ASU School of Geographical Sciences & Urban Planning

Lectures on Spatial Complexity
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Lecture 4: 24^{th} October 2011

Scaling and Size Distributions:
Rank-Size to Allometry & Economies of Scale

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Outline of Lecture 4

First a Digression on Tails: The Semantic Problem

More on Rank Size: Names and Skyscraper Heights

Firm Sizes and Incomes

Populations in SMSAs in the US

The Second Kind of Scaling: Changes in Shape: Population and Area

Interactions and Scaling

Population and Related Attributes; Super and Sub Linearity

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First a Digression on Tails: The Semantic Problem

Let us revisit the scaling model

$$f(P) = KP^{-\beta}$$

We form the cumulative frequency

$$F(P) = \int f(P)\,dP \sim KP^{-\beta+1}$$

And then the counter cumulative

$$r(P) = F(P') = \int_{P'}^{\infty} f(P)\,dP \sim KP^{-\beta+1}$$

We can simplify the counter cumulative to show that this too follows a power law: then

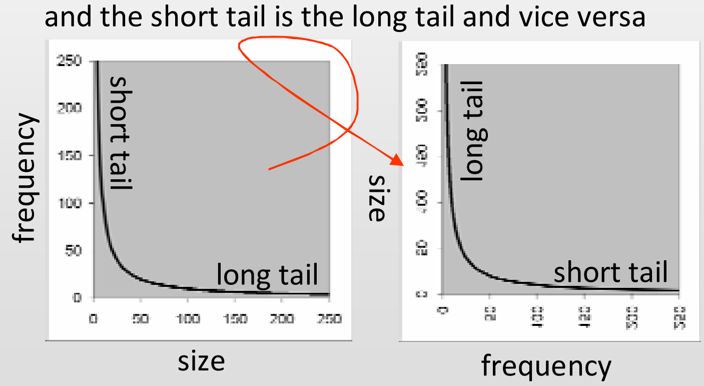
$$P(r) = Z\ r(P)^{\frac{1}{1-\beta}} = Z\ r^{\alpha}$$

So basically we get the pure Zipf Law

$$if\ \ \beta = 2 \quad then \quad \alpha = -1 \quad and \quad P(r) = Z\ r^{-1}$$

It is this equation $P(r) = Z / r$ that we can write as $r = Z / P(r)$

And now of course we can plot the function both ways around and the short tail is the long tail and vice versa



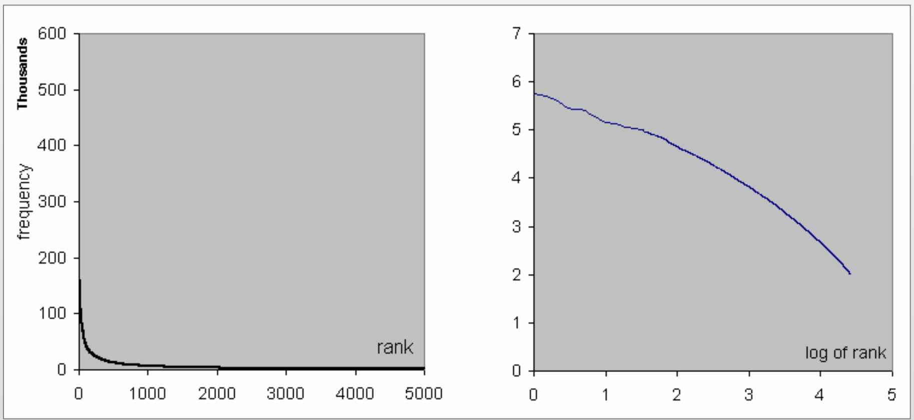
More on Rank Size: Names and Skyscraper Heights

Names – surnames also scale as a power law – let us look at some evidence as this provides some sort of intuitive sense of what such a law might mean.

We examine the electoral register for the UK in 1996

1	SMITH	560122
2	JONES	431558
3	WILLIAMS	285836
4	BROWN	264869
5	TAYLOR	251567
6	DAVIES	216535
7	WILSON	192338
8	EVANS	173636
9	THOMAS	154557
10	JOHNSON	145459

Now let us plot the graph of frequency versus rank and then also transform this to a linear scale – for all 25630 names in the data



Changes in red from 1881 to 1996

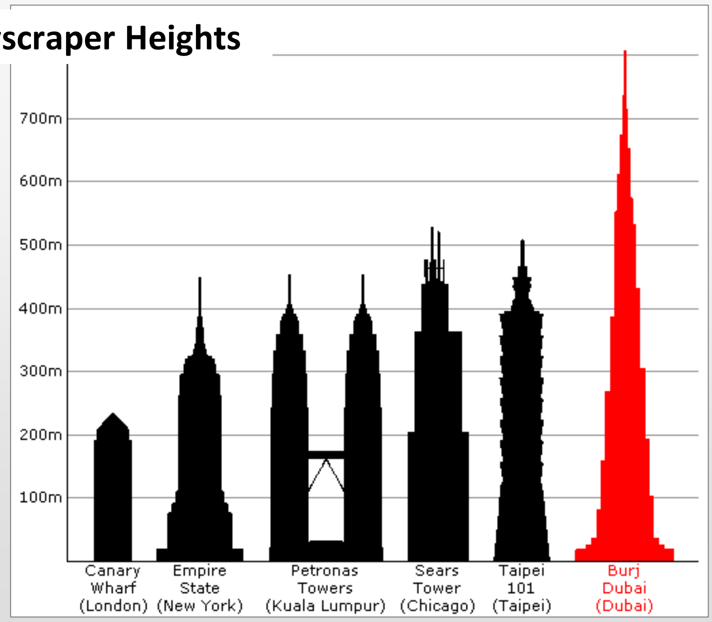
1996		***1881***	
SMITH	560122	SMITH	406573
JONES	431558	JONES	336447
WILLIAMS	285836	WILLIAMS	212602
BROWN	264869	BROWN	192061
TAYLOR	251567	TAYLOR	186584
DAVIES	216535	DAVIES	152450
WILSON	192338	WILSON	136222
EVANS	173636	EVANS	129757
THOMAS	154557	THOMAS	122449
JOHNSON	145459	ROBERTS	111602

Changes in Rank from 1881 to 1996 in the British Electoral Role

	1996	1881
BATTY	1254	957
BLAIR	500	514
BUSH	723	591
FINCHER	10769	8104
FLANNIGAN	4802	4808
HOWARD	114	111
WEBBER	575	471
WYATT	540	494

