



# Lecture 3: Basic Theories of Space, Social Physics and the Urban Economy:

The Role of Distance in London: Examples:  
Von Thunen, Population Density, Gravitation

# Outline

- Spatial Structure: Profit and Cost, Distance, Agglomeration, Accessibility
- Von Thunen and DLA
- Population Density
- Population Size Distributions
- Gravitation: The Basic Models
- The Next Lectures

# **Spatial Structure: Profit and Cost, Distance, Agglomeration, Accessibility**

We don't have time for a detailed development of theories of spatial structure but benefits and costs – profits and losses - are key to location. These define the pull and push factors of selecting any location defining agglomeration and dis-agglomeration economies, & accessibility and inaccessibility

We will define two very different approaches that lead to the same sort of structures

First from urban economics, and second the same from physical movement which we loosely call 'social physics'

## Von Thunen and then DLA

### *The Von Thunen Model*

Essentially benefits (or profits) depend on nearness to market  
which under normal profits we measure as rent payable

This is balanced against the cost of transport to the market.

This might relate to space that one might get – as one  
moves further away from a point, more space is accessible

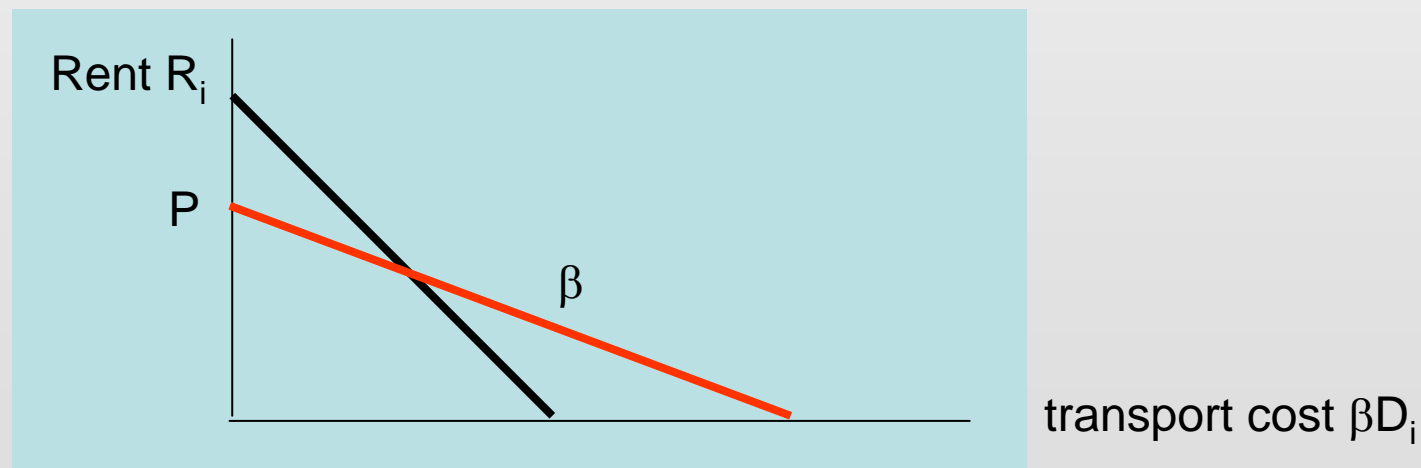
Who locates where depends on trade off of benefits versus  
transport cost

We define profit or yield at the market  $P$ , distance  $D_i$  at  
location  $i$  from the market or centre, transport cost per unit  
of distance  $\beta$  and then rent payable at  $i$  which is  $R_i$

We thus define the key equation for the costs and benefits of location as

$$R_i = P - \beta D_i$$

This is a linear equation where we can think of profit or yield as the intercept and slope as the transport cost per unit of distance. As we vary these the slope of the line will vary as



