Lecture 2: 19th October 2011

Hierarchy, Emergence, Feedback & NonLinearity

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Outline of Lecture 2

The Rudiments of Complexity Again: Function, Pattern, Interaction, Space, Scale, Size

Pattern and Hierarchy

Fractals and Space-Filling

Interactions and Networks

Evolution and Emergence

Feedback and Nonlinearity: Innovation

A Simple Network Model: Scale Free Networks

The Rudiments of Complexity Again: Function, Pattern, Interaction, Space, Scale, Size

First we will say something more about the key determinants of spatial systems

Function pertains to how systems work and hold together but we will not develop models of these workings as yet – we will simply state what we know in simple terms

Pattern pertains to the shape and structure of how functions manifest themselves either as locations and/or as networks. We focus on measures of structure such as dimension

Interaction pertains to how systems elements that exist in space are glued together – how they relate and these are configured as flows on networks

Space pertains to the extent and density of the system in question such as a city or region and we make a distinction between intraurban and interurban. In fact our ideas apply to both for cities are composed of agent and systems of cities are composed of cities which in turn are composed of agents

Scale is the way systems are configured in terms of how their size manifests itself across different spatial extents, neighbourhoods, cities, regions, nations, the globe

Size pertains to the volume or mass of a component, a city, a region and so on measured generically as population P which occupies in general greater extents of space across higher and higher scales.

Let us provide some simple mnemonics of all these characteristics. Our three types of scaling laws of course which we developed in the first lecture link all these characteristics together.

We will look at these in turn. Function relates to how the components of a system – how its locations relate to one another in terms of how populations relate. Locations intensify as people demand to be together to exchange in markets and it is usual for there to be a limited number of points where this takes place.

The density around these points is highest and the population then distributes itself around such points usually following some sort of inverse distance law as we showed in the first lecture with the gravity model.

Assume that everyone interacts with a market C. Then the distance from a point j to the market is d_j and we assume the density D_j follows an inverse square law of distance – a power law (or in fact, often a negative exponential) – and this can be written

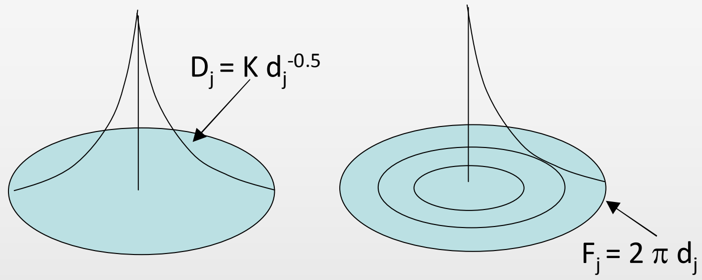
$D_j = K d_j^{-2}$

Now we can plot a density cone in familiar form around the market centre C and we note also that the number of points where people can live around C varies according to the circumference of the circle at distance d from the centre, that is the number of locations, is

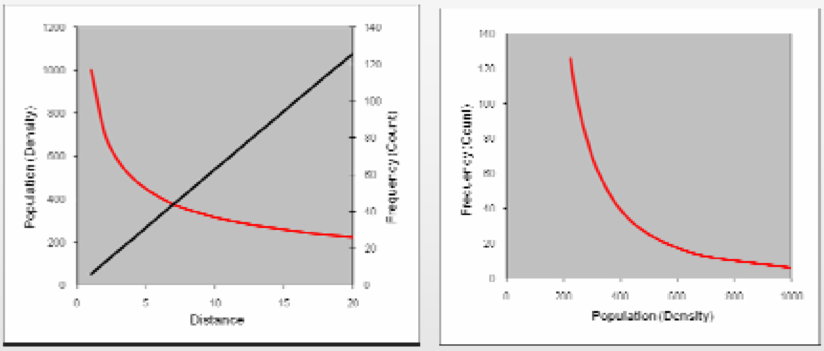
$F_j = 2 \pi d_j$

The size of each point is the density $D_j = K d_j^{-2} = P_j$

Here is the graphic



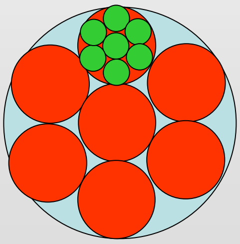
We have changed the power of distance to 0.5 because this gives us a better result. Now let us see if this satisfies our basic scaling relations – let us count the frequency of different locations and compare these against different sizes. We thus compare F_j against D_j or P_j for a simple numerical example



It is easy to show that the relation is dead simple and is (by construction) a power law, that is $F_j = G P_j^{-2.0}$ which leads directly to the rank size rule – albeit using a power of distance very different from the inverse square law. So we have establish that this functional form leads to scaling.