The size of the electoral population has increased from around 26 to 40 million

Skyscraper Heights



The conventional wisdom is that we define a tall building as being greater than 30 metres or maybe greater than 8, 10 or 12 stories

In fact, buildings greater than 30 metres and less than 100 metres are "high rise" while buildings greater than 100 metres are "skyscrapers"

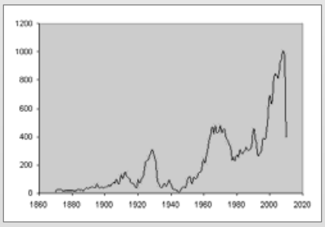
The average height of 'stories' over all high buildings is lowest in Paris at 3.27 m and largest in Dubai at 4.32 m

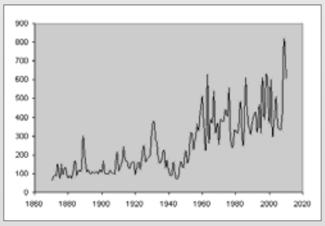
Ok let us look at the distribution of heights in different world cities – we will find the scaling in this interurban context much less in slope than that is cities – so the implication is that competition inside of cities is much less than between cities?

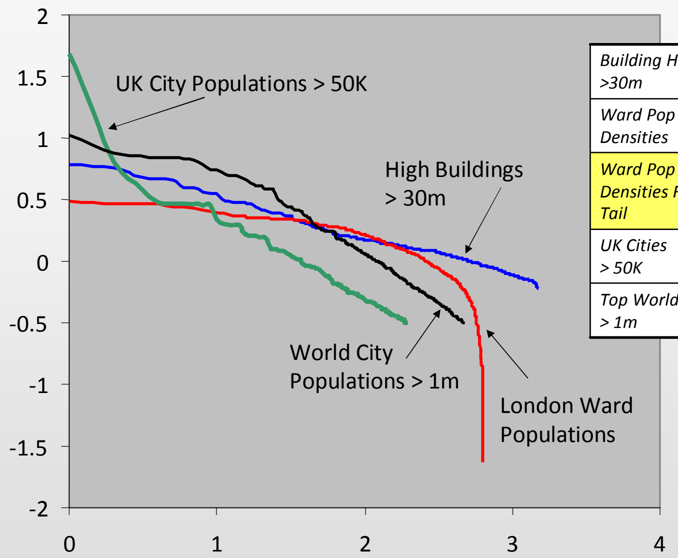
There is considerable debate (& semantic confusion) about the competitive forces and shape of the tails but for skyscrapers, interesting differences from other competitive phenomena

First, few have been destroyed – i.e. there is only ‘growth’ of new buildings; *second*, high-rise buildings are ‘qualitatively’ different from small; and *third*, buildings do not actually grow.

Here is the frequency of buildings > 30 m (left) and highest building constructed by year since 1870s (right)







Building Heights >30m	1486	0.99	***-0.31***
Ward Pop Densities	633	0.64	***-0.57***
Ward Pop Densities Fat Tail	160	0.89	***-0.20***
UK Cities > 50K	194	0.97	***-0.70***
Top World Cities > 1m	478	0.98	***-0.73***

Intra-city or intra-urban distributions tend to be flatter, less extreme, than inter-city or inter-urban

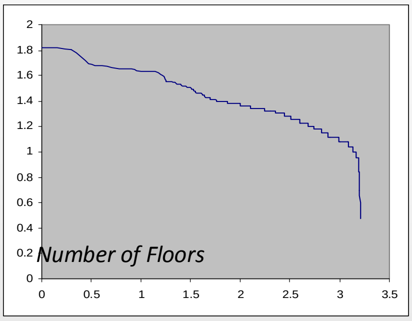
London and Hong Kong: Baseline Exemplars

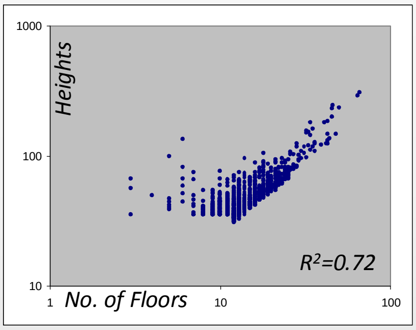
The ***Emporis*** Database: data on high rise buildings > 30 m for many cities, e.g. 8 in UK,

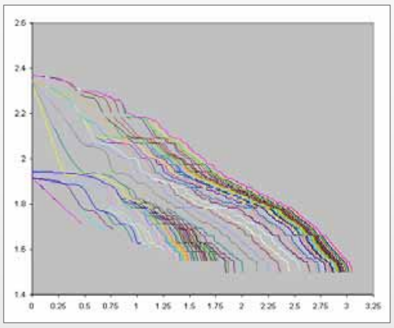
340,000 buildings world-wide with height, stories, floor area, land use type, year of build,

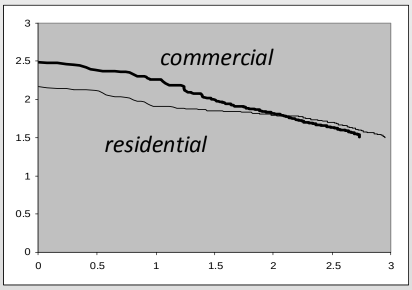
Many of these data fields are missing so a much reduced set is only usable for each city; e.g. London has 2495, but 1598 have height data.

We will look first at three distributions for each city: the scaling of height and number of stories, the prediction of height from stories, and change in scaling from the late 19th C



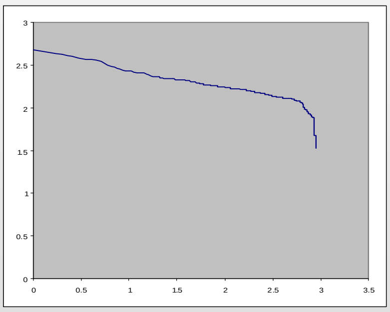


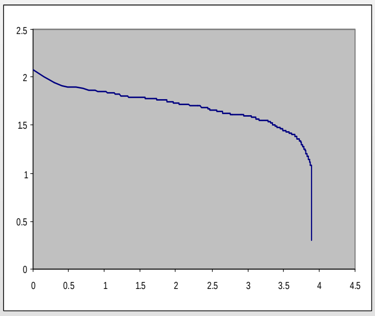




Type	Slope	r^2
All heights	-0.3169	0.9875
Heights from floor stories	-0.3186	0.8764
All floors	-0.3248	0.8433
Heights com+mixed	-0.3792	0.9945
Heights residential	-0.2428	0.9232