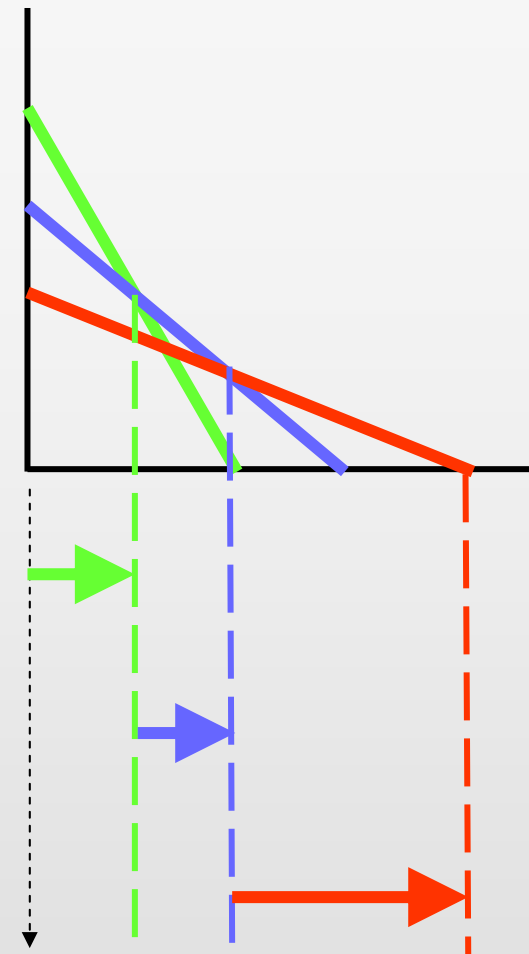
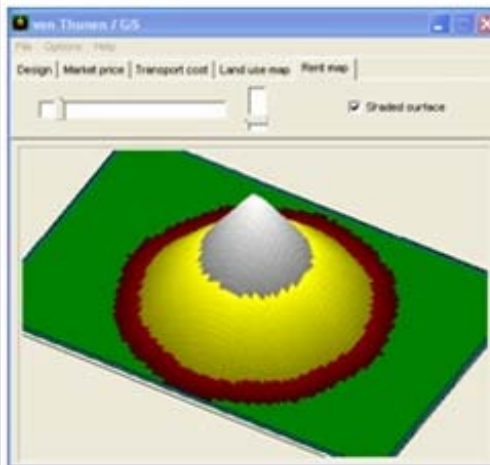
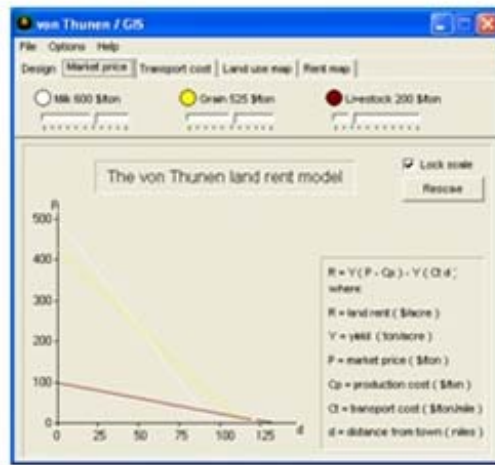
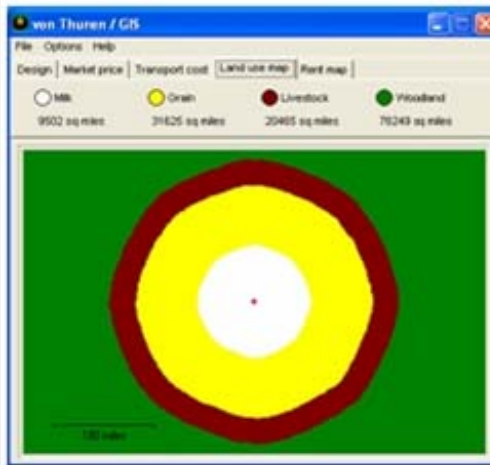
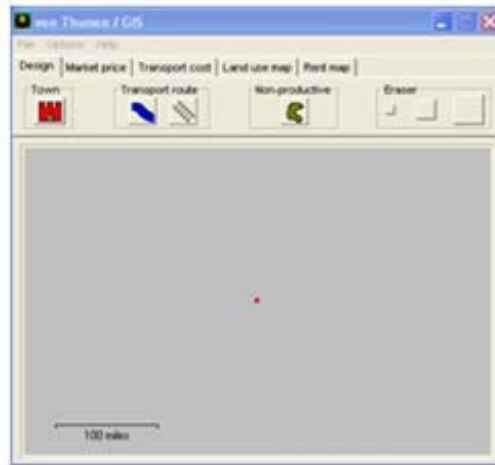
Let me load the von Thunen model and show you how this works