Now we can do exactly the same kind of exercise for a large space divided into a hierarchy of central places – we can assume a radius around the largest centre and calculate the total population, and then for successive smaller centres with smaller hinterlands, we can produce populations and then compare these against areas which are frequencies and which generate the same kind of rule.

Our lattice is then, and we can forget the spaces in between –

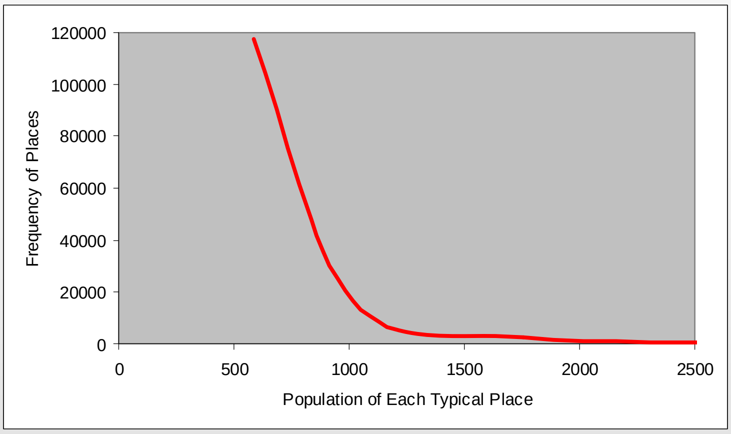


Applying the same logic as for each circular town at each level and computing total populations in the hierarchy, we derive the same sort of scaling as follows.

First we assume a maximum radius d=1000 for the biggest all embracing central place – the blue circle and this gives the following total population as the integral of the density up to d=1000; the population is approximately 15811

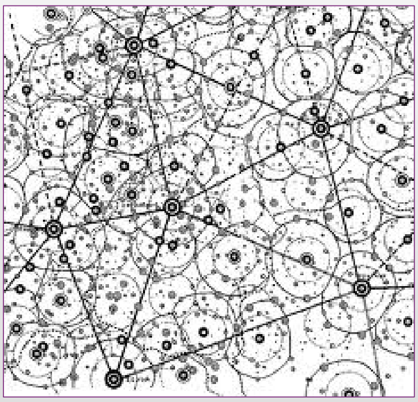
Then at the next level down we divide the area of the largest circle into say, 7 red sub-circles each with radius 1000/3 and each of gives a population of 9128. We then get 49 areas at the next level down – the green circles each with a population of 5270 and so on, down to where we fix the lowest level at 40,353,607 circular areas, each with a population of 113.

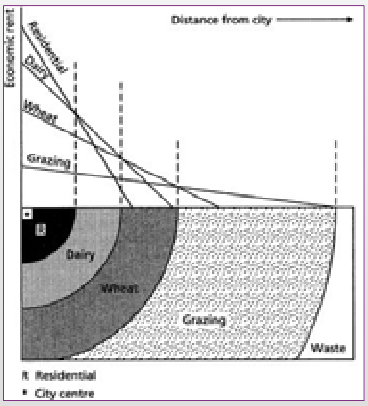
If we then graph the frequency of this hierarchy against typical population size and plot the following graph which is clearly scaling.



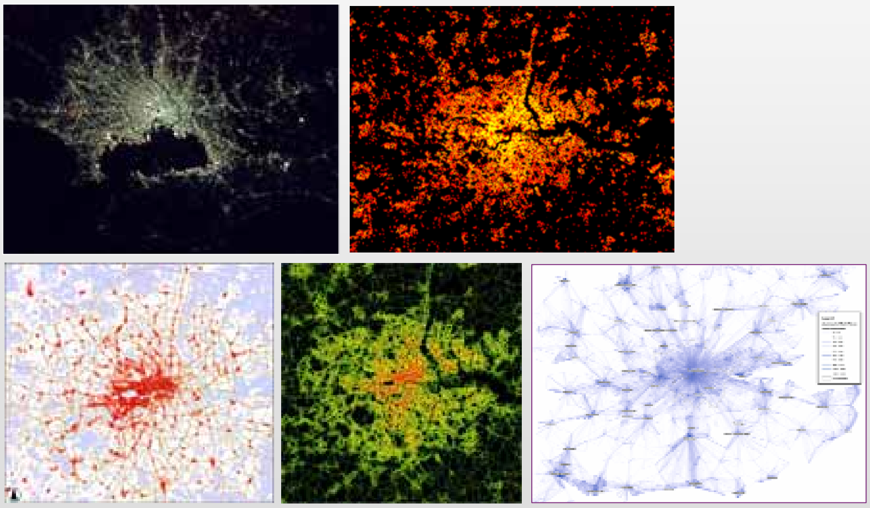
This relation is linear on a log-log scale with the power of the size around 0.28, dependent of course on the assumptions we have made about how many levels of hierarchy there are and how each successive size is subdivided

