School of Geographical Sciences & Urban Planning

Lectures on Spatial Complexity
17th-28th October 2011

Lecture 1:

Defining Complexity

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Outline of Lecture 1

About the Course – the Course Outline; 8 Lectures

What is Complexity? What is Spatial Complexity?

The Systems Approach: Order from the Top Down

The Development of (Temporal) Dynamics

Complex Systems: Order from the Bottom Up

Size and Scale: The Fundamental Scaling Relations

Defining Complexity

About the Course – the Course Outline; 8 Lectures

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Lecture 3: Simple Spatial Growth Models

Lecture 4: Scaling & Size : Rank Size & Urban Dynamics

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Lecture 6: Modeling Spatial Interaction

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Resources: the websites: *www.complexcity.info* and *www.spatialcomplexity.info*

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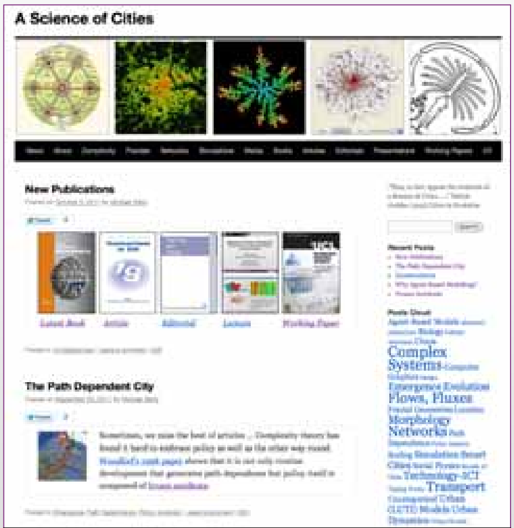

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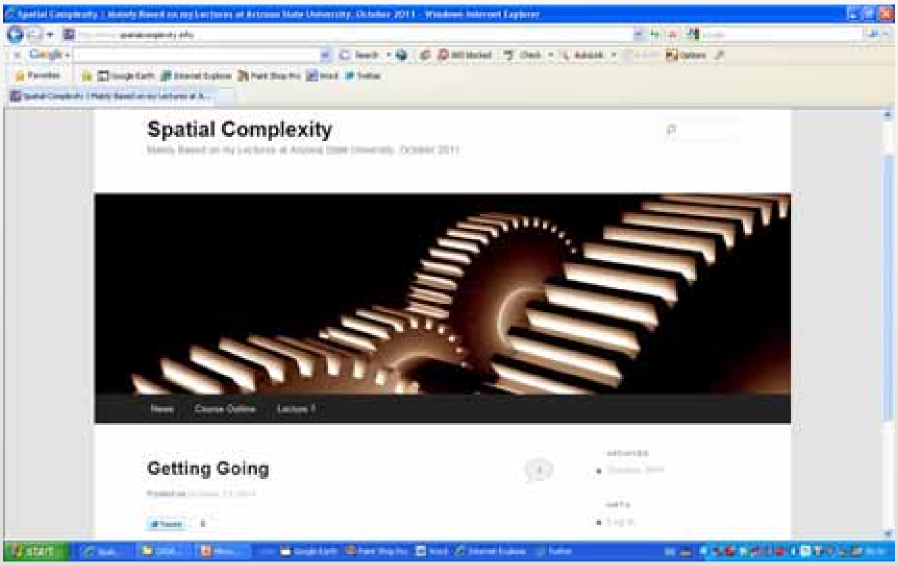

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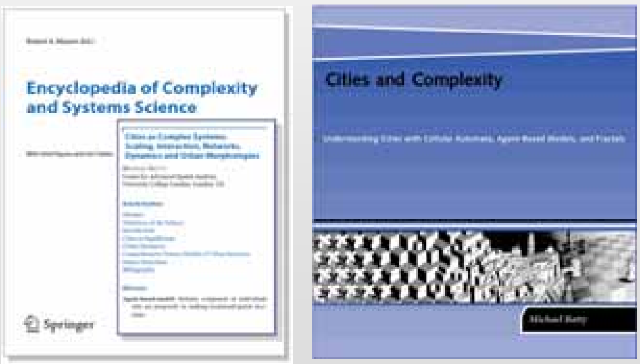
Two key references that I will refer to – one online, one in the library I hope:

My paper: http://www.complexcity.info/flows/

My book:



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What is Complexity? What is Spatial Complexity?