Go to our web site and download to save or run the model from

<http://www.casa.ucl.ac.uk/software/vonthunen.asp>

What you see is the following which I will show before we run it: a simply canvas on which you plant a market, you can adjust the benefits and cost equation for a series of different land uses – all agricultural which imply different types of production and transport of goods to market

And market clearing takes place where one land use outbids in terms of rent any other: this assumes normal profits



Land use are determined by the bid rent curves in terms of their dominance and the circular pattern of land use falls out from this – I will run the model but you should do it too

## *The Diffusion-Limited Aggregation Model*

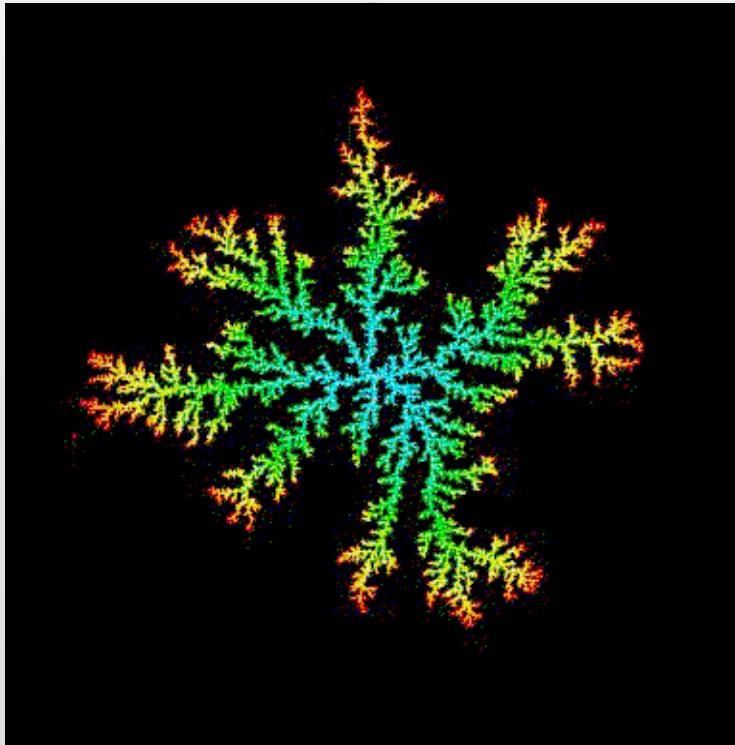
My second model is quite different – it is one that I will introduce later this week as part of Cellular Automata (CA). This is a physical model in which an agent wants to locate at a town to get economies of scale but also wants to be as far from the town as possible.

Thus the agent wants to realise agglomeration, clustering but also economies of getting as much space as possible. Thus the issues are to balance centripetal forces with centrifugal.

