School of Geographical Sciences & Urban Planning

Lectures on Spatial Complexity
17th-28th October 2011

Lecture 5: 26th October 2011

Networks, Flows & Interactions

Michael Batty

m.batty@ucl.ac.uk
twitter @jmichaelbatty

http://www.complexcity.info/
http://www.spatialcomplexity.info/

Outline of Lecture 5

A Digression: The Last Lecture

Defining Graphs and Flows

Historical Developments of Flow Networks

The Evolution of Networks: Phase Transitions and Fractals

The Structure of Networks

Back to Scaling and Preferential Attachment

An Example: Flows on the London Rail System

A Digression: The Last Lecture

As this is a 1 credit course of some 16 hours equivalent of lectures, the requirement as ASU practice is attendance.

In the very last lecture – next Wednesday – I will reserve the last hour for a brief review of the course, and then ask for some feedback from you all.

I would like you to think about how this course on Spatial Complexity in particular and Complexity in general might help you with your own research and I will initiate a discussion that asks you to make such a comment.

I would also like you to give me some constructive feedback about how the course fits your needs and how I might make it better.

To Today's Lecture then: Defining Graphs and Flows

A graph is a network of <u>nodes</u> and <u>arcs</u> (or vertices and links if you like) that are usually expressed in terms of whether or not an arc exists.

A weighted graph or network is defined as one where the arcs or links take some value

There is a direct relations between flow graphs and weighted graphs if the weights can be interpreted as flows.

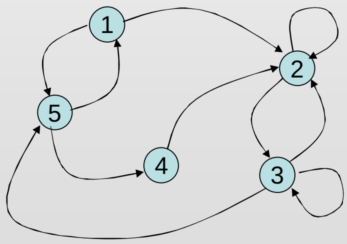
Graphs my be directed – digraphs – where the link is in either direction of both

Or they may be undirected where there is no relevance to the directionality

The graph is usually described by an adjacency matrix $[a_{ij}]$ where a_{ij} is defined as follows

$$a_{ij} = \begin{cases} 1 \text{ if there is a link from } i - j \\ 0, \textit{otherwise} \end{cases}$$

Here is a typical graph which is directed and its adjacency matrix. Measuring its connectivity is central. We will note measures in class. The degree of each node is a key idea.



$a_{ij}=$

0	1	0	0	1
0	1	1	0	0
0	1	1	0	1
0	1	0	0	0
1	0	0	1	0

A non-directed graph would have a symmetric adjacency matrix i.e. $a_{ij}=a_{ji}$

We can sum the number of arcs leaving any node and the number entering any node and these are defined as <u>outdegrees</u> and <u>indegrees</u> respectively and we define these as

$$a_i = \sum_j a_{ij}$$

$$a_j = \sum_i a_{ij}$$

Note that the indegrees equal the respective outdegrees if the matrix is non-directed, hence symmetric. Indegrees and outdegrees are either counts if this is data or possibly predicted ‘potential’ links if the graph describes a model

We can now look at weighted graphs

Centre for Advanced Spatial Analysis UCL

A weight can be put on each arc which is W_{ij}. This can be any value that describes the arc or link but in graphs or networks that relate to the movement of material or information, this might be a flow. If it is a flow then often we write this in spatial interaction terms as follows

$$T_{ij} = W_{ij} a_{ij}$$

$$O_i = \sum_j T_{ij}$$

$$D_j = \sum_i T_{ij}$$

The indegrees are the flows which flow into an origin and the outdegrees are those flows that leave an origin for a destination, defined as D_j and O_i respectively

One last thing and we have enough notation to ground what we need to say

If we add over origins and destinations to produce modelled or simulated flows T'_{ij} – potentials – then we often use these as measures of accessibility which are defined so that we can measure the potential flow – these if you like are measures that come from defining how near an origin or destination is to all other origins or destinations

We will have a lot more to say about this in the next lecture when we deal with another form of scaling, namely gravitational models – in fact our third form of scaling

For completeness, accessibilities are this defined as

$$X_i = \sum_j T'_{ij}$$

$$X_j = \sum_i T'_{ij}$$

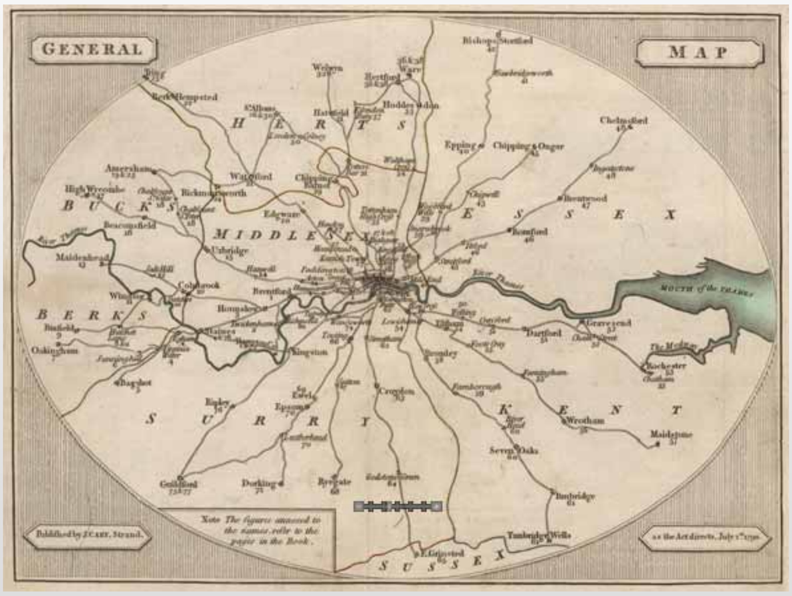
Centre for Advanced Spatial Analysis UCL

Historical Developments of Flow Networks

As a slight digression but to impress the idea that flows and networks are different sides of the same coin, it is worth noting many network representations that go back in history.

