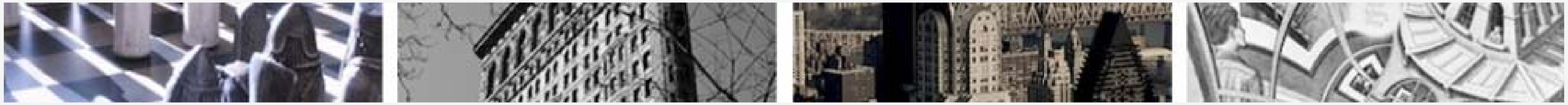


# Lecture 4:
# Land Use Transportation Models:

## Gravitation and Spatial Interaction,
## Derivation Methods

# Outline

- Gravitation: The Basic Models
- Trip Distribution: Constraints on Volume & Location
- Derivation Methods: Entropy-Maximising
- Residential Location, Modal Split
- Transportation Modelling: The Four Stage Process
- Modular Modelling: Coupled Spatial Interaction
- A Simple Example of Modularity: Lowry's Model
- DRAM-EMPAL Style Models
- Demand and Supply: Market Clearing

# Gravitation: The Basic Models

Ok, we can incorporate these ideas in the basic model of forces which was first articulated by Newton as his second law of motion – force is proportional to mass times acceleration

In more conventional terms we might write the force between two bodies as

$$F_{12} = G\frac{M_1 M_2}{d_{12}^2} \quad \textit{Or more generally} \quad F_{ij} = G\frac{M_i M_j}{d_{ij}^{\varepsilon}}$$

There is a very long history of analogies between force and social interaction going back to Newton himself. There are many references and I will add some to the web page

But let me immediately generalise this and say that we need to define many interactions – we break our system in to areas or points which we define as origins and destinations i and j

And then we measure the distance as in von Thunen not as distance per se but as travel cost or rather generalised cost $c_{ij}$

We also define the mass at the origins and destinations as $O_i$ and $D_j$ and we then write the conventional spatial interaction of gravity model as

$$T_{ij} \sim \frac{P_i P_j}{c_{ij}^{\alpha}} = K \frac{P_i P_j}{c_{ij}^{\alpha}}$$

Where K is the gravitational constant

This is the model that has been used for years but in the 1960s and 1970s various researchers cast it in a wider framework – deriving the model by setting up a series of constraints on its form which showed how it might be solved and produced various generating mechanisms that could generate consistent models

The *constraints logic* led to **consistent accounting**

The *generative logic* lead to analogies between **utility and entropy maximising** and opened a door that has not been much exploited to date between entropy, energy, urban forms physical morphology and economic structure

In particular the economic logic rather than the energy entropy logic was called choice theory, specifically discrete choice

*Now to introduce all this, we need to define some more terms*

We will refer to the size of volume of origins and destinations not as population P but as $O_i$ and $D_j$ assuming they are different from one another. We will also assume that the inverse square law on distance or travel cost does not apply and that whenever $c_{ij}$ appears it will be parameterised with a value that varies which we call $\lambda$

We will assume trips are as we have defined them as $T_{ij}$ but we will also normalise trips by their total volume T to produce probabilities

$$p_{ij} = \frac{T_{ij}}{T} = \frac{T_{ij}}{\sum_{ij} T_{ij}}$$

*Note that we use summation extensively in what follows*

# Trip Distribution: Constraints on Volume & Location

We must move quite quickly now so let me introduce the basic constraints on spatial interaction and then state various models

The constraints are usually specified as origin constraints and destination constraints as

$$O_i = \sum_j T_{ij}$$

$$D_j = \sum_i T_{ij}$$

And we can take our basic gravity model and make it subject to either or both of these constraints or not at all

*So what we get are four possible models*

Unconstrained $T_{ij} = KO_i D_j c_{ij}^{-\lambda}$

Singly (Origin) Constrained so that the volume of trips at the origins is conserved $T_{ij} = A_i O_i D_j c_{ij}^{-\lambda}$

Singly (Destination) Constrained so that the volume of trips at the destinations is conserved $T_{ij} = B_j O_i D_j c_{ij}^{-\lambda}$