We can do the same for Hong Kong, our other exemplar, and we will simply show the heights scaling for now, and then for the rest of the cities, simply the results





Heights *Number of Floors*

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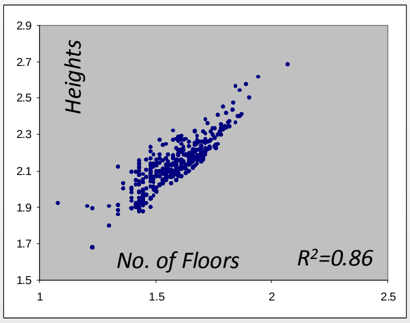

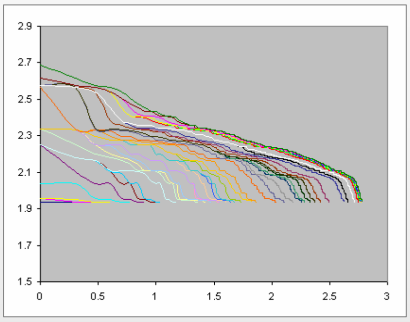

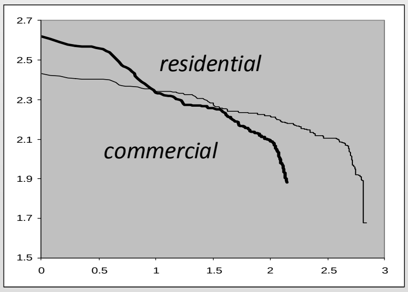

Type	Slope	r^2
All heights	-0.3046	0.6999
Heights from floor stories	-0.2686	0.7145
All floors	-0.2973	0.8018
Heights com+mixed	-0.2779	0.9804
Heights residential	-0.1679	0.9703

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The Top World Cities

We have taken the top 50 cities in terms of population starting with Tokyo (28 million) down to Melbourne (3 million)

Only 38 have good enough data, and thus we have selected these plus three other iconic cities – Dubai, Barcelona, Kuala Lumpur that have unusual high buildings.

We show this date in the table below

……………….

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Tokyo, Japan - 28,025,000 - 3 478	Santiago, Chile - 5,261,000 - 1587
Mexico City, Mexico - 18,131,000 - 1637	Guangzhou, China - 5,162,000 - 603
Mumbai, India - 18,042,000 - 1366	St. Petersburg, Russian Fed. - 5,132,000 - 962
Sáo Paulo, Brazil - 17, 711,000 - 6850	Toronto, Canada - 4,657,000 - 2883
New York City, USA - 16,626,000 -78 523	Philadelphia, USA - 4,398,000 - 703
Shanghai, China - 14,173,000 – 1222	Milano, Italy - 4,251,000 - 747
Los Angeles, USA - 13,129,000 - 1771	Madrid, Spain - 4,072,000 - 1429
Calcutta, India - 12,900,000- 527	San Francisco, USA - 4,051,000 - 1230
Buenos Aires, Argentina - 12,431,000 - 1893	Washington DC, USA - 3,927,000 - 1402
Seóul, South Korea - 12,215,000 - 3099	Houston, USA - 3,918,000 - 3292
Beijing, China - 12,033,000 - 1122	Detroit, USA - 3,785,000 - 696
Ōsaka, Japan - 10,609,000 - 1326	Frankfurt, Germany - 3,700,000 - 6632
Rio de Janeiro, Brazil - 10,556,000 - 3042	Sydney, Australia - 3,665,000 - 1190
Jakarta, Indonesia - 9,815,000 - 837	Singapore, Singapore - 3,587,000 - 6801
Paris, France - 9,638,000 - 971	Montréal, Canada - 3,401,000 - 550
Istanbul, Turkey - 9,413,000 - 2553	Berlin, Germany - 3,337,000 - 1125
Moscow, Russian Fed. - 9,299,000 - 2330	Melbourne, Australia - 3,188,000 – 723
London, United Kingdom - 7,640,000 - 2507	*Barcelona – 716 – 1605602*
Bangkok, Thailand - 7,221,000 - 949	*Dubhai 1175 – 1241000*
Chicago, USA - 6,945,000 - 2761	*Kuala Lumpur – 766 - 1 800 674*
Hong Kong, China - 6,097,000 - 8086	

	Av St	Slope	r^2		Av St	Slope	r^2
LA	4.1940	-0.6773	0.8197	Barcelona	3.9358	-0.4431	0.8993
Frankfurt	3.8447	-0.6135	0.8768	Sao Paolo	3.7728	-0.4429	0.7170
Houston	4.1404	-0.5884	0.7849	Sydney	3.8094	-0.4348	0.9561
Moscow	4.0626	-0.5654	0.8380	Beijing	3.9287	-0.4290	0.7999
San Francisco	4.0334	-0.5649	0.8409	Mumbai	3.5377	-0.4262	0.9250
Madrid	3.9703	-0.5469	0.9293	Shanghai	4.2147	-0.4122	0.8598
Detroit	4.0277	-0.5448	0.9203	Buenos-Aires	3.5224	-0.4110	0.6520
Toronto	3.4009	-0.5195	0.8406	Tokyo	4.1029	-0.4033	0.8271
Philadelphia	3.9459	-0.5156	0.8608	Mexico-City	3.9656	-0.3863	0.7881
Singapore	3.5658	-0.5128	0.8129	Santiago	3.5315	-0.3834	0.8000
St Petersburg	4.0417	-0.5078	0.7607	Seoul	3.9230	-0.3825	0.8550
All Cities-World	3.6714	-0.4874	0.8898	Istanbul	4.0284	-0.3264	0.7179
Chicago	3.5154	-0.4856	0.7909	Milano	3.3808	-0.3225	0.9796
Dubai	4.3194	-0.4786	0.8273	Jakarta	3.8144	-0.3177	0.7146
New York	3.4649	-0.4750	0.9305	Washington	4.0746	-0.3153	0.9145
Melbourne	3.7390	-0.4735	0.9006	Rio de Janeiro	3.3043	-0.3122	0.9107
Paris	3.2732	-0.4724	0.5742	Bangkok	3.7959	-0.3024	0.7780
Guangzhou	4.0083	-0.4678	0.8447	Calcutta	3.4231	-0.2715	0.6600
Montreal	3.9423	-0.4625	0.7441	Osaka	4.1274	-0.2679	0.8348
Berlin	3.4974	-0.4514	0.8737	KL	4.2134	-0.4492	0.9195

London and HK are not in this list yet ….