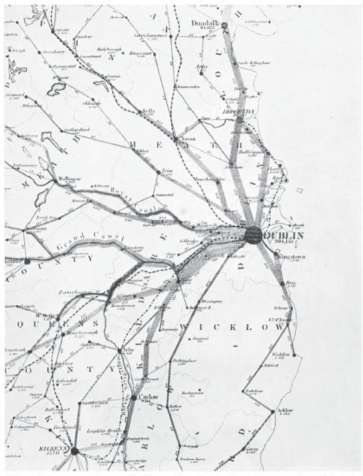
Networks of course relate to maps and it is quite likely that as far back as when maps were prepared for armies to march, network type maps were prepared. I still don't know much about the history before the 18th century but network maps were being produced as soon as stage coaches came to be scheduled. Here are two maps, one of networks in 1792 and one of flows in 1837.

The network map in 1792 is Cary's Survey of the High Roads From London

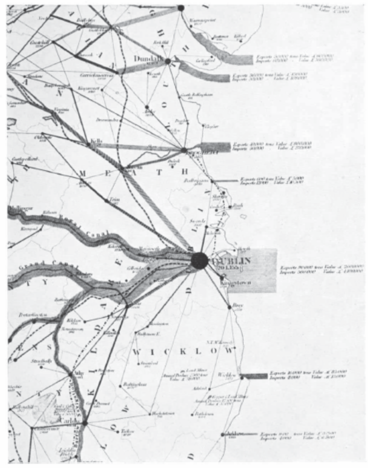


Centre for Advanced Spatial Analysis UCL

These are the flows maps from Henry Drury Harness, a surveyor in the British army in 1837 in Ireland



Harness' Passenger Conveyance Map, 1837. Original, 1 inch to 10 miles: Brit. Mus.



Section of Harness' Traffic Flow Map, 1837. Original, 1 inch to 10 miles. Brit. Mus.

Now in the 19th century there were at least two very significant flow mapping examples that related flows and networks to location theory; the first was by JOHANN GEORG KOHL, a German geographer who theorised that cities would have fractal like structures that were organised to span space.

There is a nice article on this by Peucker, Thomas K. (1968) Johann Georg Kohl, A Theoretical Geographer Of The 19th Century', **The Professional Geographer**, **20**: 4, 247-250

Here are some pictures from his book but note also that Kohl explored not only the plane but also the third dimension for cities which is some evidence that he was thinking about allometry – this is a prescient and far sighted example of early theorising of the kind we are doing here – some 160 years later – his work was produced in 1841

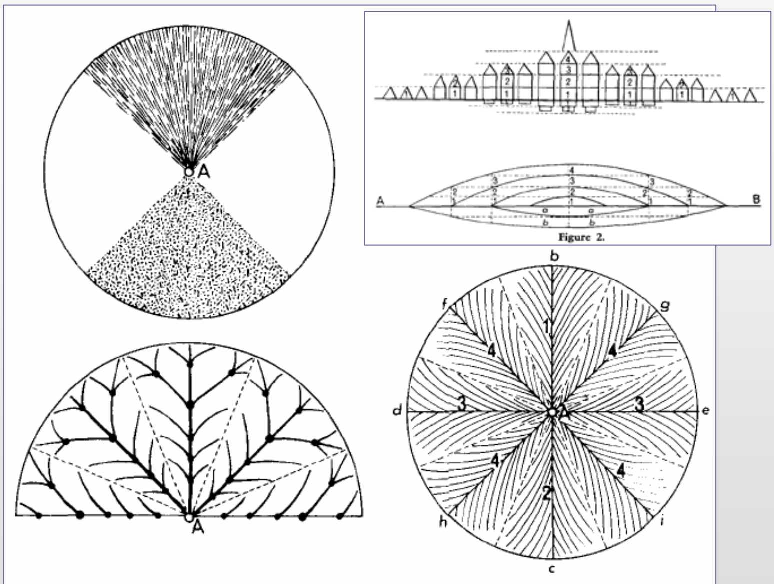


Figure 1.

Centre for Advanced Spatial Analysis

UCL

The second example is even more revealing – this is Ravenstein's map of migration currents from the 1880s where he used spatial interaction models to simulate flows.

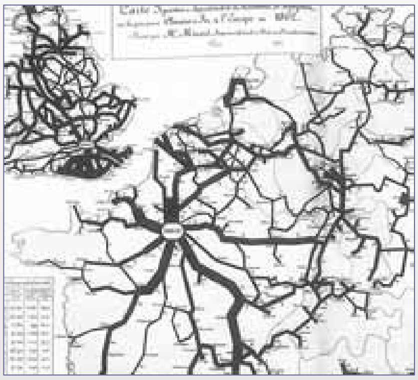
I have a much nicer map of this courtesy of Waldo Tobler which I have framed in my room at UCL but I couldn't find it on the web

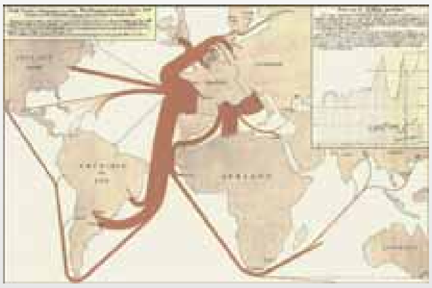
I will try and get this and if so substitute it for this one