OK now let us put up some patterns that are consistent with these sorts of structure – this sort of radial concentric massing – first the two that we showed last time

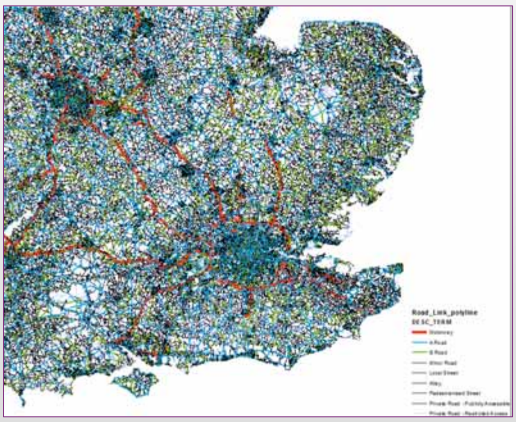


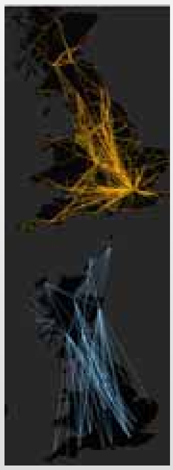


And now some morphologies that show that space is not filled completely but less so thus generating fractals



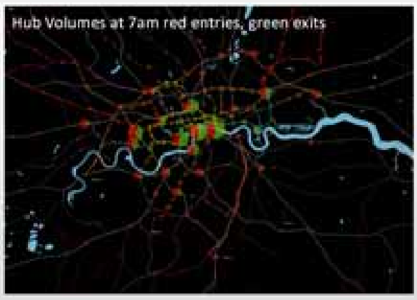
Now let us put up some interactions and their networks to show how these are support our concentric patterns around the centre. Note too that the concentric logic is weakening with polycentricty – more on that later

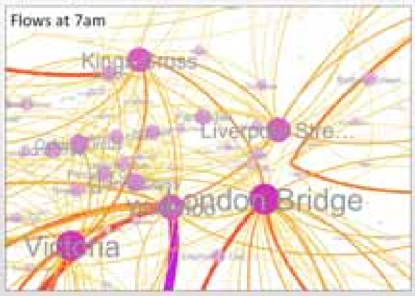




In fact I will refer you to the *A Science of Cities* blog to get some more on interaction patterns and flows by way of example. Let me see if I can log on from here and drill down.

http://www.complexcity.info/media/movies/urban-flow-networks/





Ok it worked amazingly so I am legit faculty at ASU now

I won't say anything about space, scale and size other than to remind you that in the previous (first) lecture, we ended with me identifying three scaling relations that permeate our discussion of spatial complexity everywhere.

Our ***first scaling relation***, allometry – as population grows, other attributes scale more or less than proportionately with size – this is qualitative change – it is nonlinear as much of our theory is

Our ***second scaling relation*** – the conventional spatial interaction model – relates volume of interaction or connection to distance or deterrence – to space

Our ***third scaling relation,*** the rank size rule, relates frequency of size to volumetric size (mass) as an inverse power law

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Pattern and Hierarchy

Ok a change in pace – let me tell you now about fractals – we have anticipated these a lot so far but let me impress on you the notion again of self-similarity, of modular bottom-up construction, and of hierarchy.