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We can do the same for the World's Buildings

We can of course aggregate the data we have looked at into all buildings and we have done this – there are 57000 usable heights from 340K buildings giving you a crude idea of the accuracy and error in this data set.

There are 33314 usable stories which is less than heights

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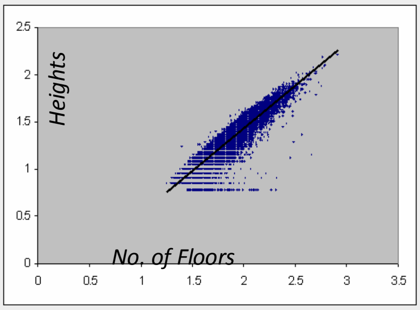

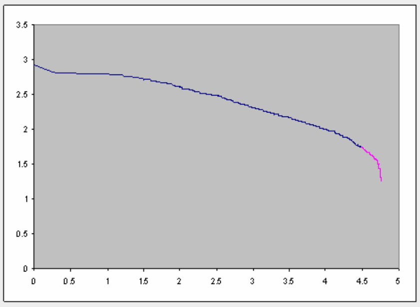

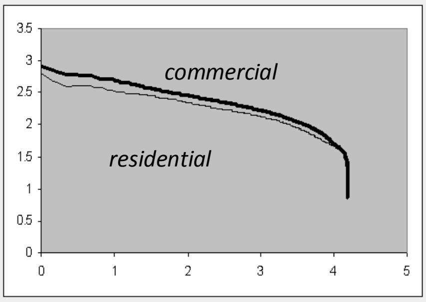

We have not done the temporal scaling relations as yet

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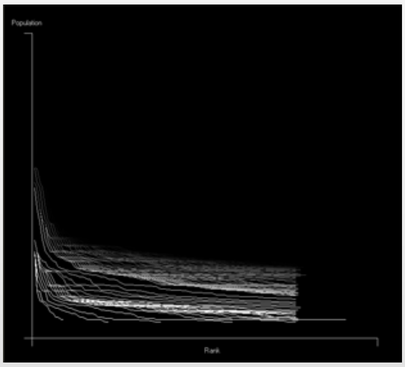
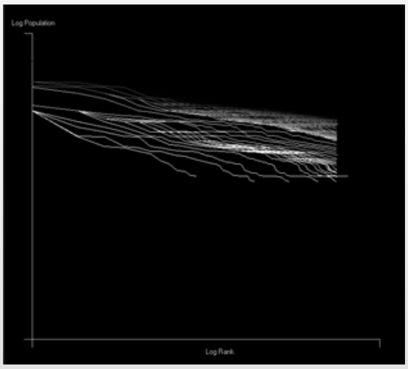
Regressions	Min	Av	No	Intercep t	Slope	r²
All buildings	3.6714					
All heights	18	70	56999	3.8932	-0.4874	0.8898
All heights less long tail	72	113	21053	3.3029	-0.3290	0.9906
All heights from floors stories	4	72	33314	3.8533	-0.5043	0.7626
As above less long tail	73	117	11850	3.0857	-0.2849	0.9398
Heights com+mixed	13	81	15464	3.8037	-0.5240	0.8546
Heights residential	12	66	16075	3.4930	-0.4581	0.9081
Heights viz floorarea			8299	0.2000	*0.3768*	0.4222
Height rank			3445	3.1134	*-0.3303*	0.9611
Floor rank			4218	6.5103	*-0.5641*	0.9511

Dynamics of Skyscraper Heights: Rank Clocks





Rank Size Relations for the Top 100 High Buildings in the New York City from 1909 until 2010

power form (left) *log form (right)*

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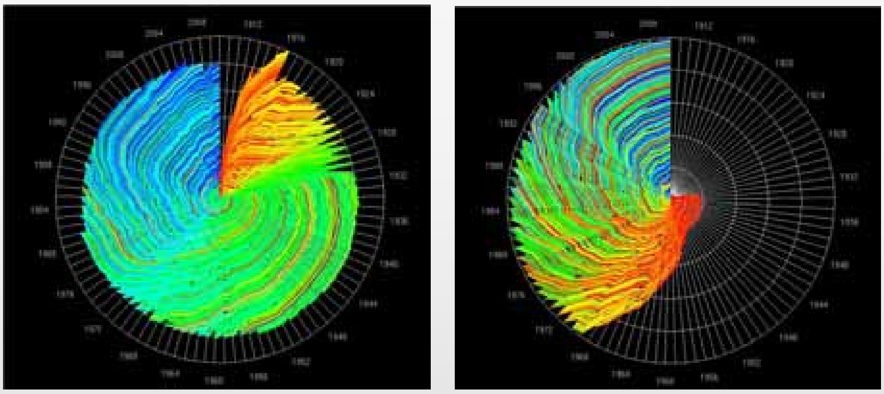
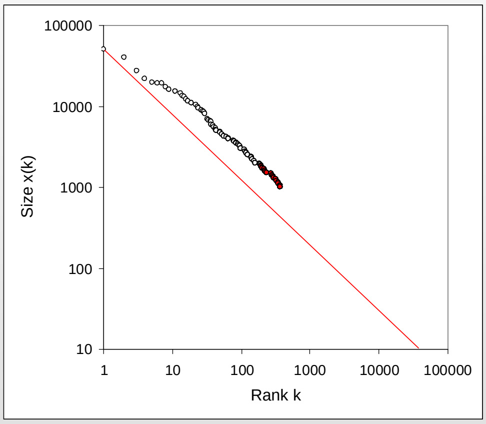


Rank Clocks of the Top 100 High Buildings in the New York City (a) and the World (b) from 1909 until 2010. There is much more work to do on all this and I am only giving you a taste of this – I will show some animations of these now RankClockUSCities.exe