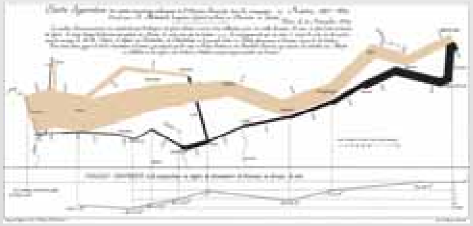
I cannot pass this by without showing the Minard maps from the mid 19th century

Centre for Advanced Spatial Analysis

UCL



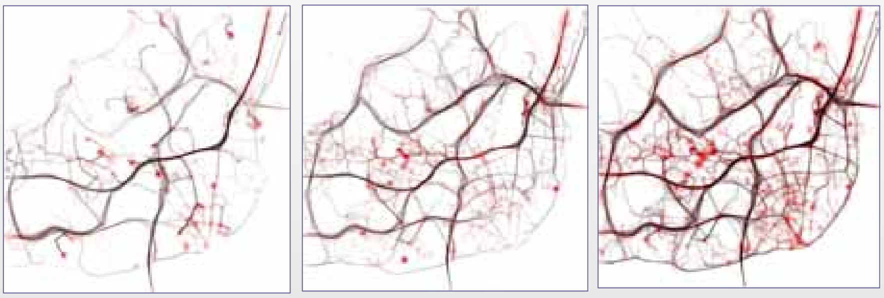


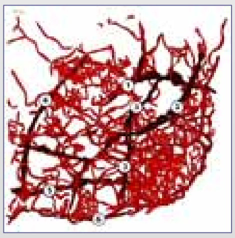


I want to bring this up to date because now there are some fantastic visualisations that are being done with networks and flows – let illustrate some of these. Here is one using Gephi I think it is called for our London tube data

Flows at 7am typical Monday last winter

Great animated maps of Lisbon showing traffic as blood flow





http://pmcruz.com/information-visualization/lisbons-blood-vessels

Centre for Advanced Spatial Analysis

UCL

Let me finish this section by pointing you are a great web site on visual complexity that has many many interesting examples of such visualisations http://www.visualcomplexity.com/

Centre for Advanced Spatial Analysis

UCL

The Evolution of Networks: Phase Transitions and Fractals

Ok let me change tack. I now want to present some examples of how networks evolve and their structure. Essentially we can think of a network as evolving and filling in with new links being added.

Our preferential attachment model is all about this and we will revisit this below.

However if we have a large number of nodes in a space and none are connected –if we begin to connect them up one at a time, the network gets denser but it takes a while before we can produce a path through the network when you have enough links to able to travel from any node to any other eg:

Here is a picture of this connecting up

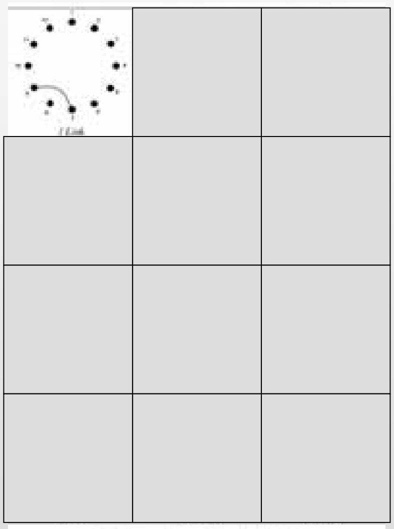


Figure 1: Increasing Connectivity in a 12-Node Graph

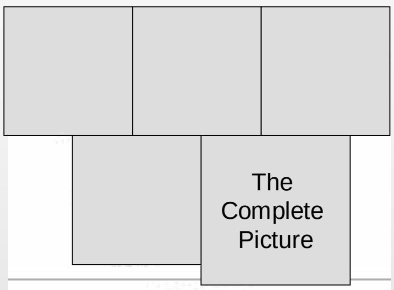


Figure 1 continued

Here is a picture of this connecting up

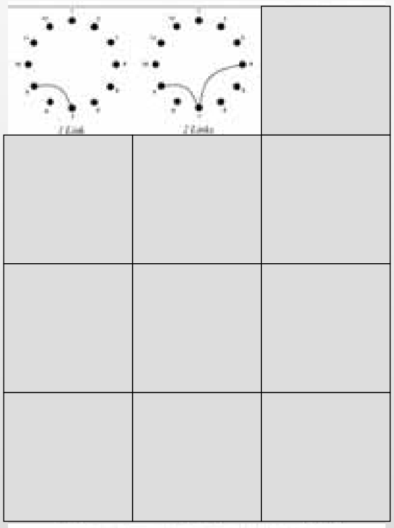


Figure 1: Increasing Connectivity in a 12-Node Graph

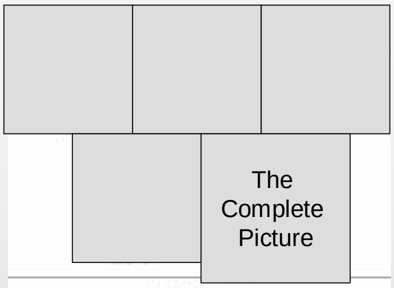


Figure 1 continued

Here is a picture of this connecting up

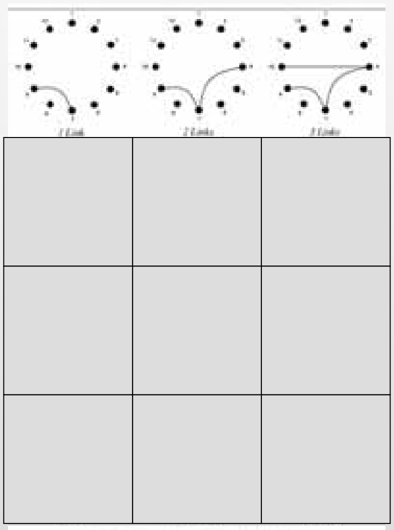


Figure 1: Increasing Connectivity in a 12-Node Graph

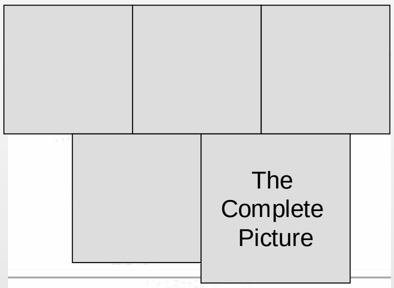
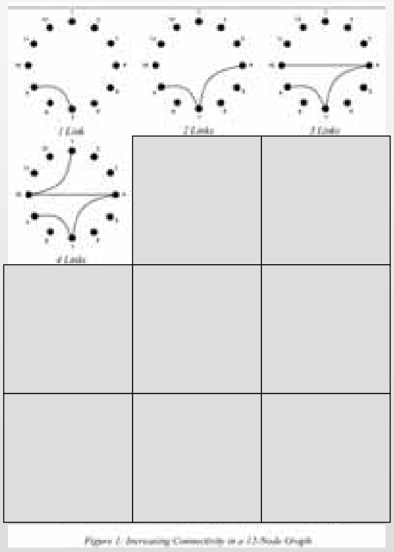
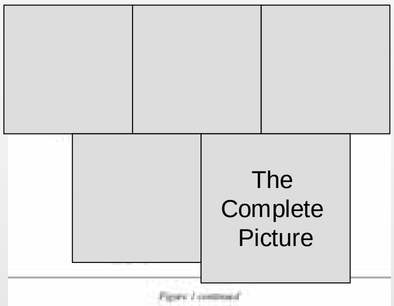