There are at least three defining issues – first there is the notion that no longer in science is there an acceptance that there are watertight theories that lead to firm and definitive predictions in the classical sense.

Classic physical theory was originally defined for closed systems either at a scale where 'we assume' that our actions do not matter – individually –as in the universe, or in a closed controlled context like a lab.

As soon as we move to the real world, this kind of predictability can no longer be accepted and during the last 50, perhaps 100 years, this realisation has weakened classical theory, despite the fact that we still hold these notions to be 'true'or 'accepted' in developing our theories and models.

Once we move to the real world all kinds of external and uncontrollable forces disrupt this classic predictability.

Complexity theory has developed because of this, because the systems we deal with have inherent unpredictability. Key concepts such as emergence, surprise in prediction, and bottom up action being central to these ideas.

Second, the idea that there are explicit interactions between many and multiple elements in the world defines a scale change in how we might deal with these elements. If there are P elements, then simply by defining possible interactions between each of these, there are P^2 connections and these are only for first order effects.

This raises the notion of interactions, networks, connections as being central to complexity

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Third, the notion is that the world is becoming more complex: in simple network terms as P gets bigger, then the number of interactions grows exponentially.

Now we cannot define, by any measure, the possible number of interactions P^2 as a measure of complexity, life is much more subtle than this but this does give a sense of how absolute complexity can be 'approached'.

In fact, it is quite likely that the very nature of these interactions is changing in time, that is that the nature of how P links to P is changing qualitatively in terms of what these interactions are – i.e. through new technologies – and also quantitatively in terms of the powers of P.

As we will see these interaction effects are central to the notion of a complex system, indeed any system.

The Systems Approach: Order from the Top Down

Cities and spatial/geographical systems have been treated as 'systems' for many years – for at least two centuries – and probably longer but their treatment has been implicit.

In fact, cities were largely regarded as being chaotic in the 19th century as the industrial city developed and the reaction that came to be called institutionalized planning – city (and regional) planning here in America – was the notion that order and organization needed to be imposed on the city from the 'top down'

In fact cities even in their 'chaotic state' were still ordered systems in one sense, except that their order was pathological. It was assumed that cities were structured from the top down, as centralized systems.

Onto the agenda came the notion in the 1950s – half a century or more ago – that cities were largely systems in equilibrium. They looked as if they were in terms of their physical form or morphology.

During the early part of the 20th century, this notion of a system as a set of elements or components and interactions between them, existing in a balanced form as an equilibrium, was developed from ideas in biology and to an extent in engineering due to mathematical biologists such as Rashevsky, Lotka, von Bertalanffy and the evolutionists such as Haldane, Huxley, Fisher and so on.

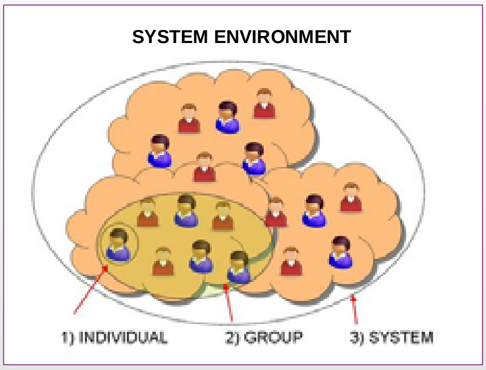
In the war years in the middle of the century, the idea that these systems were akin to mechanical-electrical systems that might be controlled, planned, became significant

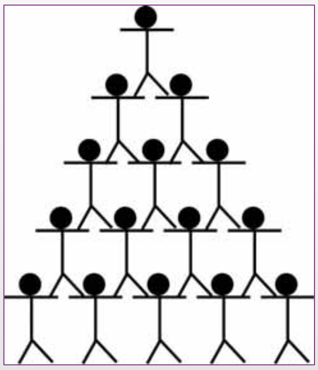
This lead to the idea of cybernetics – coined by Weiner as 'control and communication in the animal and machine' and of general systems theory, applicable to a wider range of systems due to its generic nature.