Doubly Constrained trip volumes at origins + destinations are conserved $T_{ij} = A_i B_j O_i D_j c_{ij}^{-\lambda}$

*The first three are location models, the last is a traffic model*

Ok, so what are these parameters that enable the constraints to me met – well they can be very easily produced by summing each model over the relevant subscripts – ie origins or destinations and then simply substituting and rearranging

I will do this but I will leave you to work through the algebra in your own time and many of you will know this anyway. Here are the factors which are sometimes called balancing factors

*Unconstrained* $K = T \Big/ \sum_i \sum_j O_i D_j c_{ij}^{-\lambda}$

*Origin Constrained* $A_i = 1 \Big/ \sum_j D_j c_{ij}^{-\lambda}$

*Destination Constrained* $B_j = 1 \Big/ \sum_i O_i c_{ij}^{-\lambda}$

*Doubly Constrained* $A_i = 1 \Big/ \sum_j B_j D_j c_{ij}^{-\lambda}$ $B_j = 1 \Big/ \sum_i A_i O_i c_{ij}^{-\lambda}$

# Derivation Methods: Entropy-Maximising

Now we have only dealt with constraints through consistent accounting – we now need to deal with generative methods that lead to the same sort of accounting– entropy maximising, information minimising, utility maximising and random utility maximising, and also various forms of nonlinear optimisation – in fact all these methods may be seen as a kind of optimisation of an objective function – entropy utility and so on – subject to constraints

We will define entropy maximising. First we define entropy as Shannon information and we convert all our equations and constraints to probabilities. Shannon entropy is

$$H = -\sum_i \sum_j p_{ij} \log p_{ij}$$

We maximise this entropy subject to the previous constraints – dependent on what kind of model we seek but noting now that we need another constraint on travel cost which is equivalent to energy so that we can derive a model

We thus set up the problem as

$$\max \quad H = -\sum_i \sum_j p_{ij} \log p_{ij}$$

*subject to*

$$\sum_j p_{ij} = p_i$$

$$\sum_i p_{ij} = p_j$$

$$\sum_i \sum_j p_{ij} c_{ij} = \hat{C}$$

But note that the probabilities always add to 1, that is

$$\sum_i \sum_j p_{ij} = \sum_i p_i = \sum_j p_j = 1$$

I am not going to work this through by setting up a Lagrangian and differentiating it and then getting the result. There is a lot of basic algebra involved and all I want to show is the result

For this optimisation the model that we get can be written as

$$p_{ij} = \exp(-\lambda_i - \lambda_j - c_{ij}^{-\lambda})$$

*or*

$$T_{ij} = Tp_{ij} = A_i O_i B_j D_j \exp(-\lambda c_{ij})$$

*Let us note many things*

1. This is the doubly constrained model but with an exponential of travel cost replacing the inverse power
2. We can get any of the other constrained models in the family by dropping constraints and we can do this directly

3. We can begin to explore what entropy means by substituting the probability model into the entropy equation – I will reserve this for an ad hoc seminar on entropy if you are interested.
4. We can think of this method as one in which the most likely model is generated given the information which is in the constraints
5. In terms of statistical physics, this model is essentially the Boltzmann-Gibbs distribution
6. Entropy can be seen as utility under certain circumstances
7. We solve the model – ie find its parameters by solving the entropy program which is equivalent to solving the maximum likelihood equations
8. We can then use this scheme to develop many different kinds of model – where we add more and more constraints and also disaggregate the equations to deal with groups

# Residential Location, Modal Split

Let me illustrate in two ways how we can build models using this framework

First if we say that residential location depends on not only travel cost but also on money available for housing we can argue that

1. The model is singly constrained – we know where people work and we want to find out where they live – so origins are workplaces and destinations are housing areas
2. The model then lets us predict people in housing
3. We argue that people will trade off money for housing against transport cost

And we then set up the model as follows

It is

$$\sum_j T_{ij} = O_i$$

$$\sum_i \sum_j T_{ij} c_{ij} = C$$

$$\sum_i \sum_j T_{ij} R_j = R$$

*leads to*

$$T_{ij} = A_i O_i \exp(\vartheta R_j) \exp(-\lambda c_{ij})$$

We can of course find out from this location model how many people live in destination housing zones, so it is a distribution as well as a location model

$$P_j = \sum_i T_{ij}$$

In terms of modal split we break the trips into different modes and then let the modes compete with locations for travellers

In this way we produce a combined modal split location model.