Fractals are objects that scale – they show the same shape at different scales in space and/or time

This property of scaling is sometimes called self-similarity or self-affinity

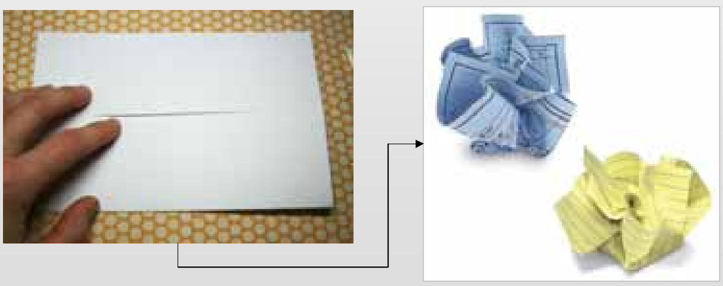
In our world of cities, we think of this scaling as being a replication of the same shapes in 2 or 3D Euclidean space

This suggests modularity in growth and evolution and processes that are uniform over many scales

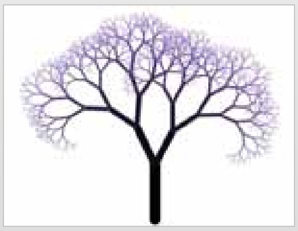
The signature of a fractal is called its dimension and usually this suggests how the fractal fills space

If we think of 0-d as a point, 1-d as a line, 2-d as a plane and 3-d as volume, then a fractal also has fractional dimension.

This means that the Euclidean world is the exception not the rule as the integral dimensions are simplifications. The best example of a fractal is a crumpled piece of paper



Other excellent examples are trees that are clearly self-similar and enact the notion of hierarchy directly into form



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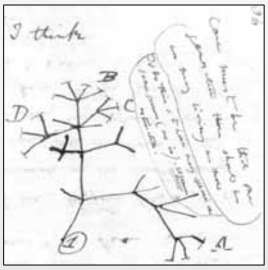
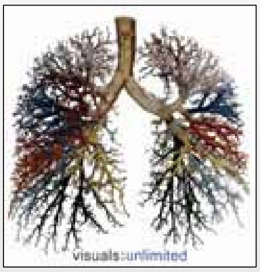
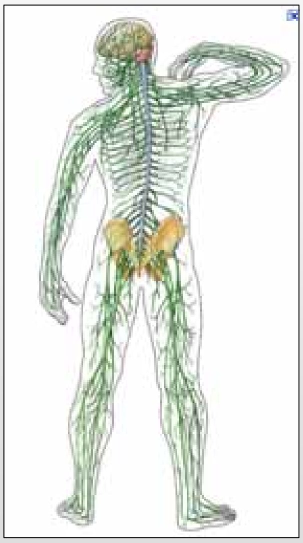
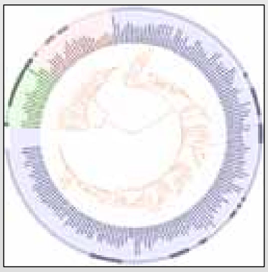
There are many wonderful examples in man and nature …









Fractals and Space-Filling

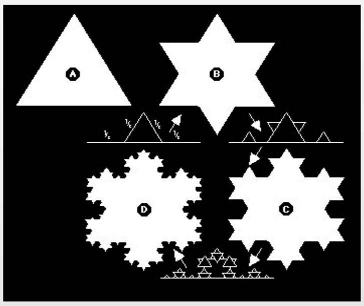
There are some basic conundrums and paradoxes with fractal geometry – the clearest one is the length of a fractal line – if a line is truly fractal, it fills space more than the line and less than the plane with a fractal dimension between 1 and 2. As it also scales – any bit of it has the same shape as an enlarged or reduced bit but the length is infinite.

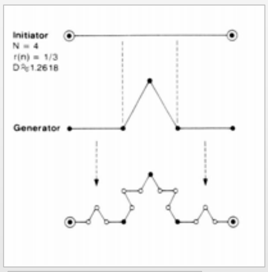
Note the famous paper in Science in 1967 by Mandelbrot – *How long is the coastline of Britain?*

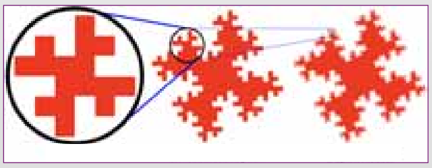
The answer of course is that it is infinitely long – it depends on the measuring stick – it depends on the scale

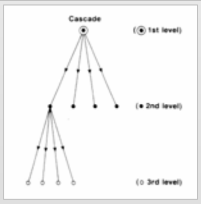
It is self-similar with detail being added at every scale – let is look at a simple model of this

We can shows this for the Koch curve. Note how we construct the irregularity by adding a scaled down piece of the curve