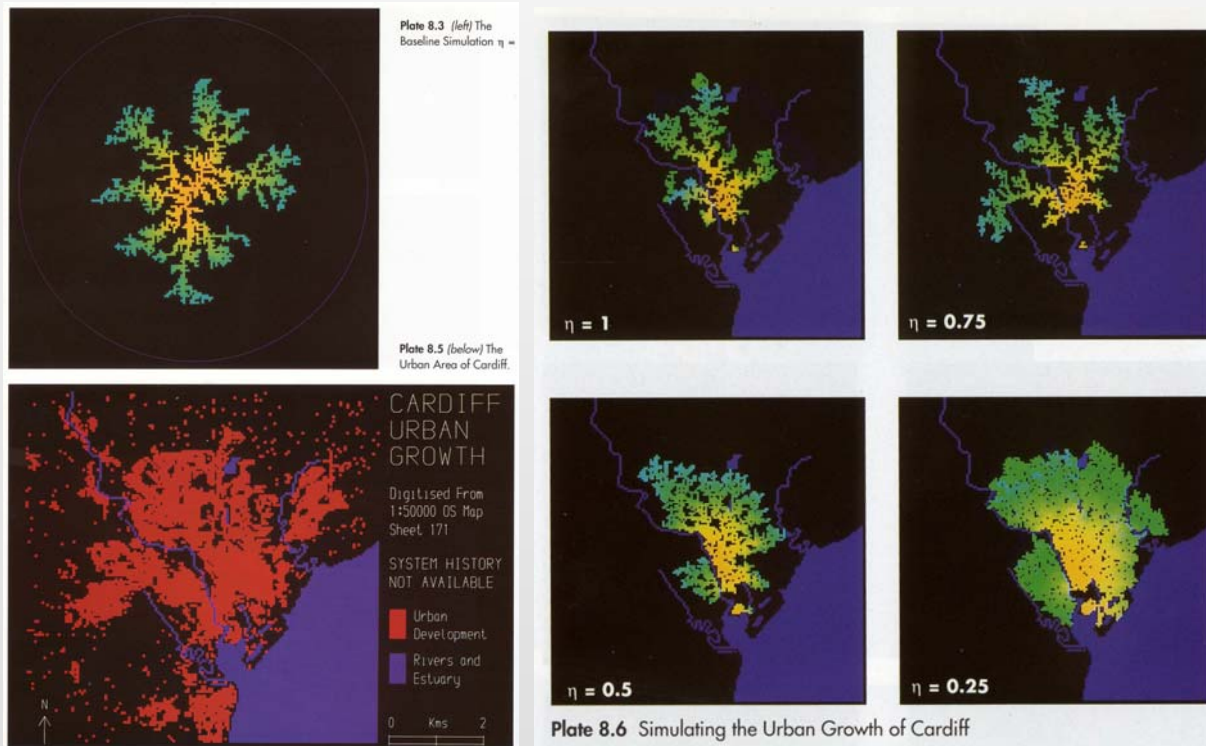
We can set up a model to show how this occurs. Plant a seed and let many agents wander randomly in a region around the seed. The rule for fixing locations are dead simple



When an agent touches another agent that is already fixed then that agent sticks and doesn't move any more  
The first agent of course to find the seeds sticks at the seed – the market centre and all agents wander randomly in space.



What we get is the following dendritic pattern: this is a model called diffusion limited aggregation, DLA where the diffusion is limited or constrained



There are many examples on the internet that you can search for yourselves: one is from the Boston group [and I will run it](#)

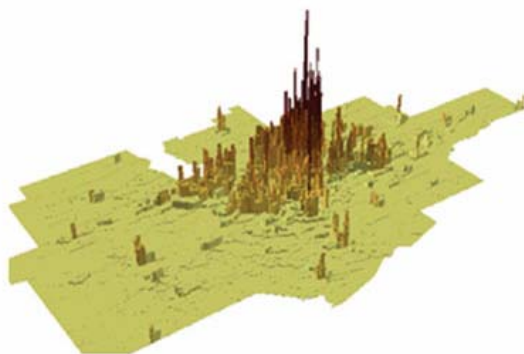
# Population Density

Tobler's first law of distance (after the geographer Waldo Tobler, 1970) states that "Everything is related to everything else, but near things are more related than distant things."

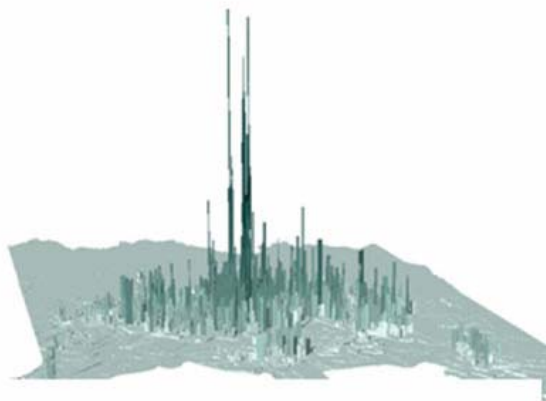
This is not unlike the assumption in von Thunen's model that interactions between places are inversely proportional to the cost of travel between them, which is much like the probability of purchasing a good is inversely proportional to the cost ([http://en.wikipedia.org/wiki/Tobler's\\_first\\_law\\_of\\_geography](http://en.wikipedia.org/wiki/Tobler's_first_law_of_geography))