Sometimes we may want the modes to be constrained and in generating specific constraints on total travellers by mode, this is equivalent to adding parameters that distort the travel costs – in fact the generic equation can be seen as one where the travel cost or energy is modified by the volume constraints

That is

$$p_{ij} = \exp(-\lambda_i - \lambda_j - c_{ij}^{-\lambda})$$

$$or$$

$$T_{ij} = Tp_{ij} = A_i O_i B_j D_j \exp(-\lambda c_{ij})$$

# Transportation Modelling: The Four Stage Process

I should conclude with saying that the transport model is part of a four stage process that involves generation, distribution, modal split and assignment and that the spatial interaction approach can be seen as either applying solely to distribution and modal split or in more integrated ways. To get some sense of this process look at the book **Modelling Transport**, by J. Ortúzar & L. Willumsen, Wiley, 3rd Ed, 2001

So far all our models have been demand model but the transport system is capacitated and in transport modelling we need to assessing trips to the network and then figure out if it is possible to meet theses assignments in terms of supply – if not we iterate to clear the transport market

The same is true of the residential market in terms of supply and we will develop all these ideas in the next talk

# Modular Modelling: Coupled Spatial Interaction

Now we have a module for one kind of interaction – consider stringing these together as more than one kind of spatial interaction

Classically we might model flows from home to work and home to shop but there are many more and in this sense, we can use these as building blocks for wider models. This is for next time too

What we will now do is illustrate how we might build such a structure taking a journey to work model from Employment to Population and then to Shopping which we structure as --

First we have the journey from work to home model as

$$T_{ij} = E_i \frac{F_j \exp(-\lambda c_{ij})}{\sum_j F_j \exp(-\lambda c_{ij})} \quad , \quad \sum_j T_{ij} = E_i$$

$$P_j = \alpha \sum_i T_{ij}$$

And then the demand from home to shop

$$S_{jm} = P_j \frac{W_m \exp(-\beta c_{jm})}{\sum_m W_m \exp(-\beta c_{jm})} \quad , \quad \sum_m S_{jm} = P_j$$

$$S_m = \sum_j S_{jm}$$

And there is a potential link back to employment from the retail sector $E_m = f(S_m)$

# A Simple Example of Modularity: Lowry's Model

Lowry's (1964) model of Pittsburgh was a model of this nature but it also incorporated in it – or rather its derivatives did more formally – a generative sequence of starting with only a portion of employment – basic – and then generating the non-basic that came from this. This non-basic set up demand for more non-basic and so on until all the non-basic employment was generated, and this sequence followed the classic multiplier effect that is central to input-output models.

Block diagrams of these types of model follow: they are hard to read – download the book and print it out

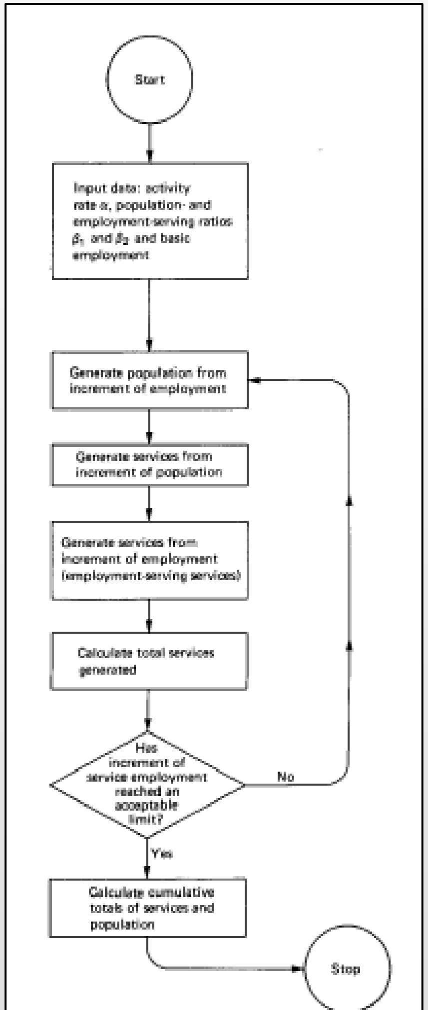



Fig. 2.1. The sequence of operations in generating population and services from basic employment.