Incomes (Top 500 US 2010) and Firm Sizes

name	income	age
William Gates III	50,000	53
Warren Buffett	40,000	79
Lawrence Ellison	27,000	65
Christy Walton & family	21,500	54
Walton	19,600	61
Alice Walton	19,300	60
S. Robson Walton	19,000	65
Michael Bloomberg	17,500	67
Charles Koch	16,000	73
David Koch	16,000	69
Sergey Brin	15,300	36
Larry Page	15,300	36
Michael Dell	14,500	44
Steven Ballmer	13,300	53
George Soros	13,000	79
Donald Bren	12,000	77
Paul Allen	11,500	56
Abigail Johnson	11,500	47
Forrest Edward Mars	11,000	78
John Mars	11,000	73
Jacqueline Mars	11,000	70
Carl Icahn	10,500	73
Ronald Perelman	10,000	66
George B. Kaiser	9,500	67
Philip Knight	9,500	71
Sheldon Adelson	9,000	76
Anne Cox Chambers	9,000	89
Jeffrey Bezos	8,800	45



I am having difficulty finding my data in firm size but I will place it in the PPT that I put online

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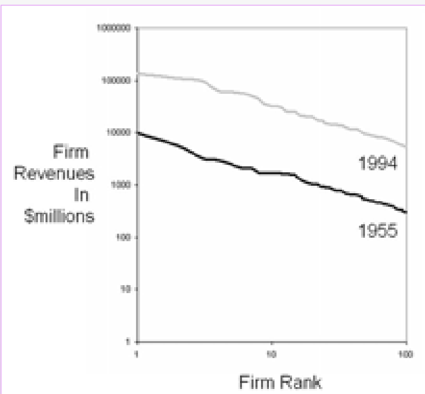
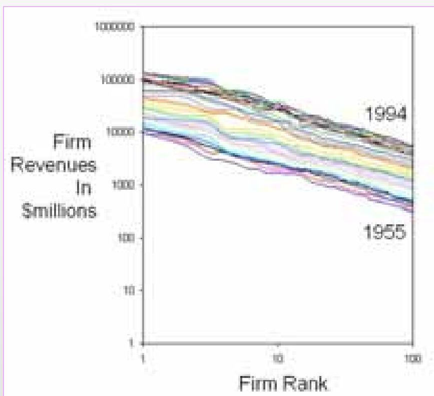
Firm Sizes: The Fortune 100, 1955 – 1995

www.cnn.money.com has the last 50 years worth of data for the top 500 firms by revenue earnings online. I have looked at the top 100 from 1955 to 1995 (because the data appears to change qualitatively in 1995), and have examined earnings/revenues and profits per earnings using the rank clock idea.

One might expect firms to behave in a more volatile way than cities. In fact of the 100 firms in 1955 only 39 are in the top 100 in 1995 and I can predict that there they will all be gone by 2020.

Here are the rank size relation, then we look at





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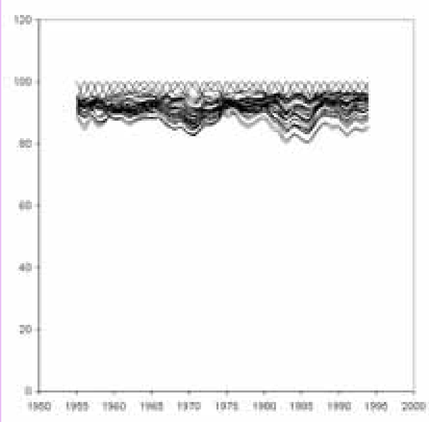

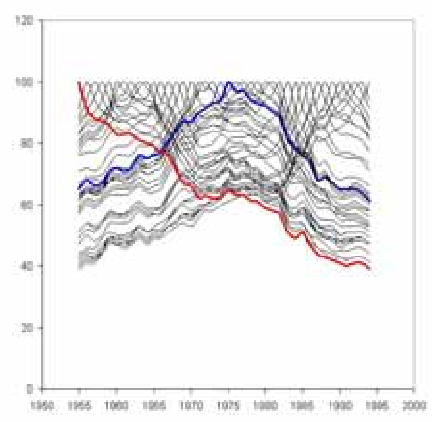

Percent Shift in Earnings/Revenues

No of Firms in Top 100 at Year t when all are in the Top 100 at Year τ

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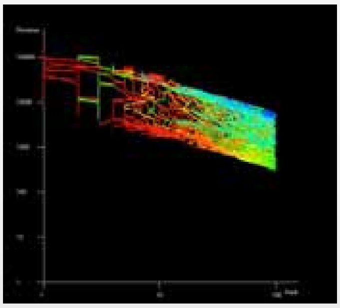

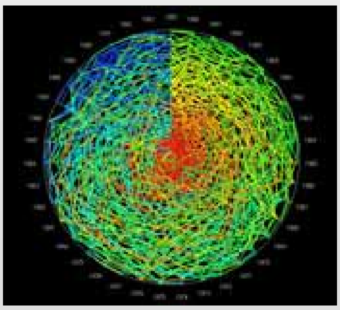

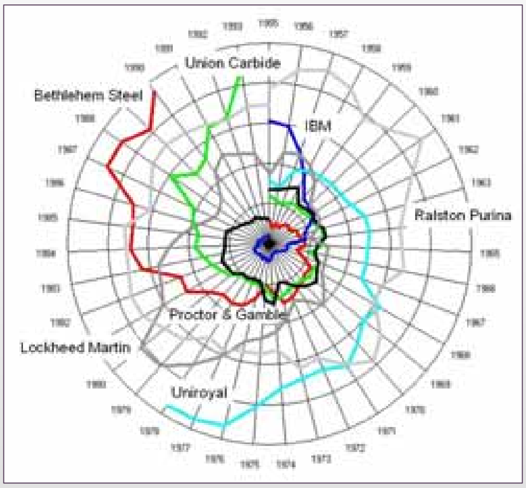

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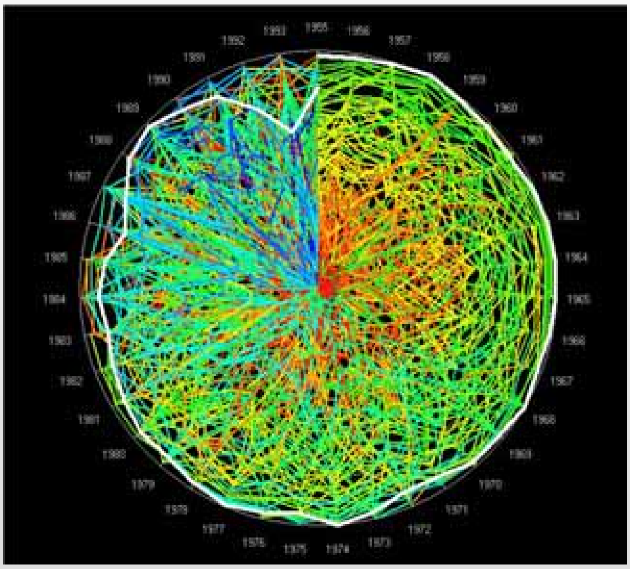
Rank-Size and Clock of Firm Profit Ratios