Applications particularly to management by people like Ackoff. West Churchman, Simon, Stafford Beer and so on were popular and by the 1960s the notion that cities could be treated as systems in the same way came onto the agenda.

Systems could be defined as

Elements with interactions – networks, well defined with respect to their environment, stable dominated by negative feedback that kept them within limits, divisible into distinct subsystems that could be arranged hierarchically. In short, as systems that were configured from the top down.

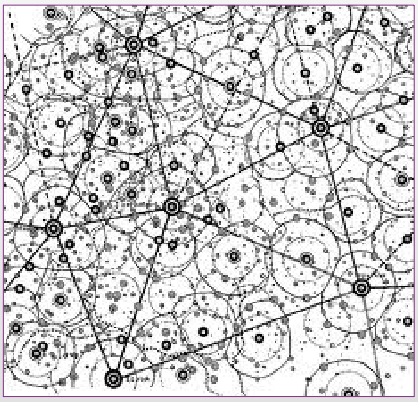


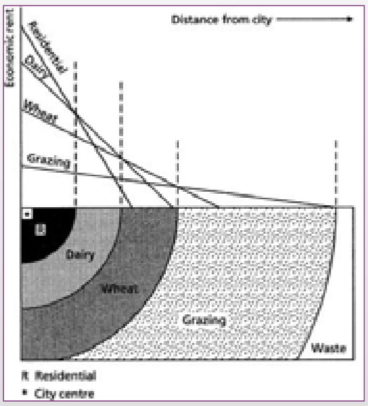


Here we have a typical set of subsystems, with its hierarchy. Around the system is its environment that is regarded as largely passive – to which its interactions can be assumed to be benign ….

A key issue is that this is rarely the case

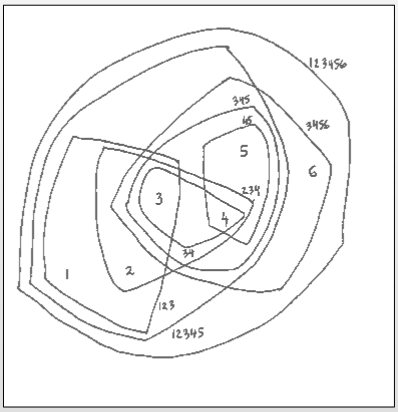
In our own field there are many many examples of hierarchies and systems – e.g. CPT and von Thunen's model

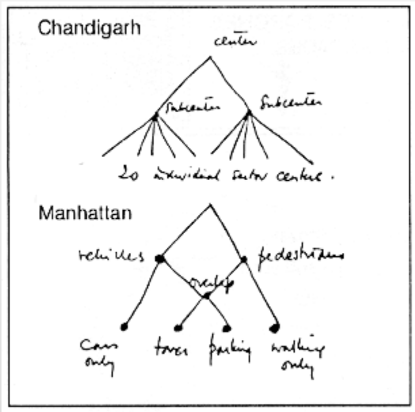




Hierarchy and size sub-systems in the environment

And in urban design, Alexander's notion that cities are sets of overlapping subsystems with the graph of these systems not being a tree but a lattice becoming significant in the 1960s.





Overlapping subsystems

The tree and semi-lattice

The Development of (Temporal) Dynamics

Almost as soon as this model was articulated, it was found wanting. First systems were clearly not in equilibrium, Feedbacks did not restore the equilibrium as in negative damping like a thermostat but feedbacks were often, in fact usually positive, building on interactions.

We have already anticipated this through our elementary comments on complexity as being based on interaction effects that can get more intricate over time, or at least change over time.