We can see this best in terms of population densities which in traditional monocentric cities decline with distance or cost from the centre)

St. Louis  
Tract Density (upper)  
Point Density (lower)



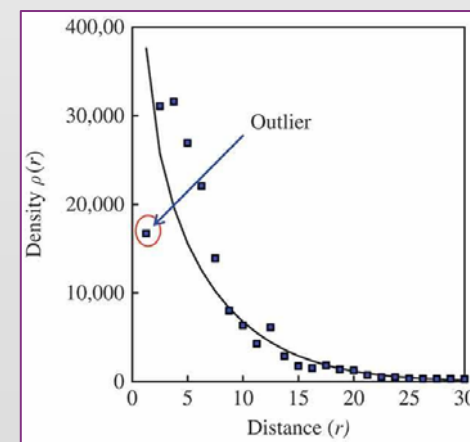
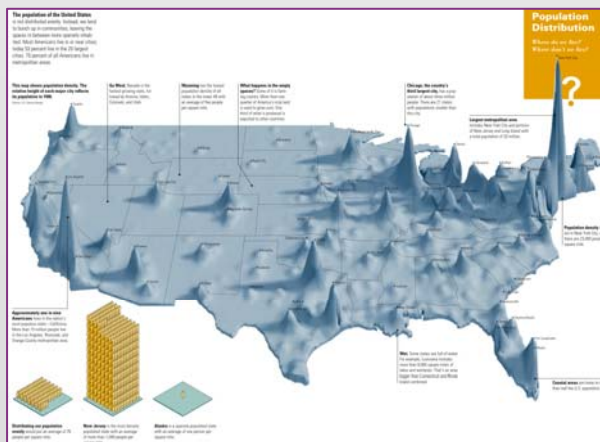
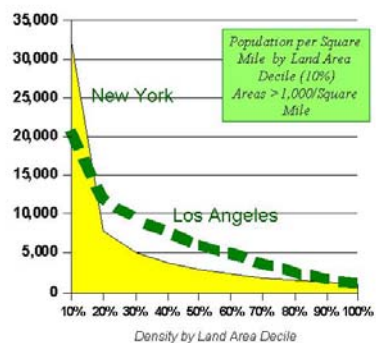
Seattle  
Tract Density (upper)  
Point Density (lower)



Boston  
Tract Density (upper)  
Point Density (lower)



Density Profiles: New York & Los Angeles



The model is straightforward. It is population density as a negative exponential function of distance  $r$