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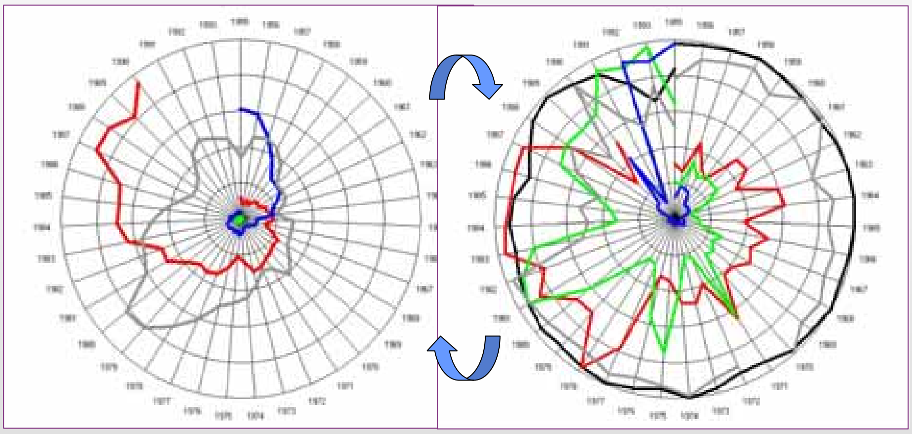
Earnings and Profit Ratios for Top Firms



Red – Bethlehem Steel: **Blue** – IBM:
Grey – Lockheed Martin: **Green** – GM

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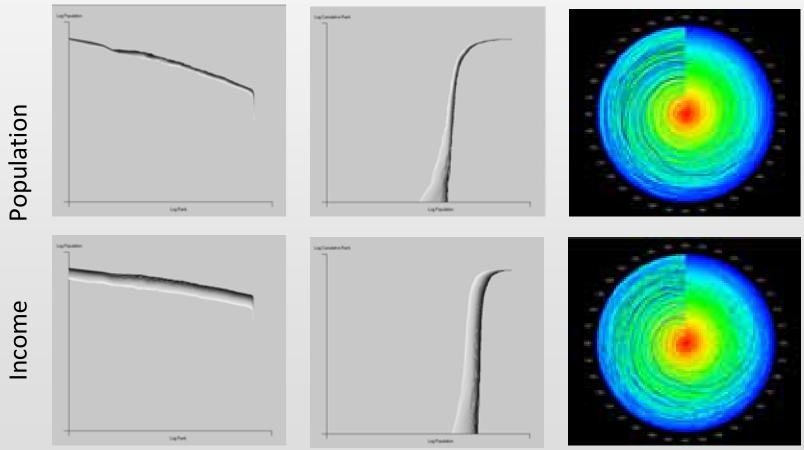
Populations in SMSAs in the US

There has been an awful lot of work done on size distributions involving income or wages. Indeed Pareto himself developed early work on this using the power law. The popular 80-20 rule emanates from this, so do ideas about the Long Tail and so on. (Note Pareto's Law is essentially the rank size rule)

But populations and wages – how do they compare in cities? I asked Luis Bettencourt about data one could get on cities and wages and he pointed me to the US Bureau of Economic Analysis on SMSAs from which I simply took their 366 regions for which population and income data are available for 37 years from 1969 to 2005 (the later regressions here are to 2008 using wages data). We used this earlier for population and land area.

We can easily plot the shifts, spaces, and clocks for these population and income data. These follow very regular scaling laws, at least in their fat tails. Here is a potpourri



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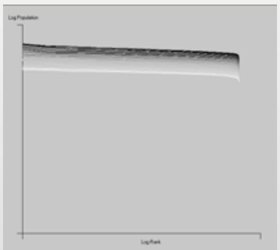
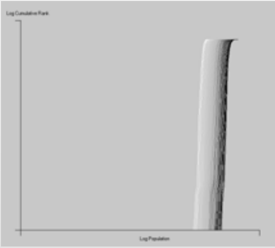
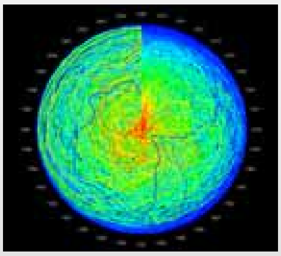
But the real interest is in per capita income/wages – i.e. *Wages / Population*. How does this rank? And if there are big shifts in rank, this shows divergence of these two variables

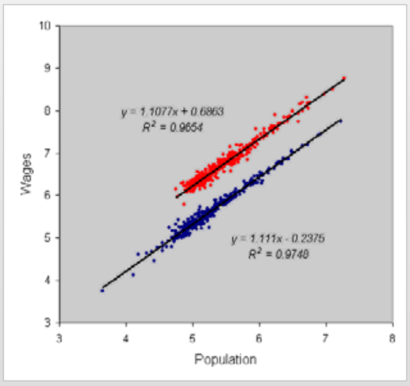


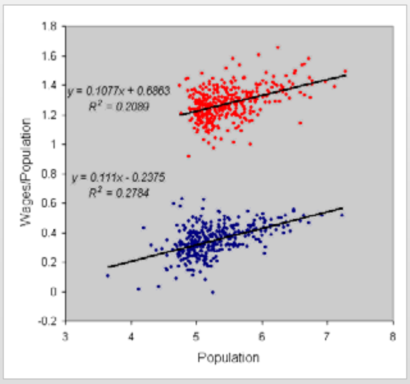




As you might expect the rank clock provides a graphic animation of this relative disorder at the micro level

Let us see if we can explain a bit of this. Population and income are very strongly correlated and superlinear as Geoff, Luis & Co suggest – below left. Thus wages/capita get greater with population and our regressions-confirm this – we just show 1969 & 2008, the end of the series





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A Digression on Pareto

Taken from (see http://en.wikipedia.org/wiki/Pareto_distribution)

The Pareto distribution, named after the Italian economist Vilfredo Pareto, is a power law probability distribution.

Pareto originally used this distribution to describe the allocation of wealth among individuals since it seemed to show rather well the way that a larger portion of the wealth is owned by a smaller percentage of the people in society.