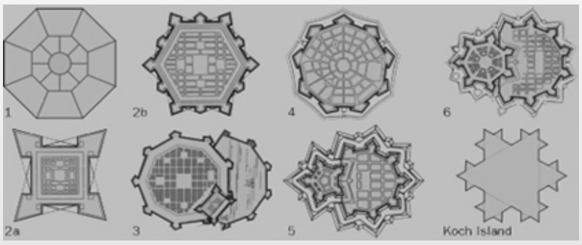


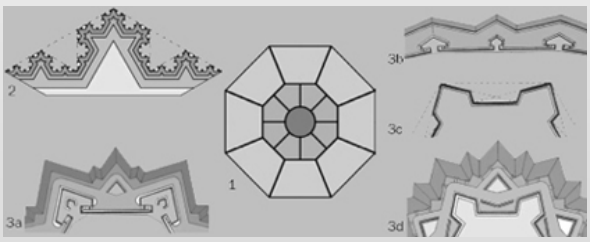


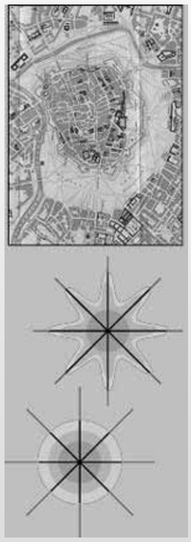
Note how the hierarchy is a feature of its construction

And note how the line is infinite but the area is finite …

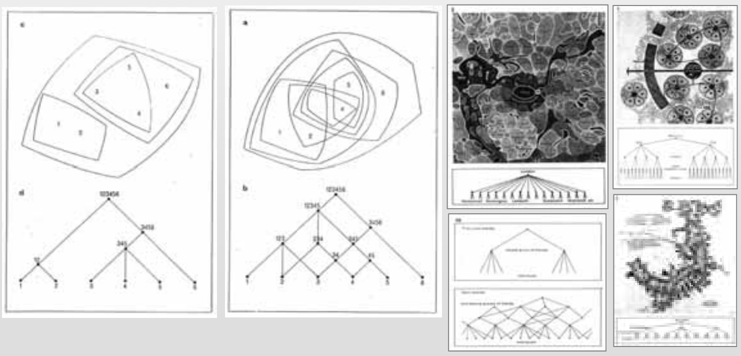
This is resonant in many ideal forms – like ideal towns as well as shapes in nature



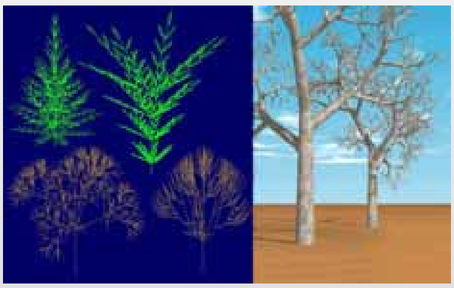




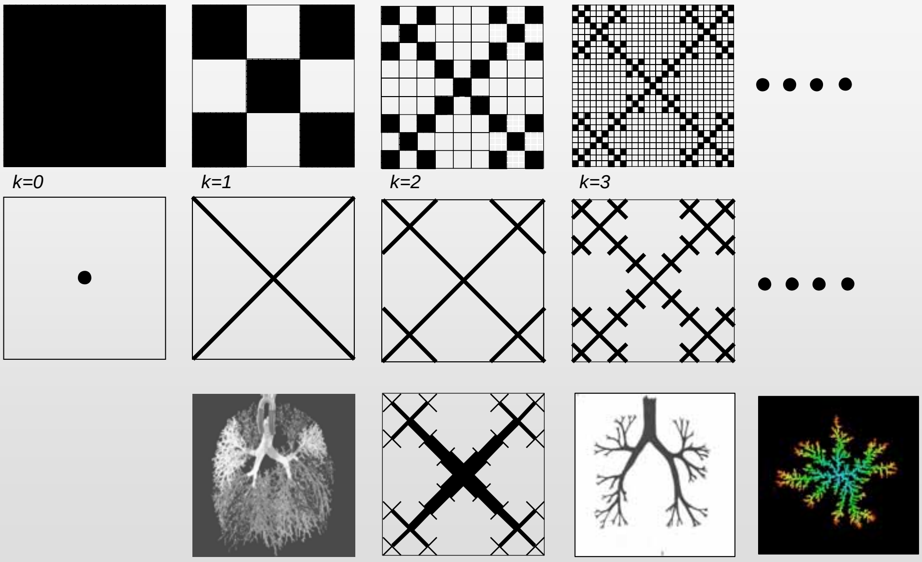
In fact as a slight digression in a famous article, the idea of loosening hierarchy was developed by Christopher Alexander who in his paper in 1966 “A City is Not a Tree” noted that complexity is really composed of overlapping sets. Here are some picture from his paper. I will post it but you can also get it from – http://www.rudi.net/books/200



More on fractal trees: Barnsley’s fern, from his book *Fractals Everywhere* is generated by a rather sophisticated mathematical system of routine and repetitive transformations called the Iterated Function System



And to summarise on hierarchy – all can be related as networks



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Interactions and Networks

Essentially the complexity of spatial systems is contained in their connections – we identified networks as being crucial to such systems at the very beginning of this course.