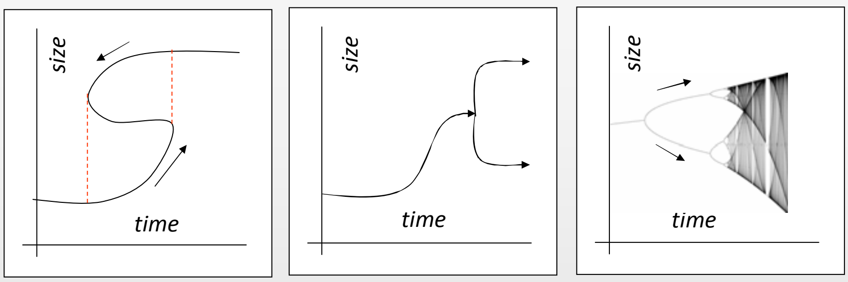
On to the agenda in systems theory came the notion that innovation and technological change rarely part of these models, was central to how systems evolved. In fact evolution became more and more central.

This changed the focus massively to one were the notion of systems developing from the bottom up became more significant but before we explain this, let us look at the sort of dynamics that was explored in the 1970s and 1980s

The idea that when systems changed – when they grew or declined – then this change was not smooth became significant. The notion that growth or change might be discontinuous – with sharp changes in directions – discontinuities became key. This lead to three related developments in the theory of dynamics:

catastrophes, **bifurcations**, and **chaos**

We will deal with these in turn and the easiest was is pictorially as three graphs



Catastrophes Bifurcations Chaos

This are simply examples of how seemingly quite deterministic systems can change through time in unpredictable and/or in discontinuous ways.

We don't have to time to explain all of this here but take it for granted that change in spatial systems is never smooth.

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Complex Systems: Order from the Bottom Up

The idea that systems evolve has been central to biology since Darwin and even before but it wasn't until the 1930s that the idea really took hold.

And only today is the notion being fully worked out and there are those who still find it hard to accept that the magic of nature and even the magic of social organization is based on the logic of multiple, fine scaled mutations which push an organism to increase its fitness for purpose without any overall central plan.

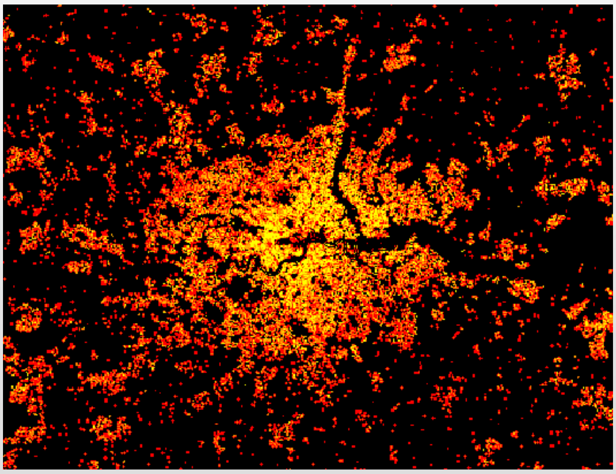
But not only the idea of biological systems but now social and economic systems have firmly come onto the bottom-up agenda. Indeed well before Darwin, Adam Smith implied as much in his comments on how the economy was organized.

In fact this logic is so basic that it is hard to see how any system of any complexity could evolve without it, from the bottom up composed of multiple changes through the generations.

In fact this is the logic that underpins complexity theory. It is manifest in the fact that most systems of the kind we are dealing with are the product of multiple, usually uncoordinated decisions which are implemented by the basic units that form the system. These units rarely communicate on a system-wide basis.

The units in social systems as we implied above are people, groups and collectives but they operate at the level of individuals acting from any larger collective. From such apparently uncoordinated action comes ***emergent patterns*** and order

I can best show you some excellent examples of such emergence which is the hallmark of a complex system in patterns of urban morphology