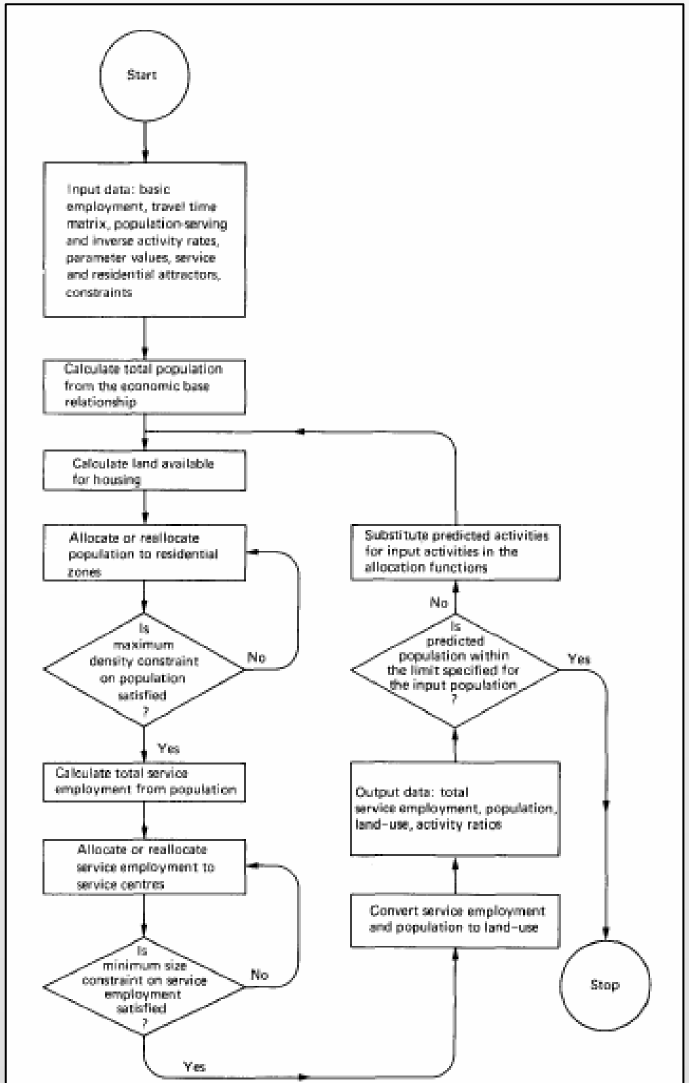



Fig. 3.2. Generalised flow chart of the Lowry model.