$$\rho(r) = \frac{K}{\exp(\lambda r)} = K \exp(-\lambda r)$$

or as an inverse power function of  $r$

$$\rho(r) = \frac{K}{r^\alpha} = Kr^{-\alpha}$$

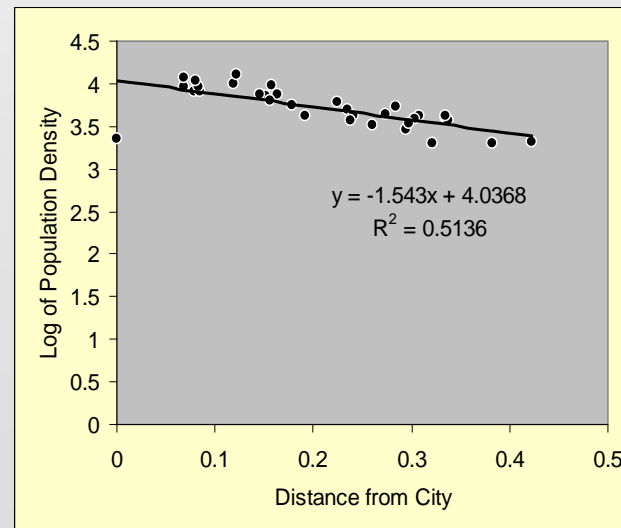
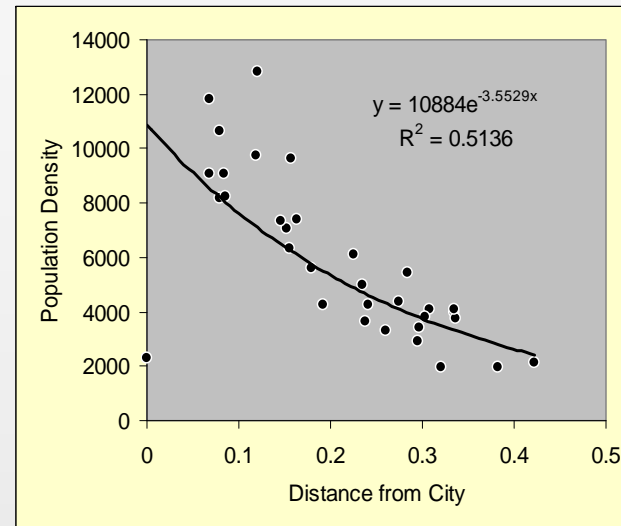
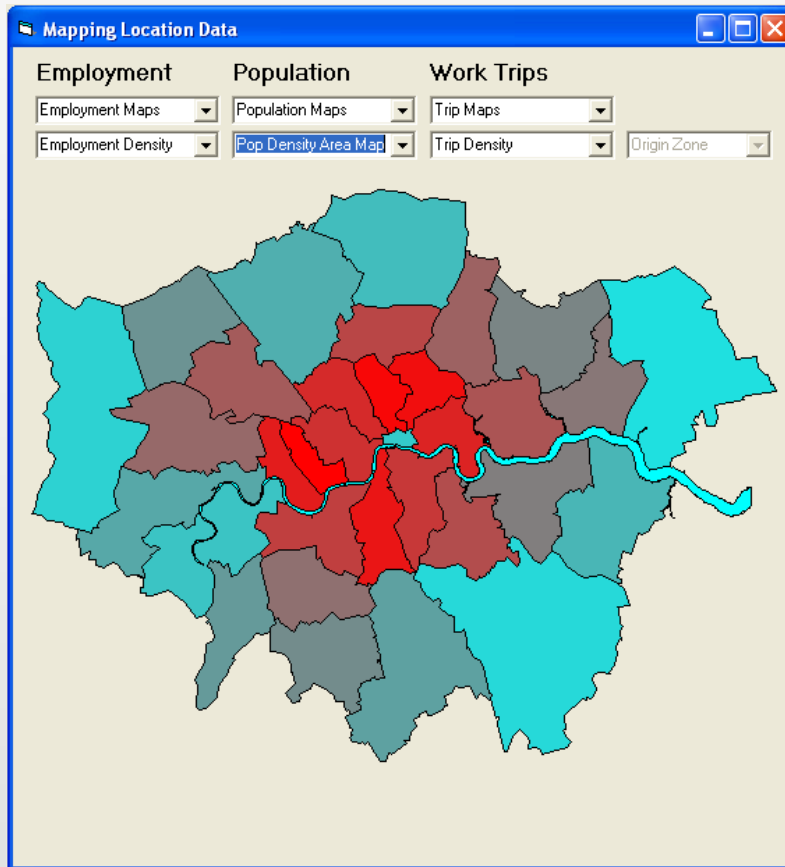
We can linearise these equations by taking logs and then fit them using regression

$$\log \rho(r) = \log K - \lambda r$$

or

$$\log \rho(r) = \log K - \alpha \log r$$

We can do this for London and the results are as follows



We can now download a spreadsheet and this will demonstrate the principles involved

Here is the web address for the spreadsheet. We will use this spreadsheet in a workshop later so you can have a go now but we will explain it in more detail next week when I will spend the first ten minutes looking at what the data implies.

But have a go at interpreting this data yourselves – if you want to you can write a program in R to do what I can show in the spreadsheet.