Figure 1 continued

Here is a picture of this connecting up

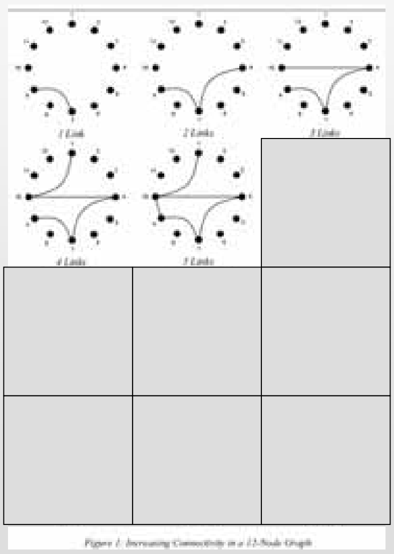


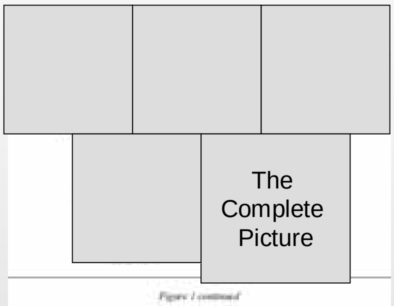


Centre for Advanced Spatial Analysis

UCL

Here is a picture of this connecting up

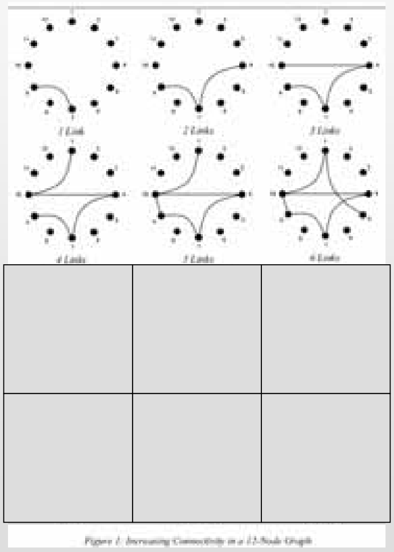


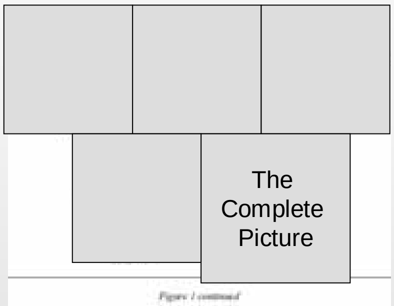


Centre for Advanced Spatial Analysis

UCL

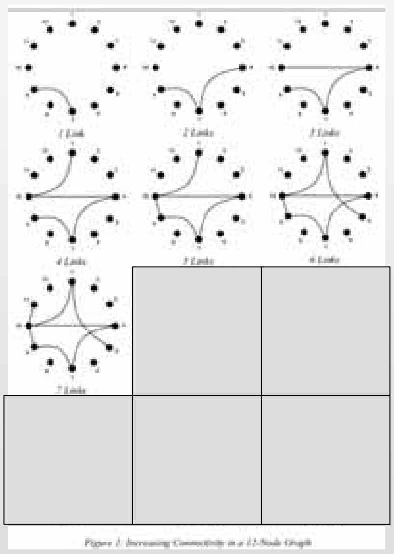
Here is a picture of this connecting up

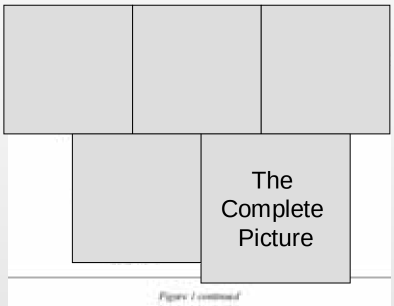




Centre for Advanced Spatial Analysis

Here is a picture of this connecting up





Centre for Advanced Spatial Analysis

Here is a picture of this connecting up

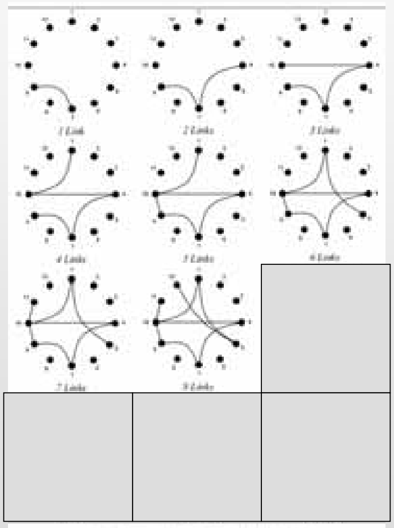


Figure 1: Increasing Connectivity in a 12-Node Graph

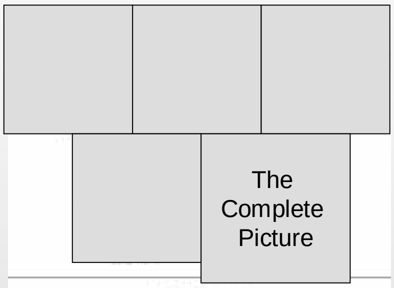


Figure 1 continued

Here is a picture of this connecting up

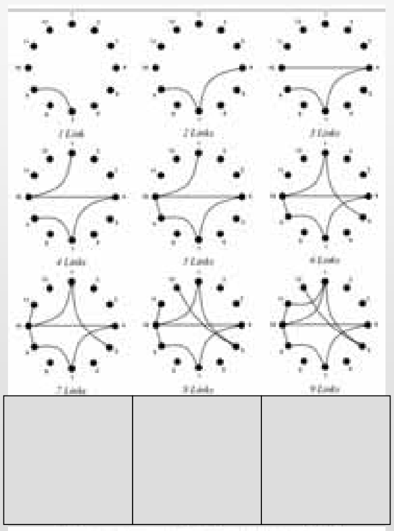


Figure 1: Increasing Connectivity in a 12-Node Graph

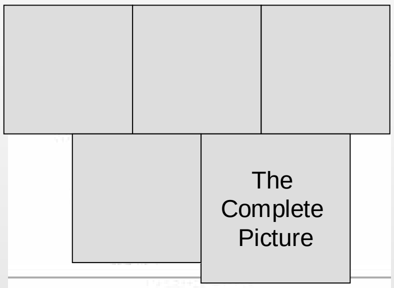


Figure 1 continued

Here is a picture of this connecting up

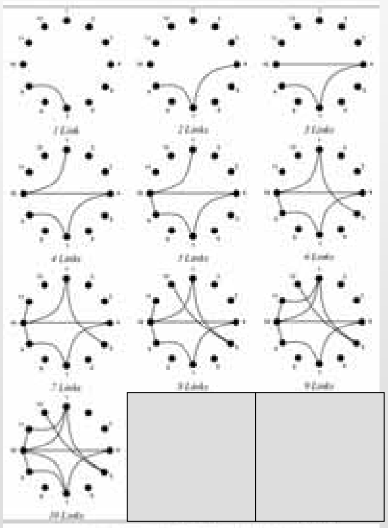


Figure 1: Increasing Connectivity in a 12-Node Graph

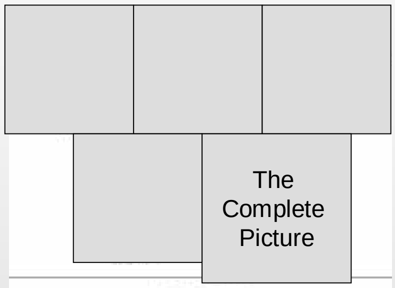


Figure 1 continued

Here is a picture of this connecting up

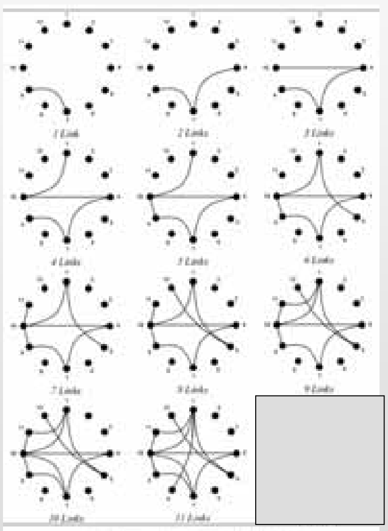


Figure 1: Increasing Connectivity in a 12-Node Graph

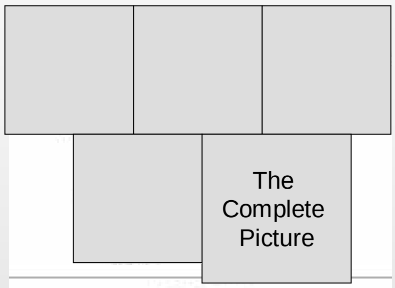
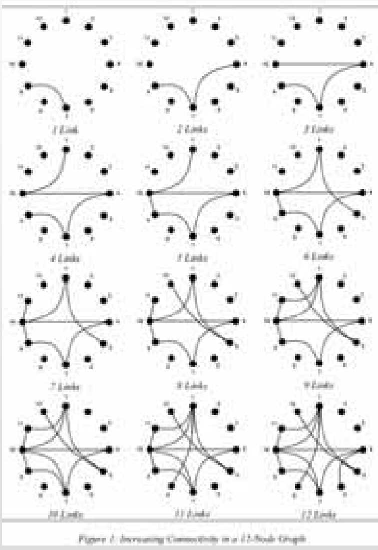
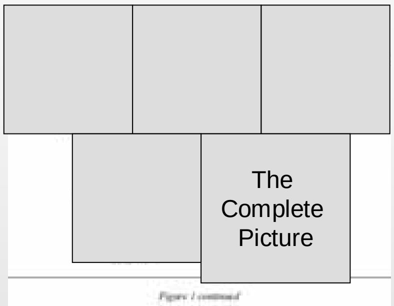