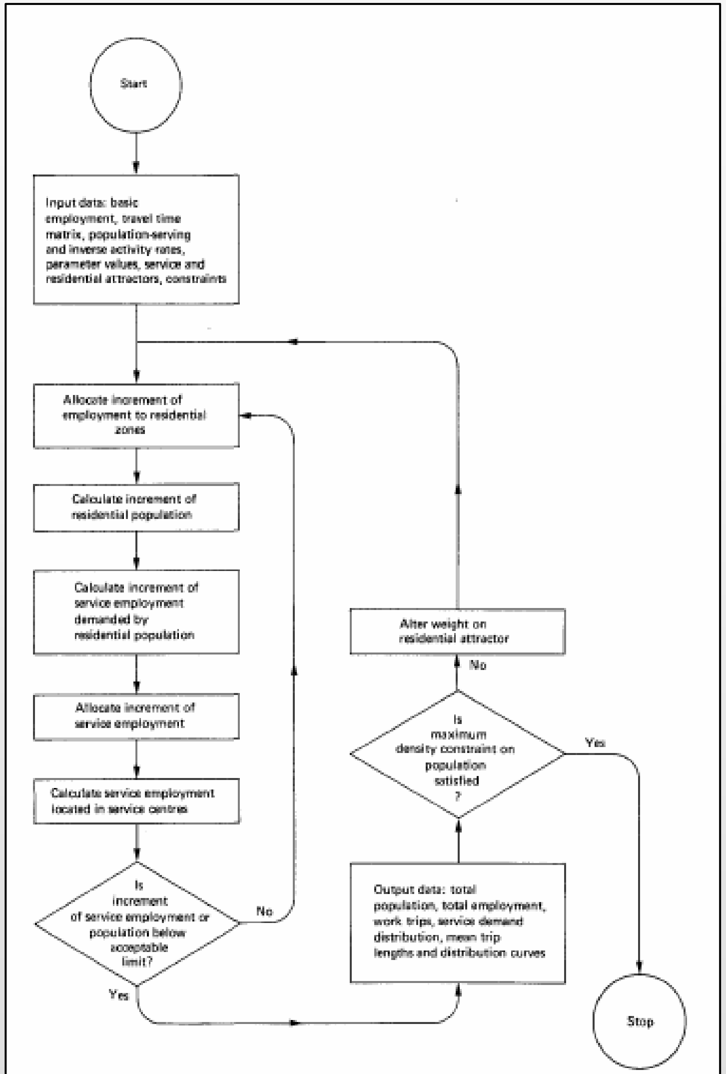



Fig. 3.3. Generalised flow chart of the Activity Allocation model.

From http://www.casa.ucl.ac.uk/urbanmodelling/

# DRAM-EMPAL Style Models

Essentially what we have here is the notion of simultaneous dependence – i.e. one activity generates another but that other activity generates the first one – what came first – the chicken or the egg?

Stephen Putman developed an integrated model to predict residential location DRAM and another to predict employment location EMPAL. In essence different models are used to do each – the employment model tends to be based on very different factors – it is a regression like model of key location factors not a flow model

# Demand and Supply: Market Clearing

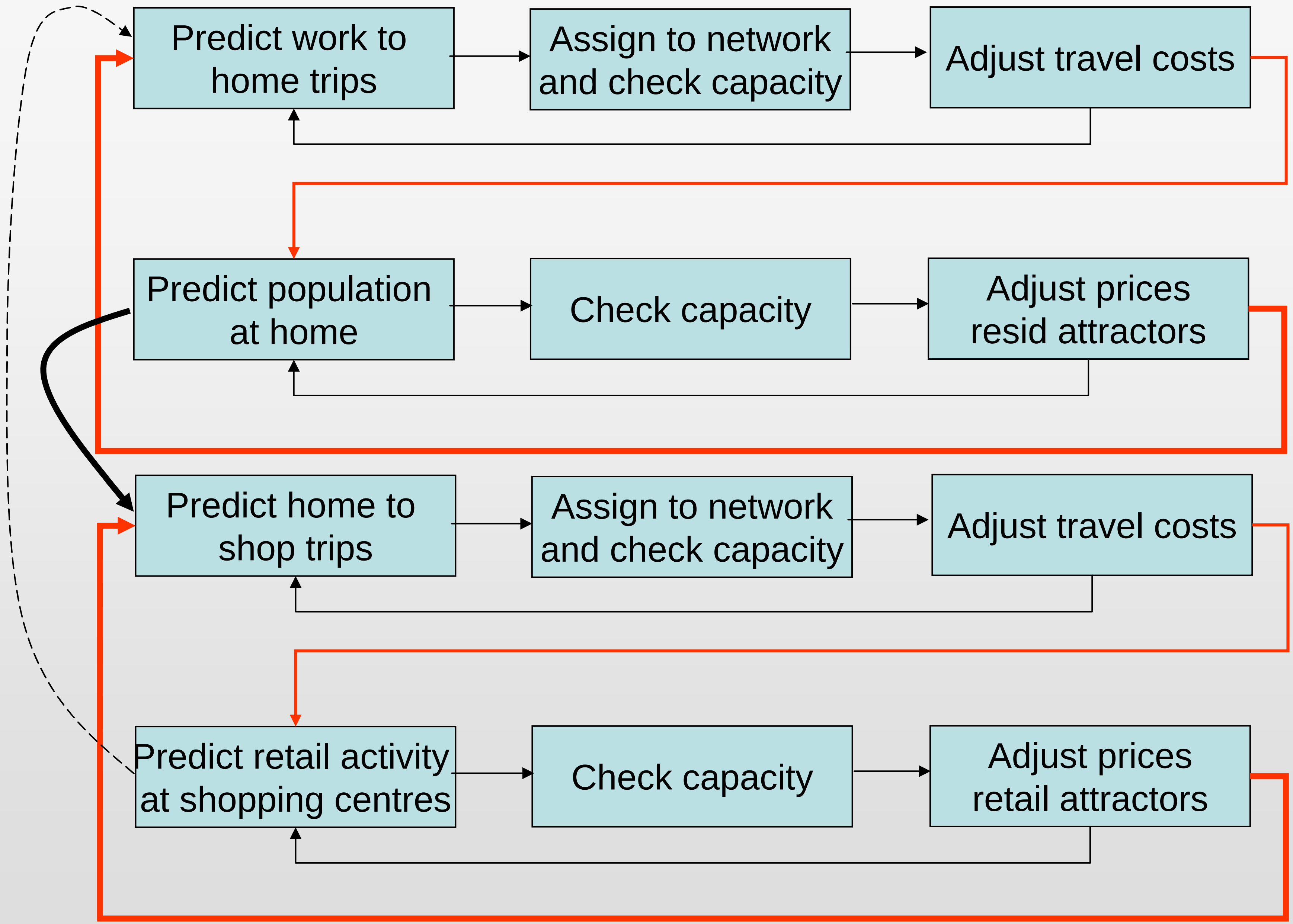
So far most of these models have been articulated from the demand side – they are models of travel demand and locational demand – they say nothing about supply although we did introduce the notion that in simulating trips and assigning these to the network, we need to invoke supply.