A good way of impressing this complexity is to see how we can construct networks using simple principles that embody ideas about fractals. We will say a lot more about networks later in this course but first let us introduce a simple model which will generate what looks like a network – more as a sequence of locations that imply how networks span space.

This is the diffusion limited aggregation model that is key to our ideas about fractals.

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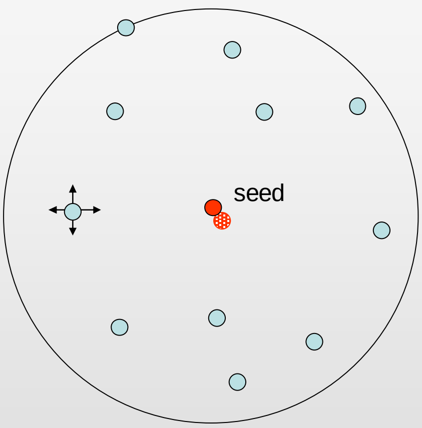
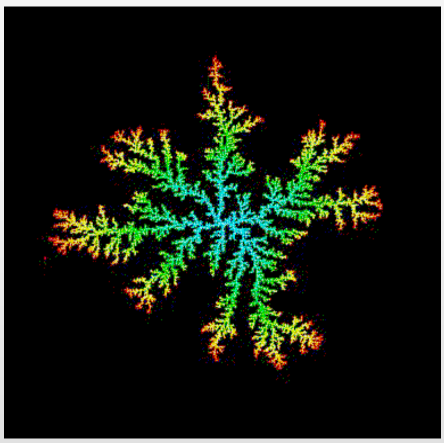
Ok, let me show you the simplest possible model of an organically growing city – based on two simple principles

- *A city is connected in that its units of development are physically adjacent*
- *Each unit of development wants as much space around it as it needs for its function.*

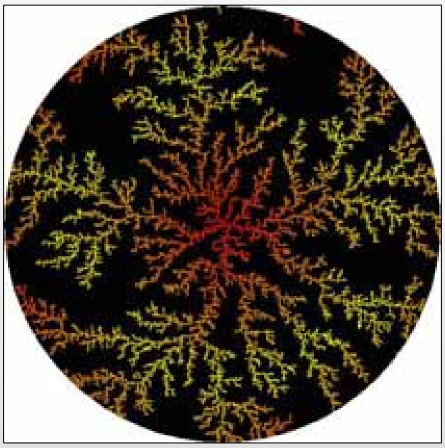
We start with a seed at the centre of a space and simply let actors or agents randomly walk in search of others who have settled. When they find someone, they stick. That is all.

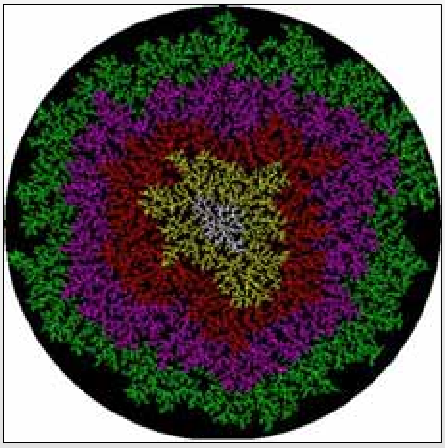
In essence, this is random walk in space which is can be likened to the diffusion of particles around a source but limited to remain within the influence of the source – the city