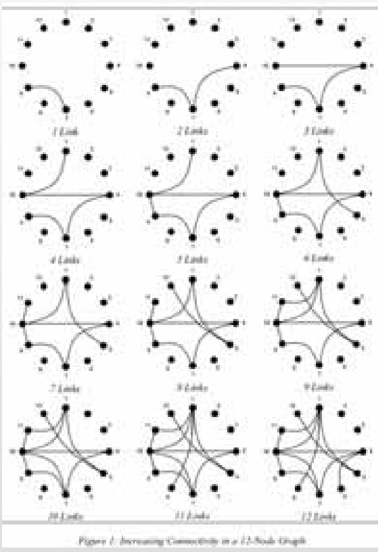
Figure 1 continued

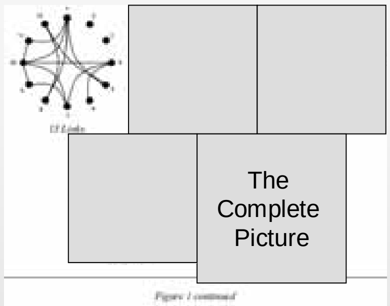
Here is a picture of this connecting up





Here is a picture of this connecting up





Here is a picture of this connecting up

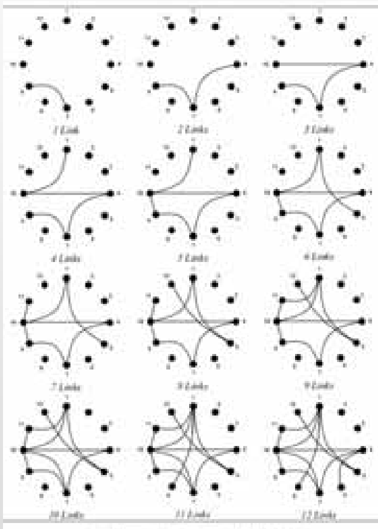


Figure 1: Increasing Connectivity in a 12-Node Graph

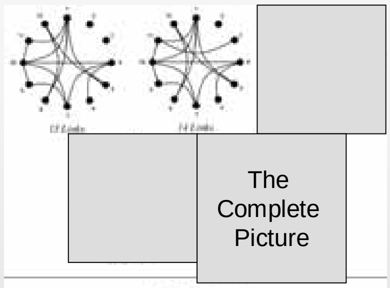


Figure 1 continued

The Complete Picture

Here is a picture of this connecting up

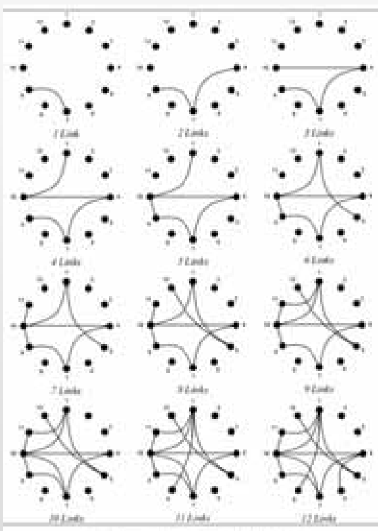


Figure 1: Increasing Connectivity in a 12-Node Graph

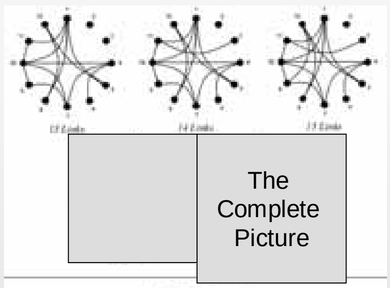
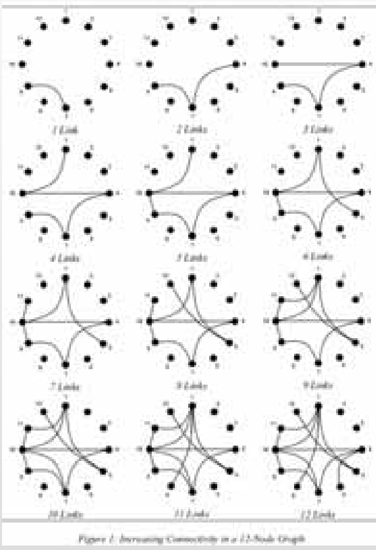
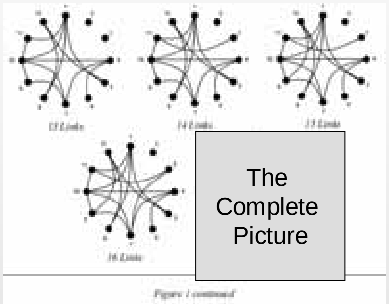