<http://www.casa.ucl.ac.uk/rits/density.xls>

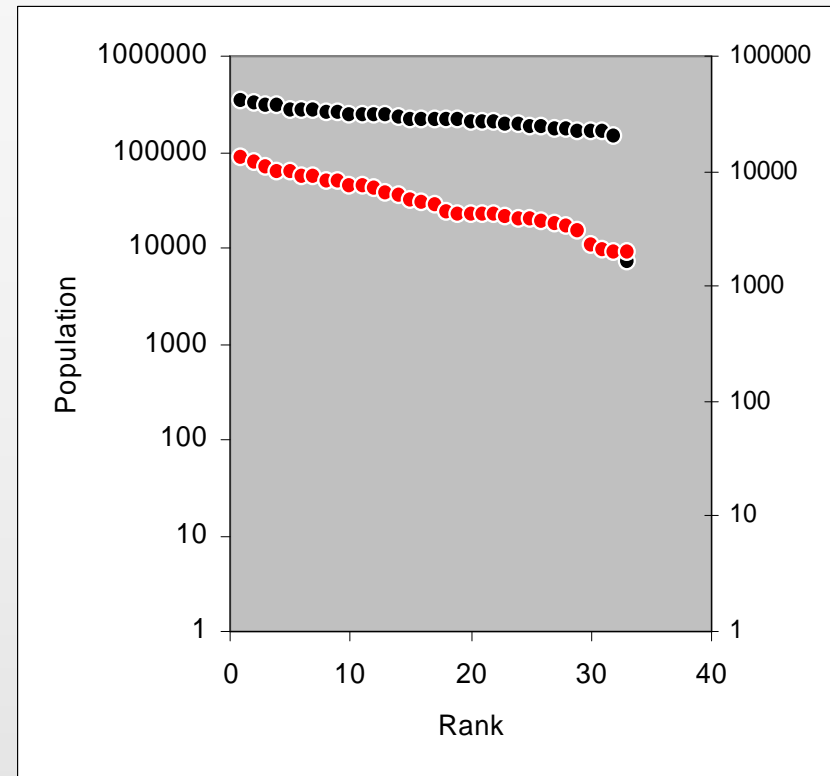
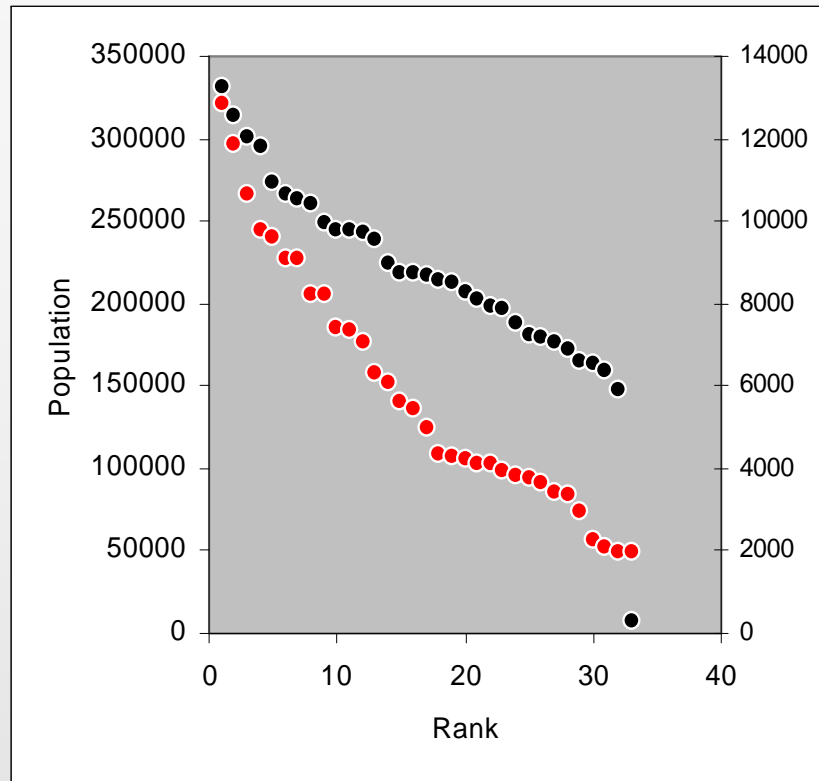
# Population Size Distributions

We can also look at population and population density in terms of their size distributions and these usually follow another power law or negative exponential

What we are doing here is getting rid of distance and then examining the frequency of the sizes. In cities, where there are many centres this would then remove the priority of one centre. Often this is another way of looking at competition in urban systems.

In fact what we usually do is rank order the sizes. Let us do this for London as this will also be one of our exercises.





This is sometimes called the rank size rule – in fact we do not form a frequency distribution as such but form the counter cumulative distribution which is easier to fit and this is what we see above. More on this in our workshop

# Questions?