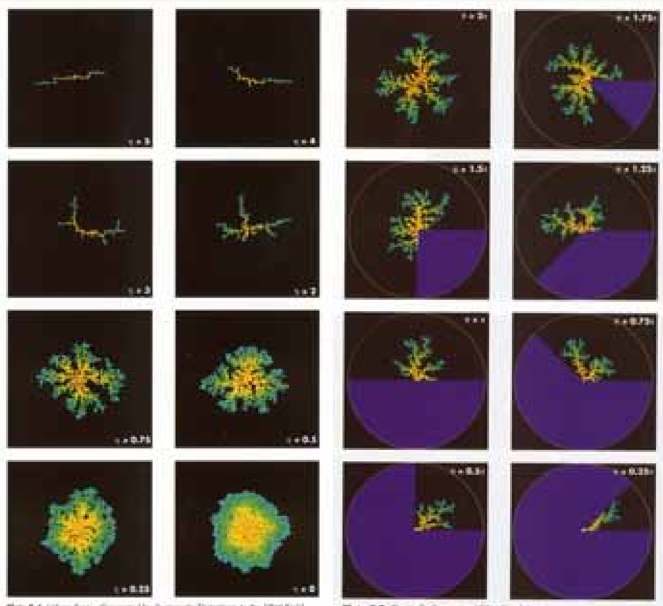


I can get access to my web site and run a little program from http://www.casa.ucl.ac.uk/Demonstrations/DLA.html





I had at this stage better give you access to my 1994 book with Paul Longley *Fractal Cities* that you can download from www.fractalcities.org; also from www.complexcity.info



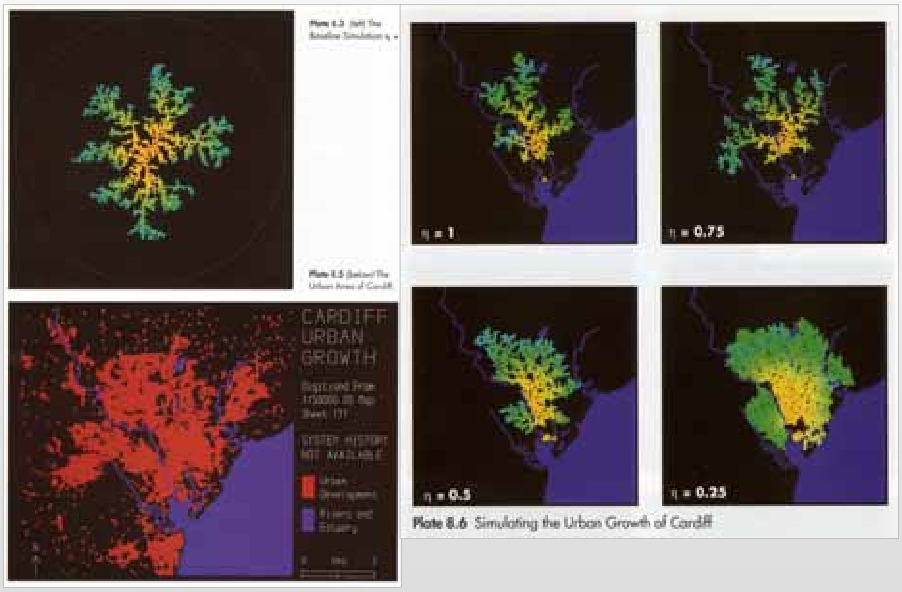


Plate 8.6 Simulating the Urban Growth of Cardiff

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Evolution and Emergence

What we have shown this morning is a kind of emergence from the bottom up – the units that generate fractal patterns are based on simple modules that are repeated as the structures grow and change and these modules tend to be reflected in the subsequent patterns. The Koch curve is an excellent example but look at some of the demos on

http://www.complexcity.info/media/demos/

We will not say any more about this today as evolution and emergence are key themes in defining complex systems and in measuring spatial complexity, and thus a lot more later.

These also imply that the scaling relations that we have identified define the signatures of such complexity.

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Feedback and Nonlinearity: Innovation

In fact all the models and ideas we have developed here imply a degree of positive feedback and this in turn generates nonlinear structures. Again we will say more on this later but to finish it is worth developing a network model of how a complex system is generated – akin if you like to our implicit DLA location model that generates a default network structure.