Figure 1 continued

The Complete Picture

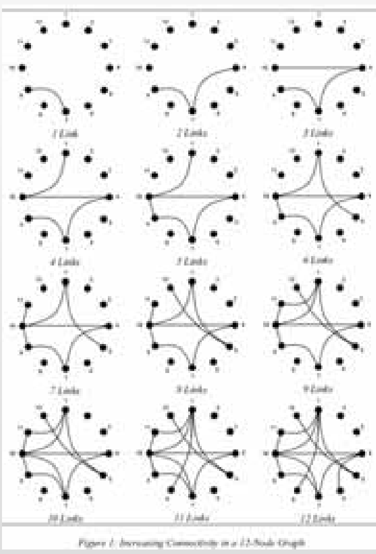
Here is a picture of this connecting up

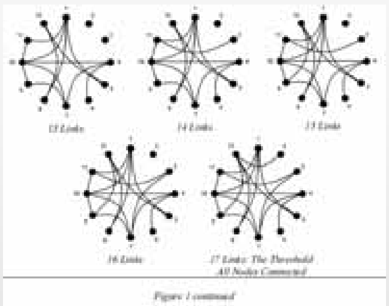




Centre for Advanced Spatial Analysis

Here is a picture of this connecting up





Centre for Advanced Spatial Analysis

Let us plot the average path length and see what happens – we get a phase transition at about 17 links – before then the path length drops quite steeply but after all are connected it doesn't change that much

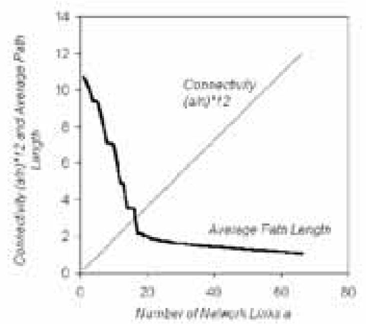


Figure 2: Link Density, Connectivity and Average Path Length
Note the breakpoint threshold at around 17 Links

The network then becomes scaling – if you want a detailed treatment then look at chapter 9 on the Fractal City in my book **Cities and Complexity**

http://www.amazon.com/Cities-Complexity-Understanding-Cellular-Agent-Based/dp/0262524791/ref=ntt_at_ep_dpt_1

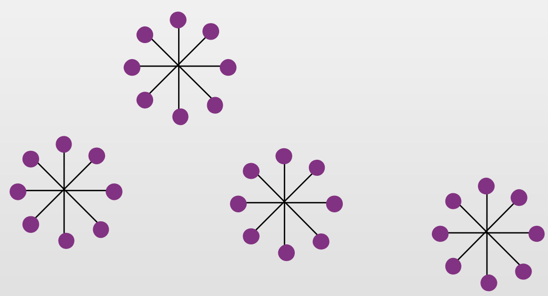
The Structure of Networks

In fact we need to mention a couple of points about the structure of networks that have come from the network science community and indeed started off this renaissance on networks.

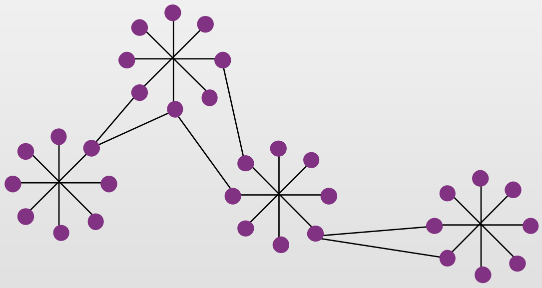
This is the idea of the small world. Basically if you form a random graph, i.e. you choose links randomly from a set of possible links between nodes, you do not get clusters in the graph.

In fact real networks always seem to generate clusters as in real communities. However clusters also need to be connected so that one can span the network with relatively shortest routes.

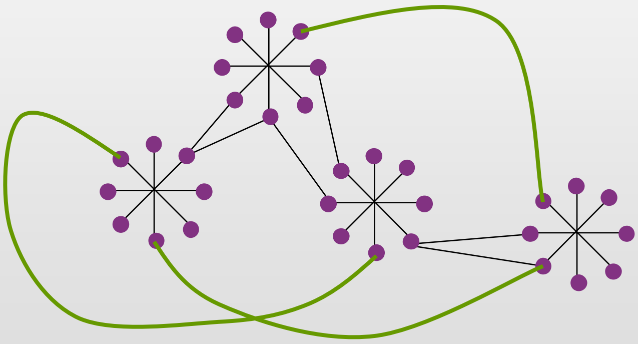
This networks with clusters and also long distance links are in fact the norm – these are sometimes referred to as small worlds after a famous article in the 1960s by Stanley Milgram. Here is how we construct such a small world.

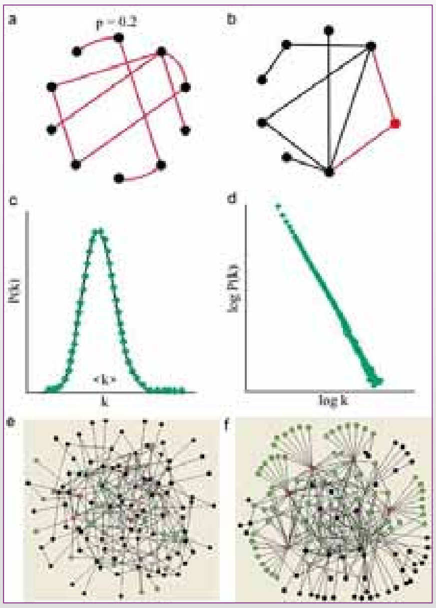


This networks with clusters and also long distance links are in fact the norm – these are sometimes referred to as small worlds after a famous article in the 1960s by Stanley Milgram. Here is how we construct such a small world.



This networks with clusters and also long distance links are in fact the norm – these are sometimes referred to as small worlds after a famous article in the 1960s by Stanley Milgram. Here is how we construct such a small world.