He also used it to describe distribution of income. This idea is sometimes expressed more simply as the Pareto principle or the "80-20 rule" which says that 20% of the population controls 80% of the wealth (and by this definition, 80% of the population have only 20% of the wealth)

The Second Kind of Scaling: Changes in Shape: Population and Area

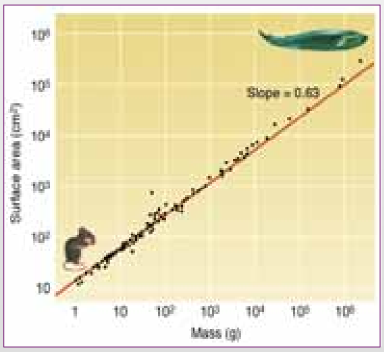
The idea of how things scale with size relates to whether or not they change in shape – that is attributes of size change differentially with respect to other attributes.

If things change linearly then they possess the property of isometry. More generally if the object changes differentially we say that this property of change is allometry.

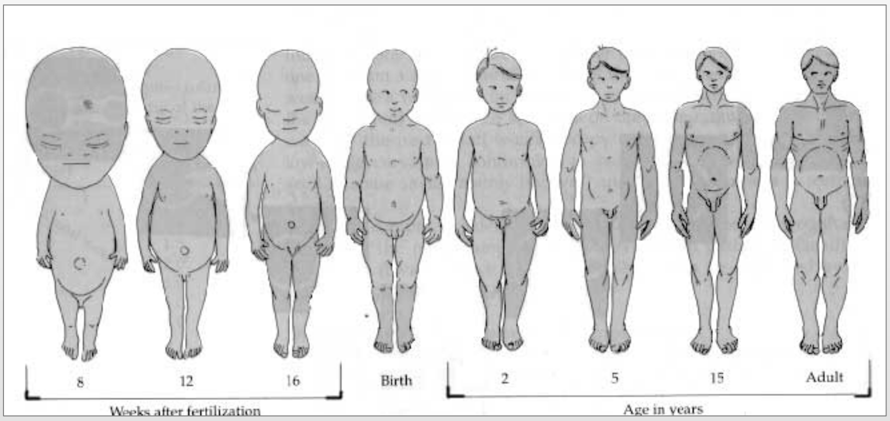
If things scale more than proportionality we say this is positive allometry whereas if they scale less than proportionality we say this is negative allometry

We write this relation as $Y_i = KP_i^{\alpha}$ where α is the allometric parameter. When $\alpha < 1$ this is negative allometry, And when $\alpha > 1$ this is positive allometry – increasing returns to scale in economic terms

By and large this relations refers to the fact that as an animal increases in mass or volume, its surface area increases at rate of 2/3 but its internal area increases faster than this



But with a species there are also change with age, and this leads to changes in shape which can be described by allometric relations



In fact the key issue in cities is how the population mass scales with the surface area and the implication is that as populations grow into the third dimension a little, then

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the populations scales more than proportionately, that is

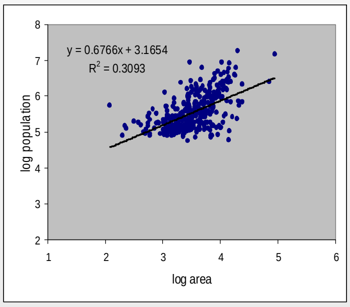
$$P_i = QA_i^{\alpha}, \alpha > 1 >> 2$$

This is likely because if the population goes up as the mass in 3 dimensions and the area goes up in 2 dimensions, a pure allometric relation would be

$$P_i = QA_i^{3/2}$$

A variety of researchers have looked at this and in general the allometry has been positive but more recent studies suggest that the allometric coefficient is lower, perhaps around 1 or in some cases even less than 1.

Here is a table of results and also the results for 366 SMSAs in 2005



In fact there is a lot of noise here and it looks like the trend is somewhat steeper from this graph if we take out the lower values of population

Table 1. Evidence of allometry between area and population size.

Date, t	Number of cities, $N(t)$	Allometric coefficient, $1+\alpha(t)$	r^{2*}	Reference
~10000BCE	359 hunter-gatherer places	1.429	0.240	Hamilton et al, 2007
~3000BCE	18 archaeological sites	1.188	0.774	Naroll, 1962
1940	412 US cities	1.333	NR	Stewart, 1947
1950	defined urbanized areas: US cities	1.167	0.706	Boyce, 1963
1950	153 US cities	1.163	0.865	Nordbeck, 1965
1950	33 US cities: Bartholomew data	1.351	0.900	Woldenberg, 1973
1950	155 US cities	1.163	0.850	Woldenberg, 1973
1951	157 UK cities	1.333	0.737	Stewart and Warntz, 1958
1953	Chinese cities	0.723	NR	Lo and Welch, 1977
1960	Swedish Cities	1.506	0.960	Nordbeck, 1971
1960	31 Ontario Canadian cities	1.149	0.990	Maher and Bourne, 1969
1960	defined urbanized areas: US cities	1.160	0.757	Boyce, 1963
1960	213 US cities	1.136	0.846	Nordbeck, 1965
1960	213 US cities	1.282	0.840	Woldenberg, 1973
1960	89 US cities: Masrel data	1.136	0.610	Woldenberg, 1973
1960	212 US cities	1.156	0.839	Lee, 1989
1961	205 UK cities	1.193	NR	Jones, 1975
1965	Swedish cities	1.538	0.790	Nordbeck, 1971
1965	318 Japanese cities	1.087	0.941	Nordbeck, 1965
1969	Satellite US cities	1.136	NR	Tobler, 1969
1970	248 US cities	1.149	0.846	Veregin and Tobler, 1997
1970	252 US cities	1.143	0.845	Lee, 1989
1973	40 historic classical buildings	1.299	0.990	Bon, 1973
1980	366 US cities	1.163	0.774	Veregin and Tobler, 1997
1980	373 US cities	1.171	0.876	Lee, 1989
1990	70 East Anglian UK cities	1.043	0.903	Batty and Longley, 1994
1990	801 Southeast UK cities	0.808	0.756	Batty and Longley, 1994
2001	386 European cities	1.014	0.740	Fuller and Gaston, 2009
2001	67 UK cities	0.946	NR	Fuller and Gaston, 2009
2001	100 highest population density metropolitan UK local authorities	0.763	0.637	Ferguson (this editorial)
2005	355 US Standard Metropolitan Statistical Area cities	0.676	0.309	Batty (this editorial)
2006	30 archetypal buildings	1.233	NR ~ 1	Steadman, 2006
2007	212 countries	0.769	0.710	Zhang and Yu, 2010
2008	~3.5m London buildings	1.296	0.835	Batty et al, 2008
2009	~122K London commercial buildings	1.199	0.838	Batty et al, 2008

* NR means not reported.