When demand and supply are in balance, then the usual signal of this is the price that is charged. In one sense the DRAM EMPAL model configures residential location as demand and employment location as supply but most models tend to treat supply as being relatively fixed, given, non-modellable

However several models that couple more than one activity together treat supply as being balanced with demand, often starting with demand, seeing if demand is met, if not changing the basis of demand and so on until equilibrium is ascertained. Sometimes prices determine the signal of this balance. If demand is too high, price rises and demand falls until supply is met and vice versa

Most urban models do not attempt to model supply for supply side modelling is much harder and less subject to generalisable behaviour

A strategy for ensuring balance is as follows for a model with two sectors – like the one we illustrated earlier


Predict work to home trips
Assign to network and check capacity
Adjust travel costs
Predict population at home
Check capacity
Adjust prices resid attractors
Predict home to shop trips
Assign to network and check capacity
Adjust travel costs
Predict retail activity at shopping centres
Check capacity
Adjust prices retail attractors

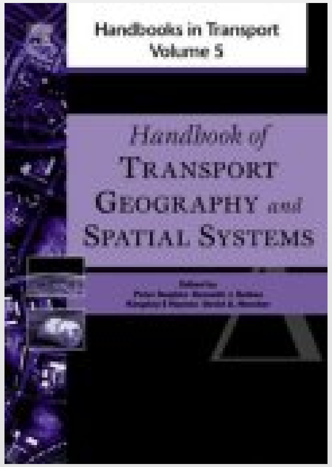
The decision to nest what loop inside what other loop is a big issue that makes these models non-unique

If the supply side is modelled separately then the way this is incorporated further complicates the sequence of model operations.

In large scale integrated models, that we will deal with next time these are crucial issues

In fact we don't have time but there is one further structural issue we will deal with when we meet next time and this involves Input-Output, in particular the Echenique Models

There is some good reading of all this material in Google Books in Button, K. J., Haynes, K. E., Stopher, P., and Hensher, D. A. (Editors) (2004) **Handbook of Transport Geography and Spatial Systems**, Volume 5 (Handbooks in Transport), Elsevier Science, New York

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# Questions?