity in cities is their action or evolution from the bottom up as millions of little decisions without co-ordinated collective action. Segregation is a case in point. Let me explain using Schelling’s model



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The Program

There are many variants of this model but it shows quite directly

- how mild preferences lead to extreme segregation
- How we cannot anticipate how local leads to global
- How this is all based on connections-interactions

I am going to set you this in the home workshop next week

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Complexity theory informs us of the need to intervene at key pressure points

Less is more, said Mies van der Rohe, and as we understand more, we should intervene less?

More is different, says Philip Anderson

Essentially as cities grow they change qualitatively and when all the worlds a city, then growth will not stop but it is change that is all important

In cities, 'what we see is not what we get' as the physical form is relatively inert.

Cities change more in what takes place and what interacts within a comparatively fixed physical form.

Less is more: the new mantra for planning

The idea that we should intervene less is an attractive notion.

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Conclusions

Short and sharp.

- We need to disentangle the wood from the trees in terms of scaling
- Develop much stronger substantive theory of how form relates to function or how economy relates to physical space
- We need much better theory per se, that meets much stronger requirements for validation, parsimony, etc.
- We need a much stronger sense of how bottom up thinking can be embedded in top down control and planning
- We need a wider synthesis based on notions about how networks deliver energy and information to cities and regions – we need to link flows to links – *geography to geometry, and then geometry to economy, and back and forth.*



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The Complexity Online Course: Register, Look at the first videos, and we will look at the **Netlogo** Stuff in the Workshop next week

Video	Duration
1.1: What is Complexity?	1:04
1.2: Our First NetLogo Model	17:53
1.3: Our Second NetLogo Model	19:31
1.4: Submit Homework 1	Homework 1
2: Dynamics and Chaos	



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Melanie Mitchell, the Course organizer's Book
Complexity: A Guided Tour is worth looking at

We will get some copies



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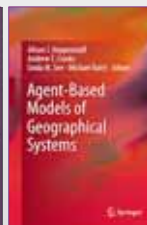


To finish, I would like to refer
you to the Blog-Web Sites and some books

<http://spatialcomplexity.info/>

<http://www.complexcity.info/>

www.casa.ucl.ac.uk



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