Interactions and Scaling

In fact as population gets larger the number of potential interaction rise as the square of the population or P^2

If we don't count self interactions and also assume symmetric interactions only count once, then the number of interactions is

$$I = \frac{P(P-1)}{2}$$

However cities do not exist on a point and thus we might assume that s deterrence effect of the areal size distance d kicks in and also that people only interaction with a fraction of those around them ξ

$$I = \xi \frac{P(P-1)}{2d}$$

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If this deterrent effect were area then as area is also distance squared then the effect might be as follows:

$$I = \xi \frac{P(P-1)}{2A} \approx \phi \frac{P}{d}$$

And we thus have something more like a linear relation. In fact it makes sense to think that anything do to with interaction will scale with population as something slightly more than the linear power but much less than the square

$$\phi \frac{P}{d} < I << \vartheta P^2$$

In short wherever we have interaction effects such as those that pertain to the generation of creative pursuits, even income generation etc, we are likely to see $I \sim P^{\beta}, \beta > 1$

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Population and Related Attributes; Super and Sub Linearity

There are some very good examples of allometry that pertain to this kind of scaling, recently due to Bettencourt, West and others. Essentially they identify scaling as being superlinear for creative pursuits which are economic and social, and sublinear for physical properties of cities such as infrastructure.

In short, as cities get bigger, wages, patent registrations and so on get larger more than proportionately and things like road space get larger less than proportionately with population

This is both good and bad – crime for example rises superlinearly as it is a classic interaction effect.

Table 1. Scaling exponents for urban indicators vs. city size

Y	β	95% CI	Adj-R^2	Observations	Country–year
New patents	1.27	[1.25,1.29]	0.72	331	U.S. 2001
Inventors	1.25	[1.22,1.27]	0.76	331	U.S. 2001
Private R&D employment	1.34	[1.29,1.39]	0.92	266	U.S. 2002
"Supercreative" employment	1.15	[1.11,1.18]	0.89	287	U.S. 2003
R&D establishments	1.19	[1.14,1.22]	0.77	287	U.S. 1997
R&D employment	1.26	[1.18,1.43]	0.93	295	China 2002
Total wages	1.12	[1.09,1.13]	0.96	361	U.S. 2002
Total bank deposits	1.08	[1.03,1.11]	0.91	267	U.S. 1996
GDP	1.15	[1.06,1.23]	0.96	295	China 2002
GDP	1.26	[1.09,1.46]	0.64	196	EU 1999–2003
GDP	1.13	[1.03,1.23]	0.94	37	Germany 2003
Total electrical consumption	1.07	[1.03,1.11]	0.88	392	Germany 2002
New AIDS cases	1.23	[1.18,1.29]	0.76	93	U.S. 2002–2003
Serious crimes	1.16	[1.11, 1.18]	0.89	287	U.S. 2003
Total housing	1.00	[0.99,1.01]	0.99	316	U.S. 1990
Total employment	1.01	[0.99,1.02]	0.98	331	U.S. 2001
Household electrical consumption	1.00	[0.94,1.06]	0.88	377	Germany 2002
Household electrical consumption	1.05	[0.89,1.22]	0.91	295	China 2002
Household water consumption	1.01	[0.89,1.11]	0.96	295	China 2002
Gasoline stations	0.77	[0.74,0.81]	0.93	318	U.S. 2001
Gasoline sales	0.79	[0.73,0.80]	0.94	318	U.S. 2001
Length of electrical cables	0.87	[0.82,0.92]	0.75	380	Germany 2002
Road surface	0.83	[0.74,0.92]	0.87	29	Germany 2002

Data sources are shown in *SI Text*. CI, confidence interval; Adj-R^2, adjusted R^2; GDP, gross domestic product.

PNAS

Growth, innovation, scaling, and the pace of life in cities

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By way of conclusion – glimpses of allometry

I want to look at allometry very briefly in terms of how shape changes in cities in the classic way. From our buildings data base for London, we have floor area; we do not have volume or surface area so we cannot get any detailed sense of how a building's volume changes as it gets bigger

However as a building gets bigger it needs more surface area than might be expected from its volume as its light needs to increase faster than the surface of its volume – hence the need to put holes into the building to maximise its surface area

Surface area thus goes up faster than 2/3.

We have explored this in a paper for London's buildings using various proxies for surface area; I refer you to the paper below for details as this is an important geometric issue.


THE EUROPEAN PHYSICAL JOURNAL B

Scaling and allometry in the building geometries of Greater London

http://www.complexcity.info/files/2011/06/batty-epjb-2008.pdf

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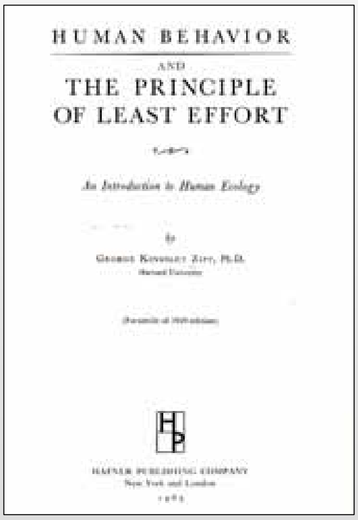
And we need to refer to a couple of classics on rank-size and allometry – Zipf's book and Stephen Jay Gould's seminal paper



Principle of least effort - Wikipedia, the free encyclopedia
en.wikipedia.org/wiki/Principle_of_least_effort
The **principle of least effort** is known as a "deterministic description of **human behavior**." The **principle of least effort** applies not only in the library context, but also ...

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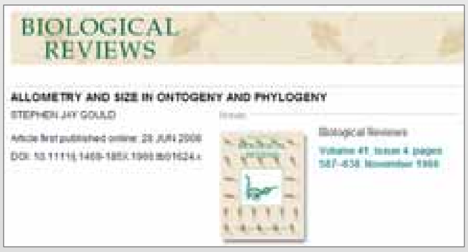

The blog will have more and more references as the course continues

Questions

www.complexity.info

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