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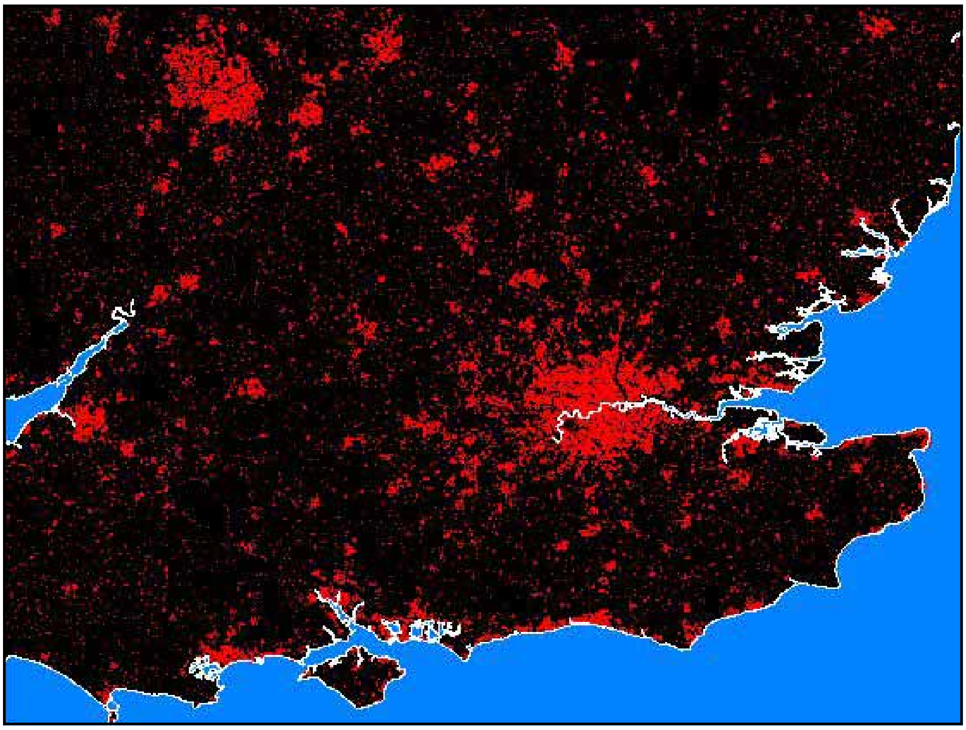

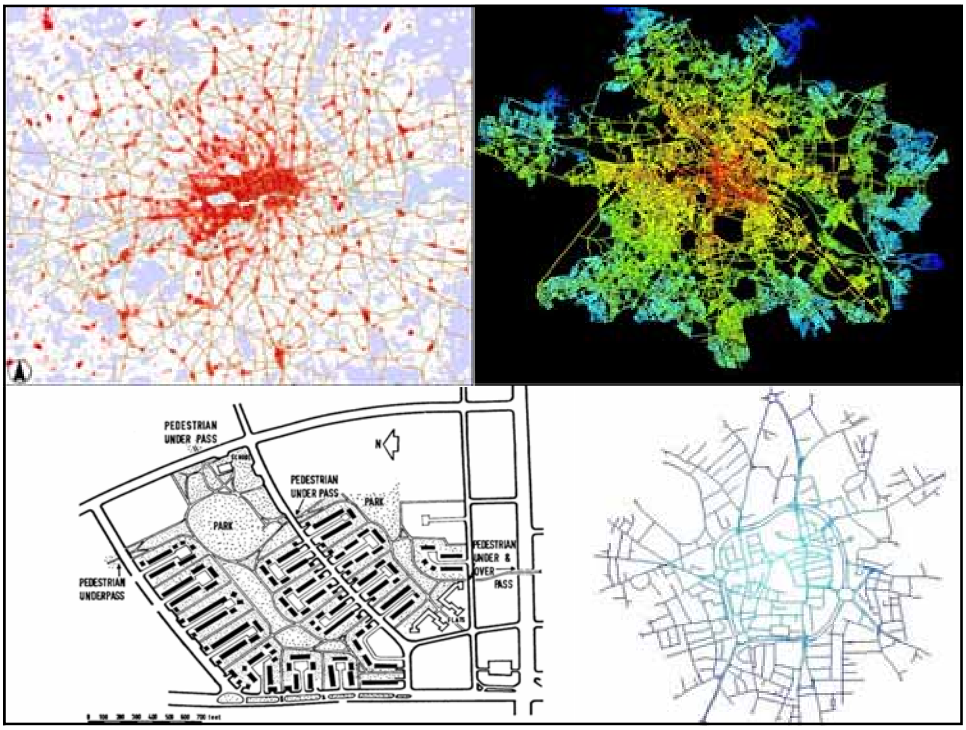

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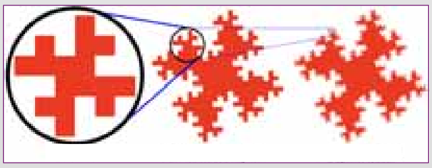
Let me make some key points about such complexity:

1. The patterns emerge as order from the product of multiple non coordinated decisions
2. Some order is established at intermediate levels – planning?
3. The ultimate structure is a product of historical accidents, frozen accidents or path dependence as it is called – history matters as to what final form we end up with
4. The patterns that often emerge are self-similar – they repeat themselves at different scales; they are fractal



We see this kind of scaling every where and we can use it as the ***signature*** of a complex system. To conclude this morning, then let me develop these ideas in terms of how we measure some of these spatial structures.

I will thus develop some fundamentals relating the mathematics of scale and size which will be central to what we develop in the rest of these lectures. These essential are scaling laws which in turn are based on power laws. This will relate to networks, interactions, flows, connectivities and self-similarity

In fact it might be worth looking at my note in *Science* 2008

The Size, Scale, and Shape of Cities, **Science**, **319**, (5864, 8 February), 769 – 771, *Full-text PDF* size: 559 Kb *http://www.complexcity.info/files/2011/06/batty-science-2008.pdf*

Size and Scale: The Fundamental Relations

We can define three related kinds of scaling that we can develop with respect to the generic size of a system that we measure by P, say population.

Now our key to size and scale is that measures of the size of the system all spin off from the number of elemental units P

We assume first that the number of interactions is P^2 but that sometimes were may be less than this number as the scale can be to any power of P. If the power is greater than 1, this is ***positive allometry*** and if less than 1 it is ***negative allometry*** and these are key relations in terms of size. If we are interested in, say, the wealth of a system, then we often find that the relation is something like $W \sim P^{1.1} = P^a$

This is our ***first scaling relation***, that of allometry. In fact our second relation can be derived from this and this is essentially the interaction between two populations. At a point we might say this is P^2 but it is more likely to be something like

$$I = K\frac{PP}{d} = K\frac{P^a}{d}$$

Where I is the interaction and d is some measure of friction due to the fact that everyone cannot interact as if they are standing on the head of a pin – this is a distance effect. K is often some scaling factor – the gravitational constant in gravity models but it might be a ratio that scales the interaction to reasonable limits, like Dunbar's Number – the average number of acquaintances, about 250.

Once we have the interaction in this form, then consider two different places *i* and *j* with a distance d_{ij} between them and we get our ***second scaling relation*** – the conventional spatial interaction model

$$I_{ij} = K \frac{P_i P_j}{d_{ij}^{\alpha}}$$

Note that α is a power that often is between 1 and 2

Our third relation is rather different and relates to the frequency of sizes. If we take a large number of cities P_i say of different sizes and we measure their frequency f_i, we can array these as

$$f_i \sim \frac{1}{P_i^{\beta}} = P_i^{-\beta}$$

This can be manipulated into our ***third scaling relation,*** the rank size rule which is key to the size distribution of objects in a spatial system. Look at Christaller's central place diagram earlier in this lecture to see that the frequency distribution of sizes accords to this – more or less.

All these relations are power laws and they all display the characteristic of self-similarity. This means that when we scale the relation we find the same functional form – that is if we multiply population $W \sim P^{1.1} = P^{a}$ by a scaling factor Z then we get the same relation; that is

$$W^{'} = (ZP)^{1.1} = Z^{1.1}P^{1.1} = Z^{1.1}W$$

Ok to finish let me simply state a way of measuring information and complexity that we will take forward later in this course

Defining Complexity

The entropy where

$$p_i = \frac{P_i}{\sum_i P_i}$$

$$H = -\sum_i p_i \log p_i$$

Let me leave you with this and think about it – we won't get to use it till much later but it is one important way of measure spatial complexity

Look at the blog – it will be up and running by Wednesday with this lecture possibly augmented

Questions

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