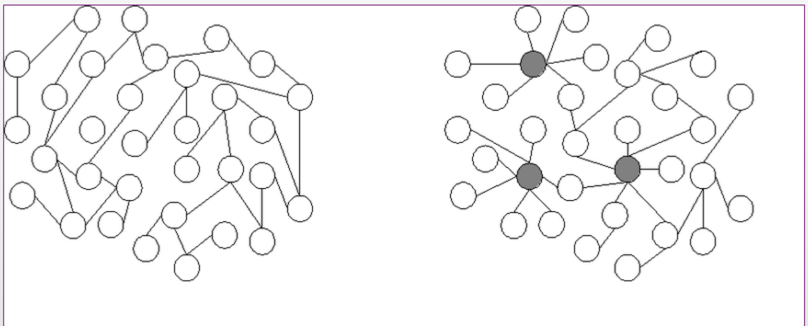




I am not going to take time out to give you a course in network science – suffice it to say that random graphs lead to Poisson scaling – a normal distribution of sizes of hubs

While preferential attach-ment leads to scale free or power law distributions.

So what about small worlds? It is not at all clear – they are more likely to be scale-free



(a) Random network (b) Scale-free network

Some more material which is useful is at
http://en.wikipedia.org/wiki/Scale-free_network

Centre for Advanced Spatial Analysis UCL

Back to Scaling and Preferential Attachment

An Example: we worked out example before but basically let us do it again with more detail – essentially we have a large number of nodes on a grid

These nodes are not activated as yet – we choose a limited number of nodes randomly across the space.

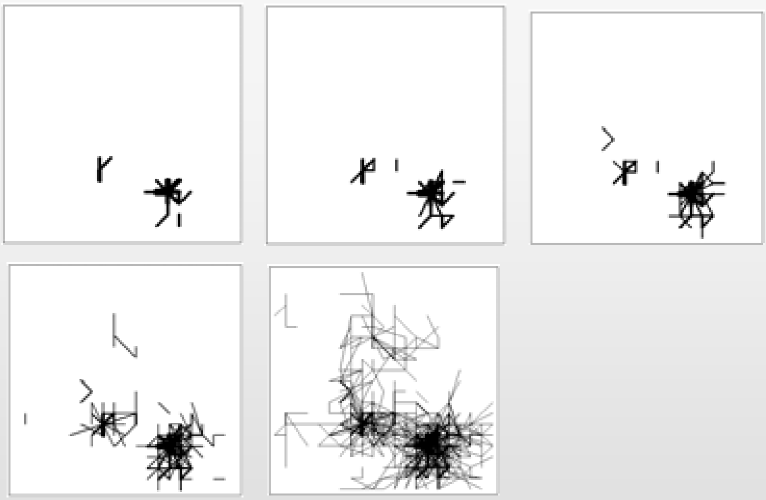
We then in each time step – consider whether these nodes connect to one another – the probability of their connection is proportional to the number of links at any one node. Of course at the start of this process there are just a bunch of nodes not connected to one will be chosen randomly. This node than increases it probability of connection a little at the next time step

This is a classic path dependence – we break the symmetry of the starting conditions and then what happens is that nodes increase their links differentially.

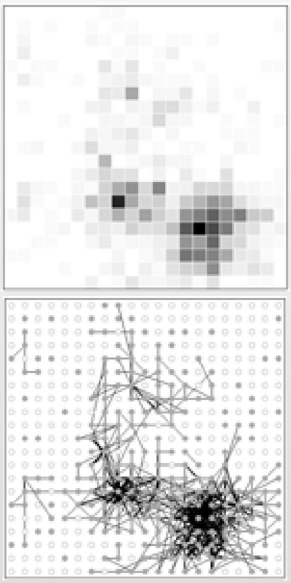
Of course, we start with a small set of active nodes and at each time step we randomly grow the system by choosing a small set of new nodes in random positions. These then become candidates for connection. Basically we have developed this model on the following 21 x 21 grid.

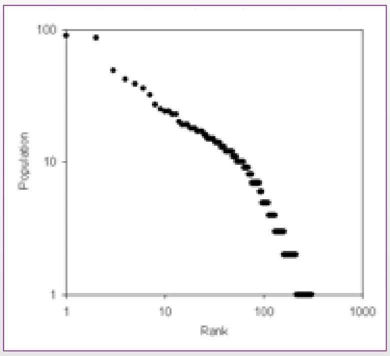
I need to give you a reference for all this where you can study it at your leisure

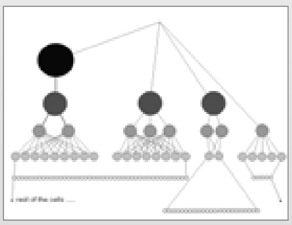
CASA Working Paper 85
Hierarchy in Cities and City Systems Michael Batty
http://www.casa.ucl.ac.uk/working_papers/paper85.pdf



Centre for Advanced Spatial Analysis







Centre for Advanced Spatial Analysis

An Example: Flows on the London Rail System

At this point, I developed some material from our Oyster card London public transport project emphasising how we computed indegrees and outdegrees of the highly structured overground and underground nodes and links.

We showed how the hub volumes varies by rank, illustrating a rank size rule of sorts for each of the 666 hubs but at each of 72 time slots (20 minutes intervals) through the day

This is the powerpoint I presented at a meeting in Glasgow and you can access it here. In the lecture I showed some slides from this

http://www.complexcity.info/files/2011/08/BATTY-Strathclyde-Networks-2011.pdf

Next time we will examine spatial interaction models.

The blog will have more and more references as the course continues

Questions

www.complexity.info

www.spatialcomplexity.info

Centre for Advanced Spatial Analysis UCL