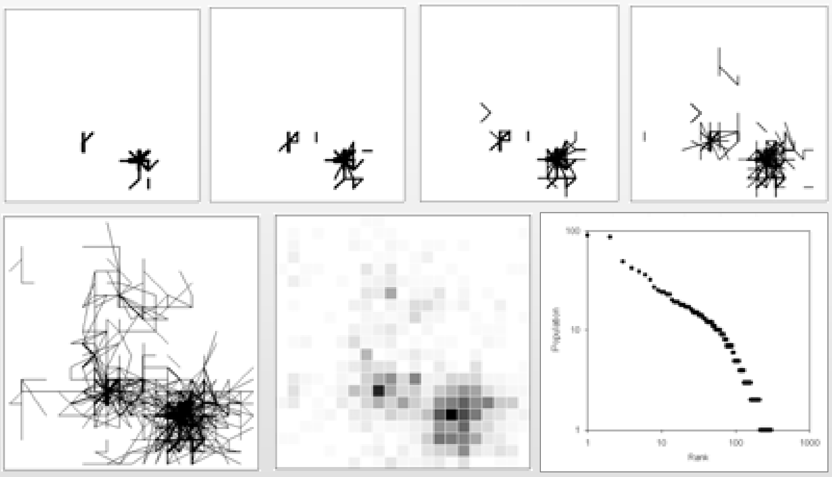
In a later lecture, we will return to these ideas but here it is worth linking these to fractal structures as they imply explicit principles of feedback, cumulative causation and the notion of the rich getting richer that often characterize the way systems sort out the big from the small and the way competition effects dominate the organization of space.

A Simple Network Model: Scale Free Networks

The model that we will introduce is based on the idea that a location or node in a network attracts links to it in proportion to its size – this is the law of proportionate effect – Gibrat's Law that we will introduce in the next lecture – that is a basic law or model rather that leads to scaling

This model is one called by the network scientists, in particular Barabasi, a preferential attachment model and it leads to a scaling of the size of nodes that is essentially equivalent to a power law in terms of their size. In short it is the rank size rule for networks

Now we have not really introduced this rule as yet for this is the next lecture – but we can sketch how the model works easily enough



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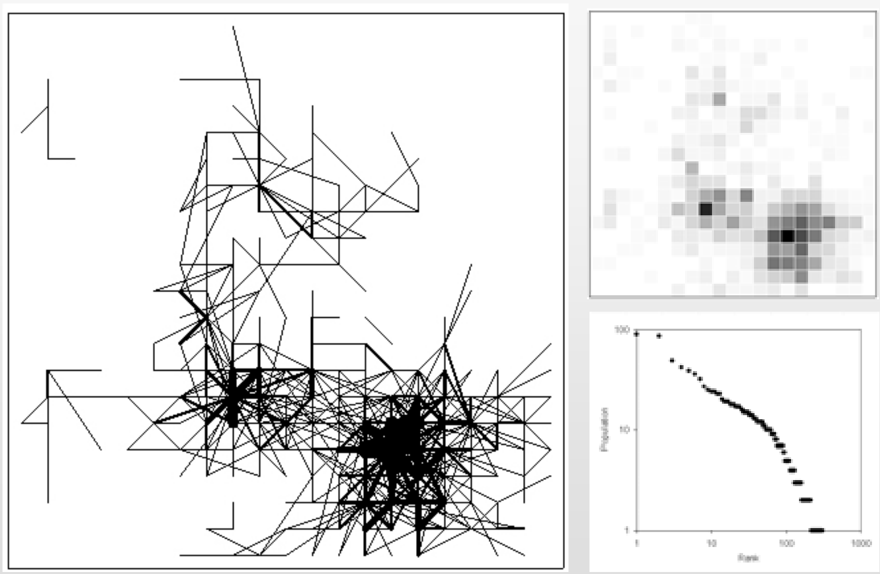

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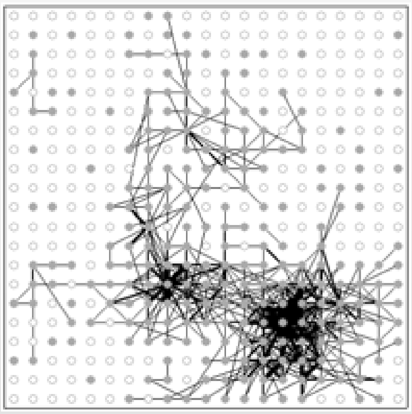

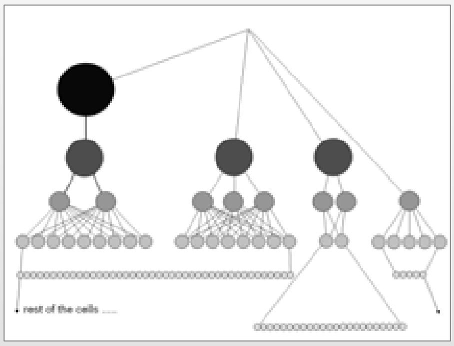

We can extract the hierarchy from this graph where we choose what group any unit goes into by assembling the hierarchy from the bottom up – according to who is linked to who

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The blog will have more and more references as the course continues

Questions

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